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PRINCIPLES OF OPERATIONS RESEARCH

Second Edition

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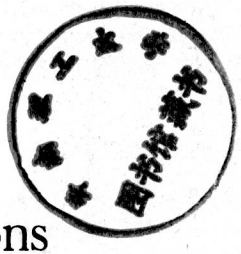
With Applications to Managerial Decisions



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To my parents
and their favorite grandchildren,
Caroline and Julie

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Preface

This book is written primarily for college students who have no previous background in operations research and who intend careers as administrators, executives, managers, or consultants in business, nonprofit enterprises, or government. The broad topic coverage also makes the text helpful for students who seek careers as teachers and researchers as well as for practitioners who desire an up-to-date review of operations research. The book is used in half-year or full-year introductory courses for undergraduates or graduates in business, economics, engineering, liberal arts, and public administration curricula.

The central goal of the book is to answer the question, "What are the *fundamental ideas* of operations research?" The text does not presuppose any *advanced* training in business administration, industrial engineering, mathematics, statistics, probability theory, or economics. Therefore, the main ideas do not rely on the reader's being expert in these areas. The text does assume, however, that the reader is not entirely naive about such subjects.

PRINCIPAL OBJECTIVES

Beginning students frequently ask, "What must I learn about operations research if I intend to become a manager rather than a specialist?" and "What must I learn about operations research given that I want to apply it to real problems?" Regrettably, a single introductory operations research course cannot completely answer either question. But such a course can better answer the first rather than the second question.

In the context of these two questions, the book's principal objectives are

- To introduce the important ideas in operations research which are both fundamental and long lasting.
- To provide those students not going beyond a single introductory course with enough understanding and confidence to appreciate the strengths and inherent limitations of the operations research approach.

- To prepare and motivate future specialists to continue in their study by having an insightful overview of operations research.
- To demonstrate the cohesiveness of operations research methodology.

Students who have been assigned this book as a text have increased their skills in formulating and building formal models of complex decision environments, in perceiving the critical issues to be resolved, and in isolating the basic phenomena that comprise the key elements of real situations. When faced with actual managerial problems, these students have learned from their grasp of the analytic fundamentals how to achieve sound and incisive evaluations of the important alternatives and how to attain the crucial insights.

COVERAGE AND EMPHASIS

Successful practitioners of operations research are quick to point out that characterizing the subject by means of its component mathematical techniques can be highly misleading. Their reason is that real-life decisions do not arise with technical labels attached, such as *linear programming* or *risk analysis simulation.* A manager must be able to recognize what existing methods, if any, can yield worthwhile insights for taking appropriate courses of action. Granting the truth of these observations, we comment on why we organized the text by standard mathematical techniques rather than by various management problems.

Accumulated teaching experience of hundreds of instructors indicates that beginning students with only limited time for this subject better understand the model-building approach when they can *readily* discern the formal structure of the decision problem. Although realism is sacrificed in the classroom by adopting this pedagogic view, students so trained effectively bridge the gap between the techniques and their applications.

To warn students, however, of the trap of thinking that operations research is synonymous with a collection of mathematical formulas, we take pains to *verbalize* the contribution that each model and technique make to decision-making. Students previously trained in science, engineering, or mathematics sometimes are surprised and even impatient that the text material is not written in a succinct mathematical style. But we suggest that thinking beyond the details of the formulas to the deeper significance of each approach, over the long pull, better prepares future managers and analysts to make successful applications.

Obviously, we had to exercise considerable judgment in selecting topics and in choosing the depth of treatment. In this regard, we were influenced considerably by more than 20 years of first-hand experience in applying operations research to actual situations in business and government. The examples in the text often are scaled-down versions of real problems that we have encountered. The specific models that we chose, their complexity, and sequential development all reflect our observations of how most students come to appreciate and understand the pivotal concepts in operations research.

Throughout, the book puts into focus the value of information derived from an operations research solution and attempts to give an accurate representation of how a hypothetical example would be applied in a real situation. The text develops the student's skill in formulating and building models and, specifically, in translating a verbal description of a decision problem into an equivalent mathematical model. The book also explains the importance and the degree of severity of a model's assumptions, the connection between the starting assumptions and the derived results, and the seriousness of the assumptions for practical purposes. Finally, the text demonstrates by means of important examples the process and usefulness of constructing analogies, finding multiple interpretations of models, and deriving significant special cases from general models.

