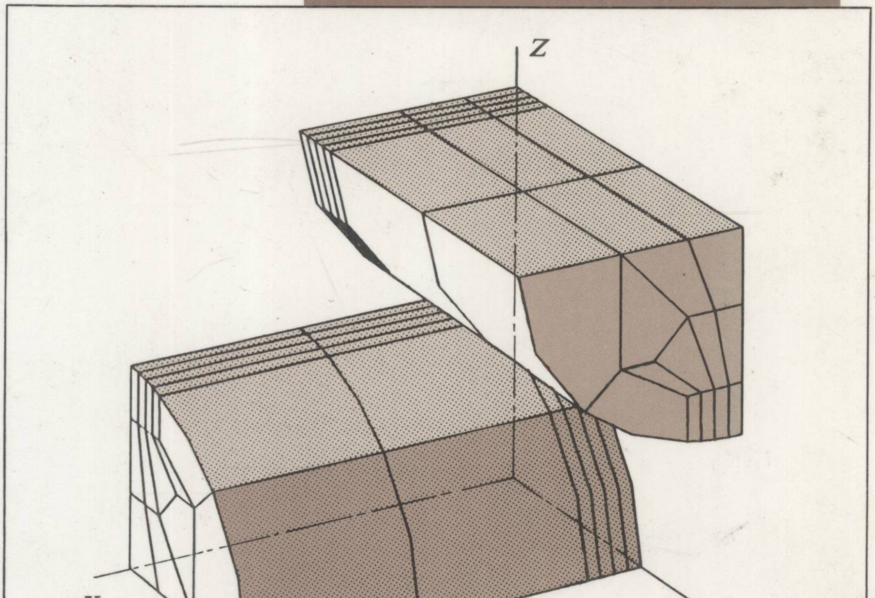
Computational Methods in Contact Mechanics

Editors: M.H. Aliabadi
and C.A. Brebbia



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Computational Methods in Contact Mechanics

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PREFACE

Modern engineering design leads to the realization of the importance of contact problems in many technological fields. Contact problems are complex and inherently non-linear due to their moving boundaries and the existence of friction along contact surfaces. Until a few years ago, researchers were engaged only in the fundamental concepts of contact problems. Today, due to the great improvement in computer technology and computational methods, it is possible to solve many complex practical contact problems accurately and efficiently.

This book is the first which presents a comprehensive review of the contact mechanics with particular emphasis on computational methods. Much attention is devoted to the physical interpretation of the contact properties as well as the numerical methodologies necessary to solve complex engineering problems. As such, the book covers formulations based on load incremental and mathematical programming approaches using both finite and boundary element methods. The mathematical modelling techniques described include the constraint method, the flexibility approach, the penalty method and the Lagrange multiplier technique.

In Chapter One, the applications of boundary element method to frictional contact problems are presented using a fully load incremental technique together with a constraint approach. Its application to fracture mechanics is also described. Chapter Two deals with the boundary element flexibility formulation for the analysis of frictionless and frictional contact problems. The formulation is described in detail and several examples are presented to demonstrate the accuracy of the method.

In Chapter Three, the Lagrange multiplier formulation for the finite element method is presented. General algorithms are described for the detection of overlapping meshes and the evaluation of contact forces. An example problem of contact stress analysis in an orthopaedic knee is presented.

Chapters Four and Five concentrate on the application of the penalty method in contact problems using the finite and boundary element methods respectively. Particular attention is paid to the numerical implementation of the method and several examples are presented to demonstrate the accuracy of the methods.

In Chapter Six, the application of the boundary element method to three-dimensional contact problem is described. The authors present a detailed formulation of the problem before proceeding to solve some classical problems.

The remaining three chapters of the book deal with mathematical programming approaches. Chapter Seven presents a formulation to deal with both small and large displacement contact problems. The chapter provides a review of the mathematical programming methods as well as giving recommendations for future development of the method. In Chapter Eight, a general solution method for three-dimensional quasistatic frictional contact problems is presented. The formulation is based on the boundary element method and mathematical programming approaches.

Finally, in Chapter Nine, the application of the finite element method to elastic-plastic contact problems is presented employing a parametric quadratic programming approach. The authors describe the numerical implementation of the method in detail and present several examples to demonstrate the accuracy of their technique.

