

TOPICS IN DIGITAL SIGNAL PROCESSING

Theory and Design of Adaptive Filters

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Theory and Design of Adaptive Filters

TOPICS IN DIGITAL SIGNAL PROCESSING

C. S. BURRUS and T. W. PARKS: *DFT/FFT AND CONVOLUTION ALGORITHMS: THEORY AND IMPLEMENTATION*

JOHN R. TREICHLER, C. RICHARD JOHNSON, Jr., and MICHAEL G. LARIMORE: *THEORY AND DESIGN OF ADAPTIVE FILTERS*

T. W. PARKS and C. S. BURRUS: *DIGITAL FILTER DESIGN*

To Sally, Betty, and Mary Anne

Preface

This book is a pedagogical compilation of fundamental adaptive filtering concepts, algorithm forms, behavioral insights, and application guidelines. The analysis and design of three basic classes of adaptive filters are presented: (1) adaptive finite-impulse-response (FIR) filters, (2) adaptive infinite-impulse-response (IIR) filters, and (3) adaptive property-restoral filters.

For the widely studied (and utilized) class of adaptive FIR filters, we develop the most popular analytical tools and distill a tutorial collection of insightful design guidelines of proven utility. For more recently developed adaptive IIR and adaptive property-restoral filters, we focus on emerging theoretical foundations and suggested applications. Our presentation is augmented by listings of FORTRAN code for the more basic of these algorithms. We also detail a real-time solution—using a Texas Instruments signal-processing chip (TMS320)—to one adaptive FIR filter problem. Thus, both practicing engineers interested in designing appropriate adaptive filters for various applications and graduate students interested in acquiring a cohesive pedagogy for initiation of basic research in adaptive filter theory can benefit from this book.

The first chapter contains a detailed description of the purpose and motivation of adaptive filtering in three practical signal-processing problems. Basically, this chapter is for those readers who do not yet know of the practical need for and utility of adaptive filters. An understanding of nonadaptive digital signal processing is assumed of the reader. More specifically, this book was written with the expectation that the reader would have a rudimentary understanding of digital filter theory (e.g., discrete Fourier transform, analog-to-digital filter transformation, and frequency-response shaping techniques of FIR and IIR digital filter design), stochastic processes (e.g., correlation functions),

linear algebra (e.g., eigenvalues and eigenvectors), and systems theory (e.g., state representations, z-transforms, transfer functions, and stability theory). If the reader does not possess such a background, the following texts are recommended as possible sources: A. V. Oppenheim and A. S. Willsky, *Signals and Systems* (Prentice-Hall, 1983) and L. R. Rabiner and B. Gold, *Theory and Application of Digital Signal Processing* (Prentice-Hall, 1975). Given such a background, the reader should find this book a self-contained primer and reference source on adaptive filtering.

Chapter 2 provides an intuitive illustration of the basic principles underlying the conversion of (mis)performance information into digital filter adaptation. A set of number guessing games with logically obvious algorithmic solutions is examined and recognized to strongly parallel digital filter adaptation where the filter parameters are effectively the numbers being guessed. These examples of adaptation are simple enough to be readily understood yet complex enough to illustrate several of the basic attributes of adaptive filters. The reader will also find these number guessing games helpful in interpreting the purpose and usefulness of algorithmic elaborations that are introduced in subsequent chapters. Essentially, this chapter is intended for those readers who believe adaptive filters are useful but do not yet have a firm idea as to the source and form of filter parameter adaptation.