To permit an instructor to select the topics that are most appropriate for the class, we have included material at several different levels of complexity. *Any material designated by an asterisk or appearing between the symbols* ▶ *and* ◀ *and set in smaller print may be skipped without loss of continuity.*

MATHEMATICS PREREQUISITES

The differential calculus is employed only in Chaps. 14, 15, and 20, and the integral calculus only in Chap. 20 and Appendix II. Nevertheless, much of the text assumes a mathematical sophistication comparable to that acquired in standard college-level introductory calculus or a no-nonsense finite mathematics course. Elementary probability theory is first needed in Chap. 16. The two appendices and much of the optional material, designated by either an asterisk or smaller print, relate to technical details and often require a noticeably higher level of mathematical maturity.

SUGGESTIONS TO STUDENTS

If you have taken courses in both the calculus and elementary statistics or probability theory, you will recall that you could understand much of the calculus text without much help from the instructor whereas you needed the guidance of the instructor to fully understand the statistics or probability theory text. This is a very common occurrence among students. You will find operations research to be more like statistics and probability theory than the calculus in that you can expect, as a matter of course, to rely on your instructor to amplify the ideas and techniques in this book. Since the concepts can be intricate, you may have to study several pages before comprehending the full idea. So be prepared for patient reading and some rereading. We have tried to avoid giving numerical examples that are so *misleadingly* simple that you are not sufficiently prepared for the exercises at the end of each chapter. As a result, you will find it advantageous to examine carefully the numerical illustrations in the chapters.

If your instructor assigns the exercises at the back of the chapters, do not wait until the last minute to begin the problems. The numerical exercises, although

straightforward, can be fatiguing if attempted in a single evening session. Most students find that formulation problems require a "gestation period," so allow yourself a few days to ponder such exercises.

Surprisingly, it is difficult to write a *verbal* description of an operations research problem that is completely unambiguous. More than once, thoughtful students have discovered vague wording in problems that we had assigned previously to other classes that experienced no difficulty in obtaining the intended solutions. Consequently, if you believe that an exercise displays a troublesome ambiguity, try to resolve it in a sensible way and make explicit on your paper the specific assumption that you adopted. The purpose of the exercises (namely, to give you practice in mathematical formulation) is well served by this procedure.

SUGGESTIONS TO INSTRUCTORS

Clearly, the entire book is too large for even a full year's introductory course. This broad topic coverage has been purposely designed to give you considerable flexibility in choosing the subjects that you want to stress. The logical organization of the chapters is shown in Exhibit A.

In a one-year introductory survey course, where the students know elementary probability theory and the calculus, you can include Chaps. 1 through 8, 10, 13, 16, 17, 19 through 22, skipping all of the starred (*) sections and optional material in smaller print. If you do not want to use the calculus, then omit Chap. 20. If the students are unfamiliar with elementary probability theory, then you can assign Chaps. 1 through 8, 10, 13, 22, and choose topics from Chaps. 9 and 11 through 15.

In a one-term introductory course on mathematical programming models, you can include Chaps. 1 through 8, 10, 13, and 22, skipping all the starred (*) sections and optional material in smaller print. In a one-term introductory course on probabilistic models, you can include Chaps. 1, 16, 17, and 19 through 22, also skipping optional and advanced material.

To assist you in selecting other combinations, the "Immediate Predecessors" of each chapter are shown in Exhibit B. In general, the chapters become progressively more difficult. Thus, although you could cover dynamic programming (Chaps. 8, 9, and 10) prior to linear programming, the reverse sequence ordinarily would be preferable, because the dynamic programming chapters assume the student already is somewhat familiar with multivariate constrained optimization models and solution techniques.

The second edition has been written to make certain topics more accessible to beginning students and to update the coverage where a subject's fundamental ideas have changed. Specifically, the *major* revisions occur in Chaps. 6, 7, 10, 13, 16, and 21. The material on networks in Chaps. 6 and 7 is now organized so that each algorithm is presented in conjunction with the model's description and application; the out-of-kilter algorithm has been added to Appendix I. A slower-paced exposition of dynamic programming models is given in Chap. 10; the new material includes worked-out numerical examples to clarify the approach. The