The Editors would like to express their appreciation to all the authors for their excellent contributions to this book. Special thanks are also due to Miss Melanie Assinder for her careful work on the preparation of the final manuscript.

Editors

CONTENTS

Chapter 1

Analysis of contact friction using the boundary element method

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Abstract

In this chapter, a boundary element formulation for solving structural problems associated with frictional contact is presented; it uses an efficient, iterative and fully load-incremental technique. Problems with any number of two-dimensional bodies in contact can be analyzed using this technique; the bodies may be conforming or non-conforming, of similar or dissimilar materials. The interface may be frictionless or frictional and may undergo slip or partial slip. Numerical solutions of both normal and tangential traction distributions can be obtained automatically for successive load increments. The technique utilizes automatic updating procedures in order to model the continuously changing boundary conditions occurring inside the contact region.

The problem of ensuring that stress intensity factor solutions for cracked bodies are accurately calculated continues to be a major consideration in design, particularly in the presence of fretting forces. In this chapter, the technique for solving problems in cracked structures is presented for configurations which require a nonlinear analysis of the contact conditions.

Stress intensity factors are evaluated at the end of each load step using the J-integral method. Results obtained show that the presence of friction significantly influences the stress intensity factors.

In order to speed up the numerical solution process, the systems of equations from the analysis are assembled in a matrix which has a particular structured form; this results in an efficient technique for solving these equations. Implementation of the algorithms is validated by solving several contact problems and the results are compared with analytical solutions. Results obtained show that an accurate account of the nonlinear behaviour, caused by the frictional effects, can be obtained only by following the loading history as well as the contact

history.

1 Introduction

In general, when solid bodies are brought together they will either be in contact at a point, along a line, or over a surface. In complex loading and geometrical configurations, a combination of the above mentioned contacts is possible. Upon application of a load on these bodies in contact, the initial contact region will change. The extent of the changes in the contact regions will depend on the magnitude of the external load, the elastic properties of the bodies and the frictional behaviour at the contact interfaces. Such dependence makes the contact problem highly nonlinear and it is only possible to analyse these problems using advanced numerical methods.

The finite element method has been used extensively to solve contact problems. One of the early applications was by Wilson & Parson[1] who used an indirect approach in which contact conditions were treated as subsidiary equations. Extension of this method to include friction was presented by Chan & Tuba.[2,3] A formulation based on flexibility matrices for contact problems was introduced by Francavilla & Zienkiewicz.[4] However, it required prior knowledge of the type of contact. More recent applications of the finite element method to contact problems can be divided into three major approaches: the Lagrange Multiplier method;[5–10] the Penalty method;[11–13] and the Mixed (or Hybrid) method.[14–15]

It may be argued from the numerical point of view that as the contact is on a boundary, a boundary-type rather than a domain-type, solution procedure such as the boundary element method (BEM) would be more suitable to the analyses of these types of problems. Furthermore, in a boundary element analysis, both the displacements and the tractions are obtained directly as part of the solution. The first application of BEM to contact problems can be traced back to Andersson.[16] Later Karami[17] and Paris[18] used both continuous and discontinuous quadratic elements to solve contact problems. Other formulations of BEM to contact problems include the penalty function approach by Yagawa, Hirayama & Ando,[19] the flexibility approach by Takahashi[20] and mathematical programming by Lee.[21] In this chapter, a boundary element method is used for the multi-body contact modelling. A more direct approach to the problem is adopted where the solution is obtained directly and explicitly from contact considerations and global equilibrium of the problem alone. This technique, based on iterative and fully incremental loading procedures, can deal with both conforming or non-conforming types of contact; and an automatic updating procedure is employed in order to model the continuously changing boundary conditions occurring inside the contact region.

For practical reasons, current research has been focused on the problems of fretting fatigue, because small crack-like flaws frequently exist in mechanical components from manufacture, or result early in the service life where surfaces are in rubbing contact (fretting). It is necessary to predict how these cracks will grow in order to ensure that the residual strength of the structure will not fall below an acceptable level over the required service life. A common location for such crack-like flaws is in fastener holes and pin-loaded lugs. A pin-loaded lug type of connection is inherently prone to fatigue because of the combination of stress concentration at the hole and fretting caused by the relative movement between the pin and the hole bore during load cycling. The combination of these effects leads to an increase of crack growth rate in these structural components. To predict crack-growth behaviour it is necessary to calculate the stress intensity factor. This requires a numerical procedure that can take into account the fact that the contact region between the pin and lug changes in a non-linear way as the crack grows.

