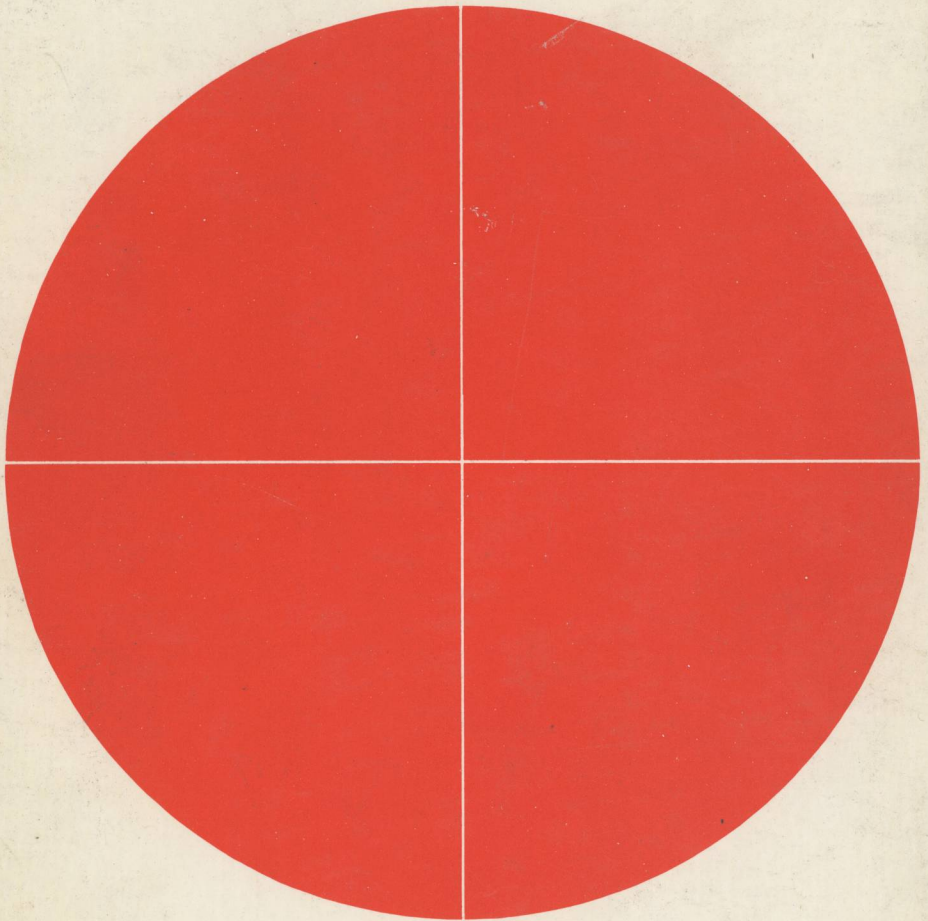


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Finite Mathematics



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FINITE MATHEMATICS

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FINITE MATHEMATICS

PREFACE

Finite Mathematics represents one of the newest and fastest growing areas of mathematics applied to practical problems. The applications for the topics of *Finite Mathematics* are for the most part in business, economics, and the social sciences. This text is designed to show students from these subject areas the underlying mathematical concepts for such problem solving.

The topics are designed for students who have taken two years of high school algebra or the equivalent. There is sufficient material for a one-semester or two-quarter course. In addition, a number of special topics are included in the supplementary sections, including a computer programming language, BASIC, and a review of systems of equations. These topics are designed to extend the students' knowledge in special-interest areas or to strengthen weak points in the students' backgrounds.

Special care has been taken to make the material understandable to students with limited backgrounds in mathematics. Concepts are clearly presented and are coupled with numerous examples (over 300) chosen primarily from business and the social sciences. Each section is also supplied with a large set of graded exercises, including those especially chosen for students in business, economics, or the social sciences, and each chapter includes a set of summary problems to review the topics studied in the chapter. The vocabulary throughout the text was selected to be appropriate for such students.

The text developments present a balance between pure manipulation, theory, and applications. The format for any required proofs is informal but mathematically correct and consistent with the students' mathematical maturity. Whenever possible, developments point out how one topic can be combined with others to form new and powerful mathematical tools for problem solving.

The answers to the odd-numbered exercises and all problems in the chapter reviews are included in the text; the remaining answers are in a teacher's manual.