Chapters 3, 4, 5, and 6 present the fundamentals of adaptive filter theory. Chapters 3 and 4 focus on adaptive FIR filters, which have been widely analyzed and applied since their conception in the late 1950s. Chapter 3 develops popular adaptive filter analysis tools for dynamic parameter estimate moment analysis. Chapter 4 is based on selections of fundamental material from the large body of useful work on adaptive FIR filter design. Chapters 3 and 4 are widely acknowledged as the traditional domain of adaptive filtering. Chapters 5 and 6 provide rudimentary descriptions of emerging theory and proposed applications of more recently "discovered" adaptive filter algorithms. Adaptive IIR filters, first proposed in the open literature in the mid 1970s, are examined in Chapter 5 from the two conceptually distinct approaches of gradient descent minimization and stability theory. The resulting algorithm forms are interpreted as logical extensions on the basic adaptive FIR filter algorithm. Chapter 5 is the most heavily theoretical of the book. This is due in part to the fact that the pedagogy of adaptive IIR filters is still in its infancy. This more intense theoretical tone is also due in part to the increase in complexity of adaptive IIR filters relative to adaptive FIR filters. Chapter 6 introduces the class of adaptive property-restoral filters, initially formulated in the early 1980s. This property-restoral class is a conceptual enlargement of the basis of the adaptive FIR and IIR filters of Chapters 4 and 5. Property-restoral adaptive filters adjust their coefficients to restore some known property of the desired output sequence without (necessarily) requiring a full sample-by-sample description of the desired output sequence, as do the more traditional adaptive filter algorithms of Chapters 4 and 5. Application of the property-restoral concept has proceeded faster than development of an appropriate comprehensive theory. Thus, Chapter 6 focuses more on

the particular engineering successes of this emerging concept rather than its abstract theoretical analysis.

Where Chapters 4 through 6 focus on theoretically derived algorithms valid under certain idealizations, practice dictates certain modifications to help ensure cost-effective, robust performance in the more harsh environment of real applications. Such issues and related implementation concerns are summarized in Chapter 7. Since the study of adaptive filters is driven by the need for (and success of) their use, Chapter 8 returns to the applications cited in Chapter 1, specifically noise cancelling, adaptive differential pulse code modulation, and channel equalization, as illustrations of the use of the adaptive filter algorithms studied in the intervening chapters.

In this book the stress is on fundamentals. Admittedly, those selected are a reflection of the biases of the authors. The possible tunnel vision imposed by such a procedure is mitigated by the citation of references examining more advanced issues.

An attribute of this book that enhances its reference text quality is the inclusion in Appendix A of FORTRAN listings of some basic algorithms. One of the major hurdles for new “students” of adaptive filtering is the conversion of algorithm equations into implementable code. In fact, our teaching experience indicates that the construction of such code is a major learning experience. The listings included are intended to aid this effort. We have chosen only the simpler versions of the algorithms examined due to their pedagogical clarity. The high data rate and minimal computation cost objectives of the majority of adaptive filtering applications support this focus on the simplest of algorithms with proven attractive properties. However, high-level language code is still somewhat removed from the coding (and hardware) necessities of actual applications. Thus, in Appendix B we detail a solution to the tone canceller problem of Section 1.1 using a Texas Instruments TMS320 general-purpose signal-processing chip.

A number of people deserve our thanks for their part in this book. Sid Burrus and Tom Parks, both at Rice University at the time, suggested this project to us as the second entry in a series of books on topics in digital signal processing sponsored by Texas Instruments. (Incidentally, their book *DFT/FFT and Convolution Algorithms: Theory and Implementation* (Wiley-Interscience, 1985) is the first in this series.) Linda Struzinsky of Cornell University typed our entire manuscript several times. Fran Costanzo and Evelyn Liebgold of Applied Signal Technology drafted the figures. George Troullinos of Texas Instruments drew the schematics and wrote the TMS320 code in Appendix B. Dennis Morgan of AT&T Bell Laboratories carefully reviewed our “final” draft and provided a number of suggestions that we subsequently adopted. We are very grateful to each of you.

This book is the culmination of our collaborations in adaptive filter research begun in the office we shared, fondly known then as The Zoo, while we were all struggling towards PhDs. In the intervening years our contacts with and employment by ARGOSystems (Sunnyvale, CA), Applied Signal Technology

(Sunnyvale, CA), and Tellabs Research Laboratory (South Bend, IN) proved indispensable to our understanding of the design aspects of adaptive filters. Also, this book would not be in its present form without the direct financial support of another company, Texas Instruments, that commissioned this manuscript. These interactions are responsible for the increased emphasis this book places on applications relative to more traditional texts.

It is the widespread success of adaptive filter applications that makes this book possible. We acknowledge our indebtedness to the efforts of the community of adaptive filtering theorists and practitioners who are responsible for these successes.

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*Chatham, Massachusetts
October 1986*

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