In this chapter, the basis of the contact mechanics and the concept of numerical modelling of contact problems are briefly described in Section 1. The basic definitions of contact modelling and numerical technique are mentioned in Section 2. For completeness, a brief formulation of the boundary integral equation and its applications is presented in Section 3; a high speed matrix solver which is required to speed up the numerical solution process is also described in Section 3. In this chapter, problems encountered in the modelling of contact problems are described, and the techniques developed to overcome these problems are presented in Section 4. An efficient and automatic, iterative and fully incremental loading procedure for both non-frictional and frictional problems is described in Section 5. In order to solve crack problems in the presence of frictional contact the J-integral method is employed here for the determination of stress intensity factors and the formulation is described in Section 6. The computer program was rigorously tested with various contact problems which may be of similar or dissimilar materials, load dependent or load independent, frictional or non-fictional and also contact problems with cracks. In Section 7, a total of six numerical examples are presented in the order of increasing complexity, including one which contains a crack. Finally discussion and conclusions are presented in Section 8.

1.1 Contact mechanics

Contact mechanics is a study of load transfer in mechanical assemblages which are of great importance in mechanical engineering. The nature of contact interaction between two contacting surfaces is complicated. The science of contact is of great practical importance, but is not well understood because of its complexity and the obscure nature of the problem. In practice, knowledge of contact can only be learnt through experiences and observations. But generally, direct observation or measurement is impossible because the area of interest is hidden under the contacting surfaces. As such, only an average effect or an overall behaviour can be obtained experimentally. The measured parameter is known as the 'coefficient of friction', from which we can deduce an overall view as to whether the two surfaces will slide or adhere to each other under given conditions.

The difficulties of the problem are further compounded by the fact that contact behaviour is sensitive to the materials of the two contacting surfaces, their texture, surface finish, local topology at the contact surfaces, rate of loading, magnitude of loading, direction of loading in relation to contact region and supports of the bodies etc.

In modern structures, many safety aspects of engineering design rely upon the frictional force. Although in many cases the frictional effect is not critical, a lack of understanding of such effects can lead to inefficient design or could endanger lives. For these reasons, the phenomenon of friction is a subject of intense experimental and theoretical research and is becoming more important in modern engineering design.

The physical mechanisms associated with friction can be referred to as the resistance to slide between two surfaces. The existence of sliding resistance is largely due to the roughness of the surfaces and the microstructure of the materials involved. Friction has a great effect on the interaction between normal and tangential tractions in a contact situation; thus if a reasonably accurate solution is to be obtained, frictional behaviour must be taken into account. For simplicity, Coulomb's friction law is usually used in most engineering calculations. The basic assumption in Coulomb's friction law is that the coefficient of static friction μ_s is assumed to be the same as the coefficient of sliding friction μ_k. The Coulomb relationship between the tangential traction and the normal pressure in sliding contact is defined as, $\mu = t_t/t_n$, where t_t and t_n are the tangential and normal traction respectively, and μ is equal to the sliding coefficient of friction, μ_k. Throughout this research, Coulomb's friction law is used (i.e. the coefficient of friction μ is constant for the entire slip zone).

1.2 Numerical modelling concept

Physically, the contact area increases in size gradually and continuously as two bodies in contact are pressed together under the influence of an external load. Numerically the process of deformation and the creation of the contact area have to be simulated discretely in the following way. Consider a node pair a, b initially in separation with a gap g_0 between them. As the load increases, nodes a and b will separately undergo deformation and at some particular load they will come into contact as the gap g_0 reduces to zero. If the external load continues to increase after nodes a, b have made contact, then a, b will be enveloped by the contact area. However, if the external load ceases to increase after node a, b have just made contact then node a, b will simply become the edge of the contact area. Numerically this is the most desirable state to be reached since contact boundaries are discretised into nodal points; therefore an edge of contact can only exist at a pair of nodes. An important criterion for an edge node-pair is to be just barely in contact with an external load which is just large enough to hold them in place. The second criterion which an edge node-pair must fulfill is that traction continuity in the normal and tangential directions must be maintained without violating any of the contact modes. It was found that such criteria can be fulfilled only if the slip contact mode is prescribed for the edge node-pair, since in this mode both normal and tangential tractions are zero. Historically, a slipping contact state at the edge of contact was first recognized by Hertz[22] during his experimental work in 1880; later the same conclusion was also reached theoretically by Galin[23] and Spence.[24]