D. C., S. S., and B. W.

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CHAPTER 1

LOGIC

1.1 INTRODUCTION

Since this text starts with a discussion of logic, it is natural to ask two questions: What is logic? What does it have to do with finite mathematics?

To answer the first question, a formal definition of logic might be:

Logic is the science of reasoning.

From this definition it is clear that a complete study of logic could be, and often is, a lifetime project. In this text we will consider one model of reasoning. It will be a *finite* model in the sense that only a finite number of cases will be allowed in analyzing reasoning patterns. It will be a *symbolic* model—that is, we will attempt to abstract patterns of valid or sound reasoning into symbolic patterns, much as a course in the algebra of real numbers abstracts the patterns of everyday arithmetic. When mathematicians first began to look at these abstract patterns, they did so because they needed logic as a tool in analyzing mathematical structures. Once they did this, they discovered that the patterns they found were exact models of other mathematical systems which on the surface had not appeared to be related to logic at all. Hence in this text we will view logic both as a tool with which to study mathematics and as a mathematical structure itself.

We have already answered the question of how logic relates to finite mathematics. It is a branch of finite mathematics. In general, finite mathematics deals with those parts of mathematics in which a finite number of

cases or only finite values of a finite number of variables are involved. This contrasts with those topics examined in calculus and calculus-related studies, which tend to deal with the infinite.

The purpose of this chapter is to demonstrate how “valid conclusions” may be reached from given information, in the hope that the technique can be extended to apply to actual situations. In order to make this transfer easier, symbols will be used to replace actual statements in much the same manner that x ’s and y ’s are used in algebra. Therefore, the name *symbolic logic* is used to describe this study.

1.2 STATEMENTS OF LOGIC

Just as the basic structure of arithmetic consists of numbers, so the basic structure of logic consists of statements.

DEFINITION

A **statement in logic** is a sentence that can be assigned a single *truth value*.

The particular logic we are going to study is a *two-valued logic*; that is to say, there are two truth values: true or false. Thus a statement is a sentence that can be assigned the truth value *true* or the truth value *false* but not both at the same time.

In English grammar this type of sentence is called a *declarative sentence*. However, not all declarative sentences qualify as statements in logic.

Because the use of language to express statements often leads to semantic ambiguities, it is sometimes difficult to assign a single truth value to a statement, thus making it difficult to classify it as a logical statement. Consider, for example, statements such as “Zsa Zsa is a beautiful woman”; “Frank is a good man”; “Skiing is fun.” It is not possible to arrive at a clear-cut true or false decision on the basis of subjective interpretations of “beautiful,” “good,” or similar value judgments.

EXAMPLE Which of the following sentences are logical statements?

- a. Today is Thursday.
- b. I have \$200 in my bank account.
- c. $2 + 2 = 5$.
- d. Friendly Mining is a good stock.
- e. Don’t buy glamour stocks.

Solution

- a. This is a logical statement, since it can easily be determined if the statement is true or false. It must be one or the other.
- b. Again, the truth or falsity of the statement can be verified, and (b) is a statement.
- c. Yes. This is a good example of a false statement since it is known that $2 + 2 = 4$ and not 5.
- d. Now the question arises: What is a good stock? There is no clear-cut decision as to true or false for this sentence unless some criteria for judging good or bad stocks have previously been defined. Thus (d) is *not* a statement in logic.
- e. No. This sentence is not declarative and does not qualify as a statement.

Therefore, the logical statements are (a), (b), and (c).

The statements illustrated in the above example are **simple statements**. Often a statement contains more than one statement as component parts. For example, "Today is Thursday and I have \$200 in my checking account" is a statement consisting of statements (a) and (b) in the above example. This type of statement is called a **compound statement**. Examples of compound statements are:

- a. If profits rise, then I will hire more employees.
- b. The tenant will pay higher rent or he will move out.
- c. The buyer selects line A and rejects line B.

The simple statements which are the components of (a) are

Profits rise.
I will hire more employees.

For (b) the component statements are

The tenant will pay a higher rent.
He (the tenant) will move out.

And for (c) the statements are

The buyer selects line A.
(The buyer) rejects line B.

If a compound statement is a logical statement, it also has to be either true or false but not both at the same time. The truth value of a compound statement will be studied in light of the truth values of its component statements.