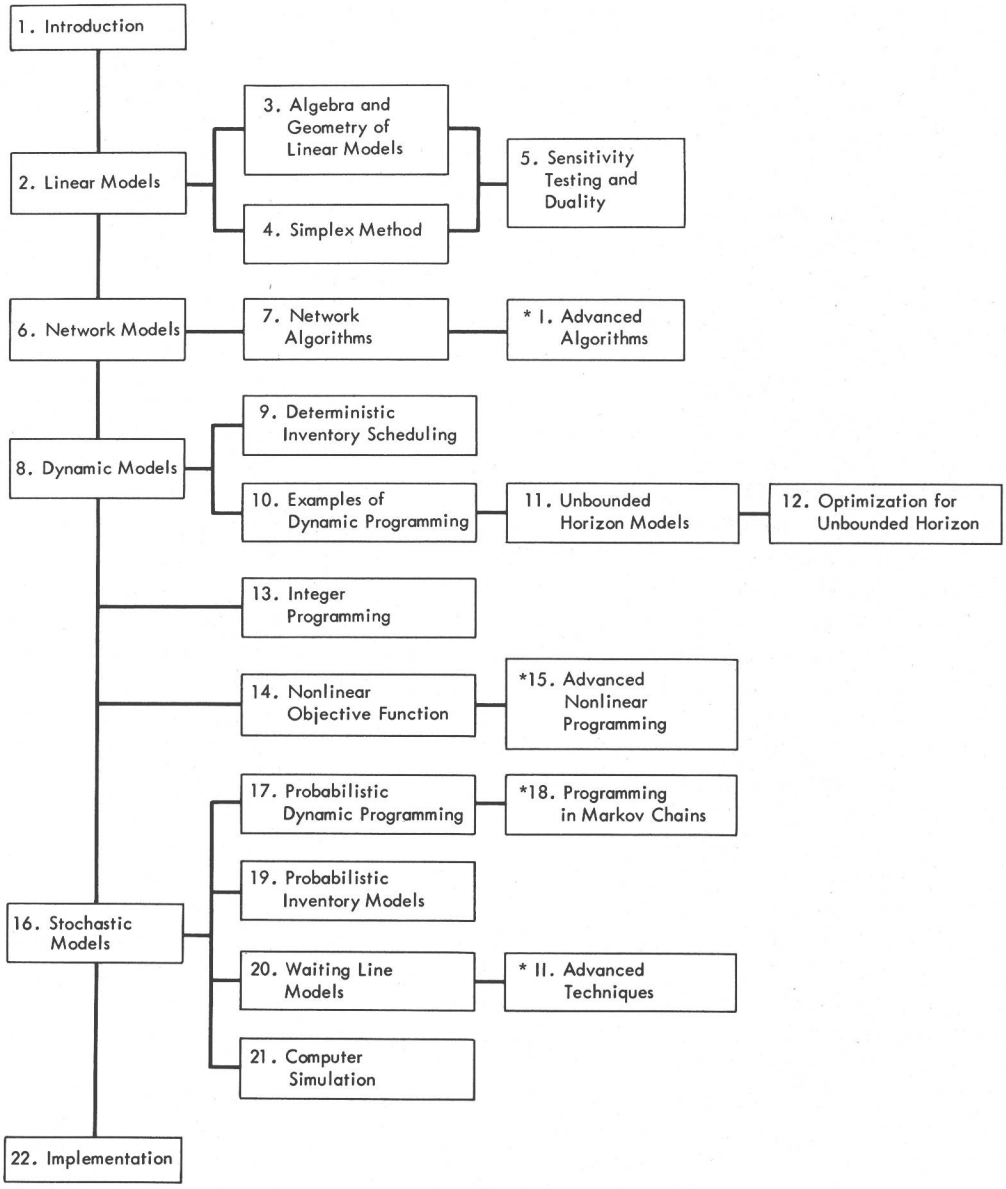

EXHIBIT A. Organization of Chapters.