Many contact problems are of the non-conforming type, that is the bodies have different initial profiles in the potential contact region. Non-conforming type problems are load dependent since the actual extent of the contact area is not known *a priori*. Generally, for problems in that category, a method has to be employed that can evaluate the load increment corresponding to each increase or decrease in the area of contact; but, within each newly developed contact area, the contact status may not now satisfy the equilibrium state. In order to satisfy both the load step and the contact state simultaneously, an iterative procedure is always required to overcome this nonlinear aspect of the problem. These problems are particularly difficult to solve when the presence of friction has extensively affected the relative movement between the contact surfaces in the tangential direction.

However, in the absence of friction the contact problem can be solved iteratively without the need for small incremental load steps. The iterative process consists of iterations for finding a contact area which must be consistent with the applied load. If the contact stresses for all the contact pairs are in equilibrium and no geometrical incompatibilities occur, either inside or outside the contact zone, the correct contact solution has been found. When friction is taken into consideration, the problem is more complicated since energy dissipation occurs due to the relative tangential displacements working against the frictional forces in the slip zone. In this case, iterative techniques are required to calculate the size of the contact area and the extent of both the sliding and adhering zone for a given external load. Load incremental procedures are also required to solve the problem in several load increments so that an accurate final solution can be obtained by following the loading history.

The size of each load increment may be obtained using a load scaling technique; in which a load increment must be scaled in such a way that a new element contact pair is either just established or just released by the total load. Since contact area is a function of external load therefore the size of a load increment, in this case, is governed by the element size. For this reason, if a load increment technique is to be used the element size must not be too large, otherwise an accurate load history will not be obtained. Alternatively, a constant-load-increment technique may be employed so that a small pre-determined, constant load is added step by step until the final load is reached. Whilst both load incremental techniques can be used to solve a nonlinear contact problem, the load scaling technique is considered to be more efficient and flexible. The constant-load-increment technique is slow, and convergence can not generally be guaranteed if the chosen load increment is too large.

2 Basic contact definitions

2.1 Modes of contact

Contact modes may be viewed as boundary constraints: they are required to be prescribed in those regions where the surfaces are in contact or potentially coming into contact. For a given contact state, the contact conditions of a contact node-pair (a and b say) may be represented by any one of the three modes shown in Table 1, where t_{t} and t_{n} are the tangential and normal tractions, and u_{t} and u_{n} are the tangential and normal displacements respectively, expressed in local coordinates. In Table 1, the **Separation** mode is defined as node pairs remaining apart; the **Slip** mode is defined as node pairs not restrained in the tangential direction, but free to slide over each other; and the **Stick** mode is defined as node pairs restrained in the tangential direction and have not experienced any slip during loading. These definitions of **Slip** and **Stick** simply represent the two extreme modes of contact in which node pairs are either in phase (adhesion) or out of phase (sliding) under the influence of the external load. However, these definitions are not sufficient to define all the frictional contact possibilities that may be involved in advancing contact regions; node pairs

Table 1: Modes of contact (gap^{ab}=original gap between a and b).

Separation	Slip	Stick
$t_t^a - t_t^b = 0$ $t_n^a - t_n^b = 0$ $t_t^a = 0$ $t_n^a = 0$	$t_t^a - t_t^b = 0$ $t_n^a - t_n^b = 0$ $t_t^a \pm \mu t_n^a = 0$ $u_n^a + u_n^b = gap^{ab}$	$t_t^a - t_t^b = 0$ $t_n^a - t_n^b = 0$ $u_t^a + u_t^b = 0$ $u_n^a + u_n^b = gap^{ab}$

which become restrained in the tangential direction for the current load increment may have undergone some slip during previous load increments. In such a situation, the concept of **Partial slip** [25] is introduced to provide a more realistic and better description of the true nature of the problem.

2.2 Local coordinate system

Variables along the boundary outside the contact zone are represented in the global coordinate system defined by the Cartesian axis X and Y as shown in Fig. 1, whereas variables inside the contact region must be examined in the local coordinate system.

