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Practical Analog and Digital Filter Design

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Practical Analog and Digital Filter Design

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***This text is dedicated to my wife who keeps me grounded,
and to my grandchildren who know no bounds.***

Preface

This book was intentionally written to be different from other filter design books in two important ways. First, the most common analog and digital filter design and implementation methods are covered in a no-nonsense manner. All important derivations and descriptions are provided to allow the reader to apply them directly to his or her own filter design problem. Over forty examples are provided to help illustrate the fundamentals of filter design. Not only are the details of analog active and digital IIR and FIR filter design presented in an organized and direct manner, but implementation issues are discussed to alert the reader to potential pitfalls. An added feature to this text is the discussion of fast Fourier transforms and how they can be used in filtering applications. The simulation of analog filters is made easier by the generation of PSpice circuit description files that include R-C component values calculated directly from the filter coefficients. In addition, the testing of IIR and FIR filters designed for audio signals is enhanced by providing sample sound files that can be filtered by using the digital filter design coefficients. Anyone with a sound card on their computer can then play the original and processed sound files for immediate evaluation.

The second difference between this book and others is that the text is accompanied by WFilter, a fully functional, Windows[®]-based filter design software package, and the source code on which it is based. The CD provides the reader with the ability to install WFilter with a few simple clicks of the mouse, and also supplies the reader with the well organized and clearly documented source code detailing the intricacies of filter design. No, the source code provided is not just a collection of fragmented functions, but rather a set of three organized programs that have been developed (with the addition of an easy-to-use graphical interface) into the organized structure of WFilter.

A basic knowledge of C programming is expected of the reader, but the code presented in the text and the appendixes is thoroughly discussed and well documented. The text does assume the reader is familiar with the fundamental concepts of linear systems such as system transfer functions and frequency response although no prior knowledge of filter design is needed.

CHAPTER CONTENTS

Chapter 1 introduces the reader to the filter design problem. An overview of WFilter is presented. Chapter 2 develops the normalized transfer functions for the Butterworth, Chebyshev, inverse Chebyshev, and elliptic approximation cases. Chapter 3 describes the conversion of the normalized lowpass filter to an unnormalized lowpass, highpass, bandpass, or bandstop filter. In addition, the calculation of the frequency response for analog filters is discussed. By the end of the third chapter, a complete analog filter design can be performed. In Chapter 4, the implementation of analog filters is considered using popular techniques in active filter design with discussion of real-world considerations. A PSpice circuit description file is generated to enable the filter developer to analyze the circuit. Chapter 4 completes the discussion of analog filters in this book.

Chapter 5 begins the discussion of discrete-time systems and digital filter design in this book. Several key features of discrete-time systems, including the notion of analog-to-digital conversion, Nyquist sampling theorem, the z -transform, and discrete-time system diagrams, are reviewed. Similarities and differences between discrete-time and continuous-time systems are discussed. In Chapter 6, digital IIR (recursive) filters are designed. Three methods of designing IIR filters are considered. In addition, the frequency response calculations and related C code for the IIR filter are developed. Chapter 7 considers digital FIR (nonrecursive) filters using a variety of window methods and the Parks-McClellan optimization routine. The special techniques necessary for FIR frequency response calculation are discussed. The implementation of real-time and nonreal-time digital FIR and IIR filters is discussed in Chapter 8. Implementation issues such as which type of digital filter to use, accuracy of quantized samples, fixed or floating point processing, and finite register length computation are discussed. The reader can then hear the effects of filtering by replaying the original and processed sound files on a sound card. Chapter 9 completes the text with an introduction of the discrete Fourier transform and the more efficient fast Fourier transform (FFT). The reader will learn how to use the FFT in filtering applications and see the code necessary for this operation.

For those readers who desire filter design references or further details of the C code for the design of analog and digital filters, nine separate appendixes provide that added information.

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Chapter 1

Introduction to Filters and Filter Design Software

Everyone has probably come in contact with one type of filter or another in their lifetime. Maybe it was a coffee filter used to separate the grounds from the liquid, or perhaps an oil filter to remove contaminants from the oil of an engine. Anyone working in an office often filters the unimportant work from the important. In essence then the act of filtering is the act of separating desired items from undesired items. Of course when we discuss filters in this text, we are not talking about coffee, oil, or paperwork, but rather electronic signals. The electronic filters we will be designing will separate the desirable signal frequencies from the undesirable, or in other applications simply change the frequency content which then changes the signal waveform.