Exhibit B

CHAPTER		IMMEDIATE PREDECESSORS
2	Linear Models	1
3	Algebra and Geometry of Linear Models	2
4	Simplex Method	2
5	Sensitivity Testing and Duality	3, 4
6	Network Models	2, 5
7	Network Algorithms	6
8	Dynamic Models	1, (7)
9	Deterministic Inventory Scheduling	8
10	Examples of Dynamic Programming	8
11	Unbounded Horizon Models	10
12	Optimization for Unbounded Horizon	11
13	Integer Programming	7
14	Nonlinear Objective Function	5
*15	Advanced Nonlinear Programming	14
16	Stochastic Models	6
17	Probabilistic Dynamic Programming	10, 16
*18	Programming in Markov Chains	12, 17
19	Probabilistic Inventory Models	(9), 16
20	Waiting Line Models	16
21	Computer Simulation	16
22	Implementation	1
*I	Advanced Network Algorithms	7
*II	Advanced Waiting Line Techniques	20

NOTE: *Chaps. 16 through 21 require probability theory.*
Chaps. 14, 15, and 20 require differential calculus.
Chap. 20 and App. II require integral calculus.
( ) indicates desirable but not essential.

explanation of models and methods for integer programming in Chap. 13 is extensively rewritten to reflect computational experience of recent years; approaches that have not worked well in practice are omitted. The material on stochastic programming in Chap. 16 is revised to be more suitable for assignment in an introductory course. Finally, the exposition of computer simulation in Chap. 21 is expanded and reorganized to give greater prominence to the model-building aspects of the approach.

The book contains more than 1000 exercises, many of which have multiple parts. New exercises are added to most chapters in the second edition. The problems are grouped into four categories. The Review Exercises are keyed closely to the text and determine whether the student has understood the conceptual developments. The Formulation Exercises consist of "word problems" and test whether the student can translate a verbal statement into a precise mathematical model. The Computational Exercises provide practice problems for applying algorithms. The Mind-Expanding Exercises are primarily for advanced students—you also may want to pick some of these problems as topics for your lectures. There is "redundancy" in the multiple parts of many exercises, and consequently, you should examine the parts to ascertain which ones you want to assign.

In comparing this book with other elementary texts in operations research, you will notice a few differences in the standard list of chapter headings. We give extended treatment at an introductory level to several topics (such as dynamic programming over an unbounded horizon, integer programming, nonlinear programming, stochastic programming, and dynamic programming in Markov chains) that are covered only briefly, if at all, elsewhere. In contrast, we have not devoted entire chapters to the subjects of replacement theory and game theory. Important replacement models are included in the dynamic programming chapters. Game theory has been omitted, with the exception of the Fundamental Theorem of Two-Person, Zero-Sum Matrix Games given in exercise 35 of Chap. 5. In practicing operations research, we have found that game theory does not contribute any *managerial insights* to real competitive and cooperative decision-making behavior that are not *already* familiar to church-going poker players who regularly read the Wall Street Journal. (This is not to say, however, that the intricacies of real competitive economic behavior, such as price wars, advertising campaigns, mergers, and acquisitions, have yet become phenomena fully understood by management scientists.)

ACKNOWLEDGMENTS

Over the years since the first draft of this text was written, I have received many helpful suggestions from students and their instructors who have used the book. The list of persons who have provided sage counsel is now far too long to record here, but I must offer special thanks at least to Richard W. Cottle (Stanford), Eric V. Denardo (Yale), Hamilton Emmons (Case Western Reserve), Donald Erlenkotter (University of California, Los Angeles), Charles H. Falkner (University

of Wisconsin), Arthur M. Geoffrion (University of California, Los Angeles), Richard B. Hoffman (Arthur Young & Co., Washington, D.C.), Charles L. Hubbard (Georgia State), Charles H. Kriebel (Carnegie Mellon), Rudolph P. Lamone (University of Maryland), Richard P. O'Neill (Louisiana State), Alan J. Rolfe (IDA, Arlington, Va.), and Matthew J. Sobel (Yale). I am deeply indebted to Professor Robert E. Machol (Northwestern) whose friendly advice prompted many of the revisions in this new edition. And I again want to acknowledge Professor Arthur F. Veinott, Jr. (Stanford) who has continued to deeply influence my own thinking about the principles of operations research.

In great measure, the managerial relevance of the book has been enhanced through my association since 1960 with McKinsey & Co., as a Consultant to the Firm, and particularly through the personal guidance of Dr. David B. Hertz, Douglas Watson, Warren M. Cannon, and D. Ronald Daniel.

A fine Instructor's Manual for the second edition was patiently prepared by Arthur S. Estey and Richard A. Ehrhardt (both of Yale), and a debt of gratitude remains to John Chamberlin, John M. Harrison, and Michael Saunders who wrote the previous Manual.