Often a complicated statement can be analyzed easily if a symbol is used to replace the statement. Lowercase letters $p, q, r, s, \dots$ are often the replacement choices. For instance, let p stand for the statement "Profits rise" and q stand for "I will hire more employees." Now the statement "If profits rise, then I will hire more employees" can be written "If p then q ."

EXAMPLE Let p be the statement "This course is a degree requirement." Let q be the statement "John must pass this course." Translate each of the following symbolic statements:

- a. If p then q
- b. If q then p
- c. p or q
- d. p and q
- e. q only if p

Solution

- a. If this course is a degree requirement, then John must pass this course.
- b. If John must pass this course, then this course is a degree requirement.
- c. This course is a degree requirement or John must pass this course.
- d. This course is a degree requirement and John must pass this course.
- e. John must pass this course only if it is a degree requirement.

It should be evident that the words, *and*, *or*, and *if...then* were used to make compound statements from simple statements. These are not the only words which serve this purpose, but they are very important and will be carefully studied in subsequent sections of this chapter.

EXERCISES 1.2

Determine which of Exercises 1–10 are statements of logic.

1. The stock market crashed on Black Monday.
2. A fixed income is a disadvantage in times of inflation.
3. Real estate is a good business.
4. Either buy the coat or leave the store.
5. Professor Jones teaches business law and has a private law practice.
6. If you fly to Denver, you can attend the meeting.
7. The Los Angeles Rams won the football game and the San Francisco 49ers lost.
8. It is very pleasant here.

9. I'll fire you if you disagree with me.
10. If it rains on Tuesday and if the meeting is canceled, I will call you and we can go to lunch.
11. List the compound statements in Exercises 1–10.

Let p , q , and r be the following statements:

- p : Government spending is increasing.
 q : Taxes are higher than ever.
 r : There is a freeze on wage increases.

Translate each of the symbolic statements in Exercises 12–18.

12. p and q
13. If p then q
14. q and r
15. If p then r
16. r if q
17. If q and r then p
18. p only if r
19. Assume that p is the *true* statement “Jane is ill” and q is the *true* statement “Jane goes to work.” What do you think the truth value of each of the following compound statements is? (An *actual* determination of the truth value of compound statements according to the rules of logic will be discussed in subsequent sections of this chapter.)
 - a. p and q
 - b. p or q
 - c. If p then q
 - d. If q then p
20. Assume that p : “Jane is ill” is a *true* statement and q : “Jane goes to work” is a *false* statement. What do you think the truth value of each of the following compound statements ought to be?
 - a. p and q
 - b. p or q
 - c. If p then q
 - d. If q then p

1.3 MODIFIERS, CONNECTIVES, AND TRUTH TABLES

Since a statement must be either true or false, the modification of a statement by the word *not* or the words *it is not the case that* changes the truth value of the statement.

Thus “Today is Thursday” must be either true or false. If it is true, then “Today is *not* Thursday” is false, and if it is false, then “Today is *not* Thursday” is true.

Symbolically, if p represents a statement, then *not* p is written $\sim p$, where $\sim$ is called a **modifier** of the statement p .

Truth tables are useful devices for testing truth values of statements. Use T to mean true and F to mean false.

p	$\sim p$
T	F
F	T

The table illustrates that if p is true, then not p ($\sim p$) is false, and if p is false, $\sim p$ is true.

Words that make a compound statement from simple statements are called **connectives**. From the discussion in the preceding section, the words *and*, *or*, and *if . . . then* serve such a purpose and are connectives.

The symbol $\wedge$ is used to mean *and*. If p and q are statements, then p and q is written $p \wedge q$.

The symbol $\vee$ is used to mean *or*. Thus p or q is written $p \vee q$.

EXAMPLE Consider statements p : "This car has a ninety-day warranty" and q : "Financing is available." Write a symbolic statement for each of the following:

- This car has a ninety-day warranty, and financing is available.
- Financing is available or this car has a ninety-day warranty.
- This car does not have a ninety-day warranty and financing is not available.
- It is not the case that this car has a ninety-day warranty or that financing is available.

Solution

- $p \wedge q$
- $q \vee p$
- $\sim p \wedge \sim q$
- $\sim(p \vee q)$ (The words *it is not the case that* usually modify the entire compound statement; therefore, the parentheses are used.)

Since a compound statement is also a logical statement, it must have a truth value, and this truth value in turn depends on the truth values of the component statements and on the definition of the connectives which form the compound statement.