There are many types of electronic filters and many ways that they can be classified. A filter's frequency selectivity is probably the most common method of classification. A filter can have a lowpass, highpass, bandpass, or bandstop response, where each name indicates how a band of frequencies is affected. For example, a lowpass filter would pass low frequencies with little attenuation (reduction in amplitude), while high frequencies would be significantly reduced. A bandstop filter would severely attenuate a middle band of frequencies while passing frequencies above and below the attenuated frequencies. Filter selectivity will be the focus of the first section in this chapter.

Filters can also be described by the method used to approximate the ideal filter. Some approximation methods emphasize low distortion in the passband of the filter while others stress the ability of the filter to attenuate the signals in the stopband. Each approximation method has visible characteristics that distinguish it from the others. Most notably, the absence or presence of ripple (variations) in the passband and stopband clearly set one approximation method apart from another. Filter approximation methods will be discussed in further detail in the second section.

Another means of classifying filters is by the implementation method used. Some filters will be built to filter analog signals using individual components mounted on circuit boards, while other filters might simply be part of a larger digital system which has other functions as well. Several implementation methods will be described in the third section of this chapter as well as the differences between analog and digital signals. However, it should be noted that digital filter design and implementation will be considered in detail starting in Chapter 5, while the first four chapters concentrate on filter approximation theory and analog filter implementation.

In the final section of this chapter we discuss WFilter, an analog and digital filter design package for Windows[®], which is included on the software disk. WFilter determines the transfer function coefficients necessary for analog filters or for digital FIR or IIR filters. After the filter has been designed, the user can view the pole-zero plot, as well as the magnitude and phase responses. The filter design parameters or the frequency response parameters can also be edited for ease of use. In addition, for analog filters, the Spice circuit file can be generated to aid in the analysis of active filters. After digital filters have been designed, they may be used to filter wave files and the results can be played for comparison (a sound card must be present). Further discussion of WFilter and the C code supplied with this text can be found in Appendix B.

1.1 FILTER SELECTIVITY

As indicated earlier, a filter's primary purpose is to differentiate between different bands of frequencies, and therefore frequency selectivity is the most common method of classifying filters. Names such as lowpass, highpass, bandpass, and bandstop are used to categorize filters, but it takes more than a name to completely describe a filter. In most cases a precise set of specifications is required in order to allow the proper design of a filter. There are two primary sets of specifications necessary to completely define a filter's response, and each of these can be provided in different ways.

The frequency specifications used to describe the passband(s) and stopband(s) could be provided in hertz (Hz) or in radians/second (rad/sec). We will use the frequency variable f measured in hertz as filter input and output specifications because it is a slightly more common way of discussing frequency. However, the frequency variable ω measured in radians/second will also be used as WFilter's internal variable of choice as well as for unnormalized frequency responses since most of those calculations will use radians/second.

The other major filter specifications are the gain characteristics of the passband(s) and stopband(s) of the filter response. A filter's gain is simply the ratio of the output signal level to the input signal level. If the filter's gain is greater than 1, then the output signal is larger than the input signal, while if the gain is less than 1, the output is smaller than the input. In most filter applications, the gain

response in the stopband is very small. For this reason, the gain is typically converted to decibels (dB) as indicated in (1.1). For example, a filter's passband gain response could be specified as 0.707 or as -3.0103 dB, while the stopband gain might be specified as 0.0001 or -80.0 dB.

$$\text{gain}_{\text{dB}} = 20 \cdot \log(\text{gain}) \quad (1.1)$$

As we can see, the values in decibels are more manageable for very small gains. Some filter designers prefer to use attenuation (or loss) values instead of gain values. Attenuation is simply the inverse of gain. For example, a filter with a gain of $1/2$ at a particular frequency would have an attenuation of 2 at that frequency. If we express attenuation in decibels we will find that it is simply the negative of the gain in decibels as indicated in (1.2). Gain values expressed in decibels will be the standard quantities used as filter specifications, although the term attenuation (or loss) will be used occasionally when appropriate.