The secretarial skill and encouraging dispositions of Ms. Sheila Hill (Stanford) and Ms. Marcia Wheeler (Yale) triumphed admirably over the seemingly never-ending strain of typing redrafts and meeting deadlines for the first edition. The limited pleasures of struggling over rewrites for the second edition were shared by Ms. Wilma Golden, Ms. Sara Martin and Ms. Ellen Mester (all of Yale), at last count. And we all can be thankful for the unsparingly, but deftly, employed red pencils of editors Mr. Will Harriss (RAND) and Mr. Kenneth Cashman (Prentice-Hall).

Amazing as it may seem, I still am lost for words in trying to adequately express appreciation to my wife Ruthie for keeping a "togetherness" at home far longer than a reasonable planning horizon. Hence, I resort (hopefully only this once in the book) to a cryptogram that remains unambiguously decipherable only to her: $\frac{1}{\epsilon} \to \infty$.

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New Haven, Connecticut

Second Edition

PRINCIPLES OF OPERATIONS RESEARCH

With Applications to Managerial Decisions

Contents

† Each chapter is preceded by a list of its contents.

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CHAPTER 1

The Art and Science of Executive Decisions

1.1 BEFORE GETTING STARTED . . .

You may well be curious to know how a subject with so abstruse a name as *operations research* could beget a several hundred page introductory text purportedly dealing only with principles. The ambiguous term *operations research* came into prominence around World War II. At that time, it was an apt description of the subject matter. Unfortunately, the name stuck, even though present-day applications of operations research are considerably more diverse than they used to be.

Now there is a worldwide confederation of professional societies named Operations Research. The staffs of many industrial organizations bear the title. So do departments in leading universities, which have gone on to sanctify the term by granting advanced degrees bearing its name. These vested interests are so well entrenched that the name *operations research* is unlikely to be supplanted in our lifetime.

Disgruntled though we may be, saddled as we are with a title that is undescriptive if not downright misleading, we nevertheless must show our respect. After all, the scientists who originated the term were on the winning side of the war. (Who knows what might have happened if the other side had invented the approach first?)

1.2 BY ANY OTHER NAME

Numerous synonyms for operations research are in common use. The British like *operational research*. A frequent American substitute is *management science*. (The popularity of this name is fostered by yet another international professional society called the Institute of Management Sciences.) As a beginning student, fortunately,

you can afford to assume a lofty indifference to the whole matter, leaving this semantic bone of contention for your seniors to wrangle over.

For convenience, and with reasonable accuracy, you can simply define operations research as a scientific approach to problem-solving for executive management. An application of operations research involves:

- Constructing mathematical, economic, and statistical descriptions or models of decision and control problems to treat situations of complexity and uncertainty.
- Analyzing the relationships that determine the probable future consequences of decision choices, and devising appropriate measures of effectiveness in order to evaluate the relative merit of alternative actions.

It is sometimes believed that operations research refers to the constant monitoring of an organization's ongoing activities—and, in fact, decision and control problems often do concern certain daily "operations" of the organization. Examples of this sort include production scheduling and inventory control, facility maintenance and repair, and staffing of service facilities, to name a few applications.

But many operations research studies treat other kinds of decisions that bear on daily operations only indirectly. These studies usually have a planning orientation. Illustrations include determining the breadth of a firm's product line, developing a long-term program for plant expansion, designing a network of warehouses for a wholesale distribution system, and entering a new business by merger or acquisition.

It is bad enough that the word "operations" inadequately describes the diversity of present-day applications. To make matters worse, the word "research" creates the false impression that the method is a "blue-sky approach." On the contrary, in the past two decades operations research has proved time and again to be a powerful and effective approach for solving critically real management problems. You will learn most of the reasons in this chapter, and you will know the full story after reading the main chapters of this book.

Better decisions in a complex and uncertain environment. A preferable term to describe the subject of this book is *decision analysis*. An emphasis on making decisions or taking actions is central to all operations research applications.

Decision analysis separates a large-scale problem into its subparts, each of which is simpler to manipulate and diagnose. After the separate elements are carefully examined, the results are synthesized to give insights into the original problem. You may wonder why such complex decision-making problems arise in the first place.

One reason is that in today's economy, technological, environmental, and competitive factors typically interact in a complicated fashion. For example, a factory production schedule has to take account of customer demand (tempered by the likelihood of a price-cut by competitors), requirements for raw materials and intermediate inventories, the capacities of equipment, the possibility of equipment