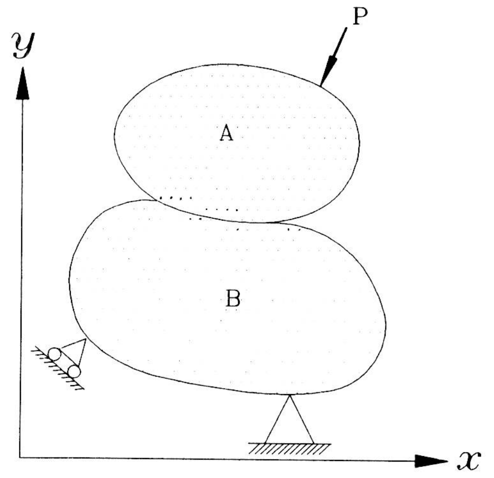

Figure 1: Global coordinate system

A transformation of the global coordinate system to the local coordinate system is therefore needed at each node pair in the potential contact area. The transformation is done by a rotation through an angle θ to the average normal direction.

Consider a node pair, a and b, with positions at (x^a, y^a) and (x^b, y^b) on two boundaries which are potentially coming into contact. The unit normal vectors **n** and the unit tangent vectors **s** at a and b are denoted by $\mathbf{n}^a$, $\mathbf{n}^b$ and $\mathbf{s}^a$, $\mathbf{s}^b$ respectively, as shown in Fig. 2. An average normal $\tilde{\mathbf{n}}$ can be defined by

$$\tilde{\mathbf{n}}^a = \frac{\mathbf{n}^a - \mathbf{n}^b}{| \mathbf{n}^a - \mathbf{n}^b |} = -\tilde{\mathbf{n}}^b \tag{1}$$

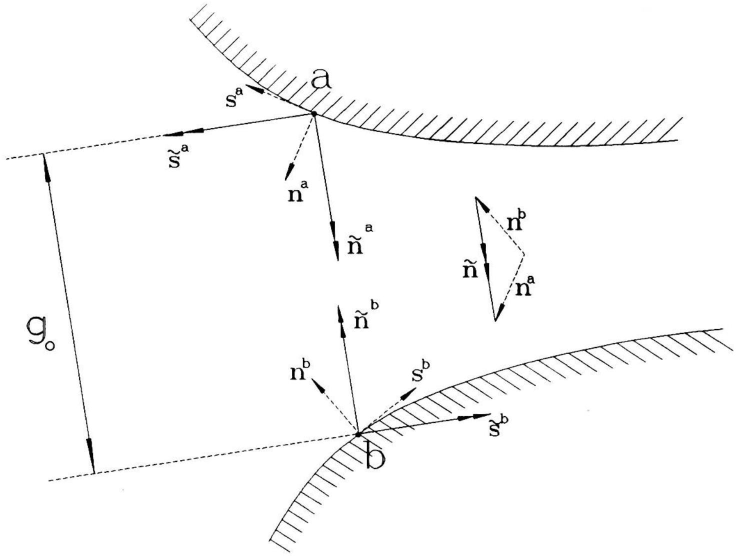

Figure 2: Local coordinate system

2.3 Definition of normal gap

The normal distance between a node pair is defined by an initial normal gap g_0 as shown in Fig. 3, from which the normal gap g_0 is calculated as follows:

$$\begin{aligned} g_0 &= | (y^b - y^a) | . \cos(\theta) + | (x^b - x^a) | . \sin(\theta) \\ &= | \mathrm{d}y | . \cos(\theta) + | \mathrm{d}x | . \sin(\theta) \\ &= | \mathrm{d}y | . \tilde{n}^a_y + | \mathrm{d}x | . \tilde{n}^a_x \end{aligned} \tag{2}$$

where $\tilde{n}^a_y$ and $\tilde{n}^a_x$ are the normal and tangential components of the average normal vector $\tilde{\mathbf{n}}^a$.

During deformation under the influence of an external load the node pair will be moved relative to each other and the surfaces brought into contact by closing the normal gap g_0; the two nodes may not be coincident. Physically $g_0 \geq 0$ since interpenetration of bodies is not allowed; contact occurs at $g_0 = 0$. Inside the contact zone, the tractions t^a_n and t^b_n, normal to the contact surfaces, must be maintained in compression in order to keep the node pair in contact, that is

$$t^a_\mathrm{n} = t^b_\mathrm{n} \leq 0 \tag{3}$$