$$\text{attn}_{\text{dB}} = 20 \cdot \log(\text{gain}^{-1}) = -20 \cdot \log(\text{gain}) = -\text{gain}_{\text{dB}} \quad (1.2)$$

1.1.1 Lowpass Filters

Figure 1.1 shows a typical lowpass filter's response using frequency and gain specifications necessary for precision filter design. The frequency range of the filter specification has been divided into three areas. The passband extends from zero frequency (dc) to the passband edge frequency f_{pass} , and the stopband extends from the stopband edge frequency f_{stop} to infinity. (We will see later in this text that digital filters have a finite upper frequency limit. We will discuss that issue at the appropriate time.) These two bands are separated by the transition band that extends from f_{pass} to f_{stop} . The filter response within the passband is allowed to vary between 0 dB and the passband gain a_{pass} , while the gain in the stopband can vary between the stopband gain a_{stop} and negative infinity. (The 0 dB gain in the passband relates to a gain of 1.0, while the gain of negative infinity in the stopband relates to a gain of 0.0.) A lowpass filter's selectivity can now be specified with only four parameters: the passband gain a_{pass} , the stopband gain a_{stop} , the passband edge frequency f_{pass} , and the stopband edge frequency f_{stop} .

Lowpass filters are used whenever it is important to limit the high-frequency content of a signal. For example, if an old audiotape has a lot of high-frequency "hiss," a lowpass filter with a passband edge frequency of 8 kHz could be used to eliminate much of the hiss. Of course, it also eliminates high frequencies that were intended to be reproduced. We should remember that any filter can differentiate only between bands of frequencies, not between information and noise.

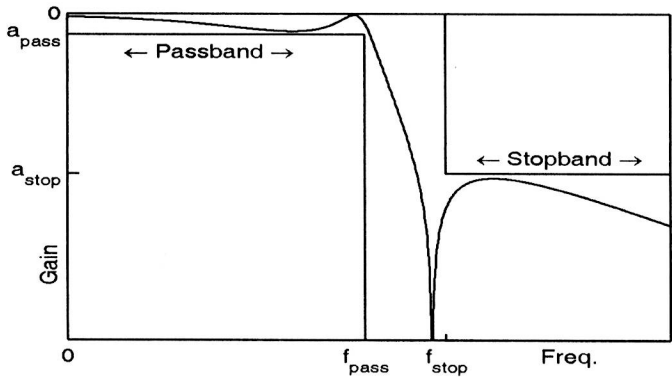


Figure 1.1 Lowpass filter specification.

1.1.2 Highpass Filters

A highpass filter can be specified as shown in Figure 1.2. Note that in this case the passband extends from f_{pass} to infinity (for analog filters) and is located at a higher frequency than the stopband which extends from zero to f_{stop} . The transition band still separates the passband and stopband. The passband gain is still specified as a_{pass} (dB) and the stopband gain is still specified as a_{stop} (dB).

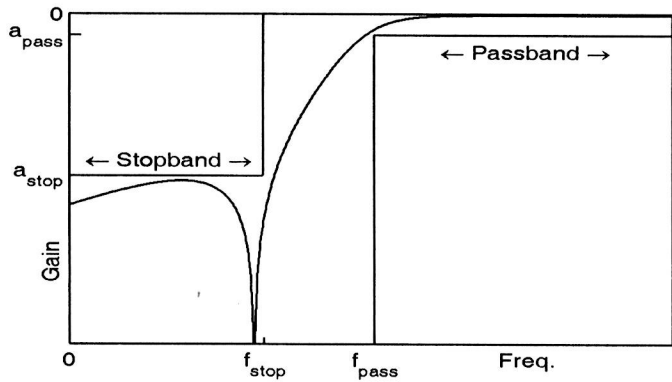


Figure 1.2 Highpass filter specification.

Highpass filters are used when it is important to eliminate low frequencies from a signal. For example, when turntables are used to play LP records (some readers may remember those black vinyl disks that would warp in a car's back window), turntable rumble can sometimes occur, producing distracting low-