In order to examine the truth value of a compound statement, we must consider all the logically possible arrangements of true and false of the component statements. For example, consider a compound statement with components p and q . The logical possibilities are: both p and q are true; both p and q are false; p is true and q is false; and p is false and q is true. Thus there are four possible entries in a truth table, as shown below.

p	q
T	T
T	F
F	T
F	F

When three or more different statements are used to form a compound statement, more logical possibilities exist; this situation will be discussed in a subsequent section of this chapter.

We are now ready to consider the definitions of some compound statements. Why the statements are defined in the way they are is not easy to justify. Decisions for rules of games are made in such a way as to make the game interesting and to make the rules workable and reasonable, but two rules should not be in conflict with each other.

The first connective to be defined is *and*, $\wedge$. Consider the statement "Sunday the football team plays in Oakland and we will attend the game." If the game is played in Oakland and we attend, then both components are true, so let us consider the compound statement to be true. Thus if p is the statement "Sunday the football team plays in Oakland" and q is the statement "We will attend the game," we can symbolize the statement

$$\begin{array}{c} p \wedge q \\ \text{T T T} \end{array}$$

What happens if the team is playing in Oakland but we decide not to attend? Or worse yet, if we decide to attend but the team is playing elsewhere?

$$\begin{array}{c} p \wedge q \\ \text{T ? F} \\ \text{F ? T} \end{array}$$

A reasonable definition would assign the value *false* to such a combination and would certainly assign *false* to the event that neither p nor q is true:

$$\begin{array}{c} p \wedge q \\ \text{F F F} \end{array}$$

The above reasoning should make the following definition acceptable: We define $p \wedge q$ to be true only when p and q are *both* true. The truth table appears below.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

The connective $\wedge$ is also called a **conjunction**.

EXAMPLE Let p be the *true* statement “This car has a ninety-day warranty” and q be the *false* statement “Financing is available.” What is the truth value of the conjunction of p and q —that is, $p \wedge q$?

Solution

p	q	$p \wedge q$
T	F	F

Since $p \wedge q$ is true only when p and q are *both* true, $p \wedge q$ is false.

EXAMPLE Make a truth table for the statement $\sim p \wedge q$.

Solution

(1) p	(2) $\sim p$	(3) q	(4) $\sim p \wedge q$
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	F

Column (2) has the opposite truth value of column (1) since it lists $\sim p$. Then columns (2) and (3) are considered for the conjunction in column (4).

The second connective to be defined is *or*, $\vee$. This connective is also called a **disjunction**. The truth table defining the disjunction appears below.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

We define $p \vee q$ to be true as long as at least *one* of the component statements is true.

Does this definition of *or* seem to make sense? All an *or* statement really promises is that at least one of the components is true. For example, “I must deposit money in my account, or my check will bounce”; “In the presidential election of 1948, either Tom Dewey was elected or Harry Truman was elected”; “On Saturday evening I will go to a show, or I will watch the movie on television.”

In the election example it would not be possible for both statements to be true simultaneously, whereas in the other two examples both statements could be true. Maybe I will go to an early show on Saturday and then come home and watch the TV movie.

A situation in which both components of a disjunction may be true is considered to be an **inclusive disjunction**. Thus the truth table for $\vee$ is the truth table for the inclusive disjunction. If it is not possible for both components to be true at the same time, the disjunction is said to be **exclusive**. The development of a truth table for the exclusive disjunction is left as an exercise.

EXAMPLE Let p be the *false* statement “The labor dispute has been settled” and q be the *true* statement “Everyone returns to work on Monday.” Determine the truth value of each of the following:

- a. $p \vee q$ b. $\sim p \vee q$
c. $p \vee \sim q$ d. $p \wedge q$

Solution

a.

p	q	$p \vee q$
F	T	T

b.

p	$\sim p$	q	$\sim p \vee q$
F	T	T	T

c.

p	q	$\sim q$	$p \vee \sim q$
F	T	F	F

d.

p	q	$p \wedge q$
F	T	F

EXAMPLE If p and q represent the same statements as in the above example but with undetermined truth values, make a truth table for the statement “It is not the case that the labor dispute has been settled or that everyone does *not* return to work on Monday.”

Solution First, symbolize the statement: $\sim(p \vee \sim q)$. Next, list the possible cases: