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Small Signal Analysis of Power Systems

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Alpha Science

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Preface

This manuscript, "Small Signal Analysis of Power Systems," is the result of a collaborative effort between the Indian Institute of Science, Bangalore, India, and the University of Illinois at Urbana-Champaign supported under the United States-India co-operative science program. The corresponding agencies were the Department of Science and Technology in India and the National Science Foundation in the United States. The faculty involved were Professors D. P. Sen Gupta and K. R. Padiyar from India and Professor M. A. Pai from the United States. The title of the project was NSF INT 93-02565 "Small Signal Stability of Electric Power Systems." Besides the several research papers that have been produced on this topic by the investigators, a central purpose was to produce a manuscript that documents the state of the art in this field. In this manuscript we discuss the areas where small signal analysis is heavily used in power systems.

In Chapters 2 and 3 (Professor Sen Gupta) the emphasis is on the nature of oscillations and their physical meaning in relation to the synchronous machine, which is modelled in detail. Chapters 4 and 5 (Professor Padiyar) discuss the phenomena of sub-synchronous resonance in power systems and its control as well as the modelling of flexible ac transmission systems (FACTS) controllers. Chapters 6 and 7 (Professor Pai) discuss the framework of multimachine small signal analysis to include static var compensators (SVC) and FACTS controllers. Small signal analysis with both these devices is discussed. We also discuss briefly the selective computation of eigenvalues, its mathematical basis, and the practical algorithms used in industry.

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Chapter 1

INTRODUCTION

1.1 Overview

In this chapter we give a brief introduction to the role of small signal analysis in power system dynamics stability and control.

Power systems differ from other physical systems in terms of the attributes associated with their large-scale nature. They are nonlinear, as most dynamic systems are, and they are very large in terms of the number of dynamic variables. Typically, a large interconnected system may have 10,000 buses or nodes. Roughly, at about 1,000 of these nodes there are dynamic devices connected, typically synchronous machines and their associated controls such as the excitation system, turbine-governor, and the fuel source (boiler, hydro or nuclear). Depending on the time frame of the phenomena to be studied, the number of state variables at each node can range from 2 to 20. Thus, the total number of state variables could range from 2,000 to 20,000. These state variables are in the frequency range of $1-3$ Hz. At the other nodes, there are loads whose dynamics are largely ignored except for special studies. We also ignore the 60 Hz transients in the transmission lines and the stator of the machines. Thus, we have a set of nonlinear differential equations coupled by algebraic variables through the network constraints. In mathematical language, the description is of the form

$$\dot{x} = f(x,y,u) \quad (1.1)$$

$$0 = g(x,y) \quad (1.2)$$

where x, y are the state and algebraic variable vectors, respectively, and u is the input vector. These inputs typically correspond to reference settings for turbine power, reference voltage at generator terminals, and reference voltage for other control devices such as the static var compensator.

The mathematical modeling of the power system corresponding to (1.1) and (1.2) is not the subject matter of this monograph. It is covered well in the literature elsewhere [1.1]–[1.5]. Once the model is derived, the problems of simulation, stability, and control are of importance. This topic is again very extensive and well researched. In this monograph, we cover only some aspects of these problems. Specifically, we concentrate on stability and control issues. Power systems have evolved over the years in a fairly robust manner. Although nonlinear simulation is of interest, control mechanisms to stabilize the system are always designed through linear models. Hence, the areas of interest to us are low-frequency oscillations, sub-synchronous resonance, small signal stability, and computation of selected eigenvalues in an otherwise large system. To complement these, we review synchronous machine models, circuit-based interconnection of components, and (as an aid for further research) present the flexible ac transmission systems (FACTS) device models.

1.2 Chapter Overviews

Chapter 2 presents an overview of the synchronous machine models. There are quite a few models with different notations in the literature. The purpose of this chapter is to bring these under a common framework and give a physical basis to them.

Chapter 3 discusses low-frequency oscillations and the design of power system stabilizers. In the years prior to 1970, most power engineering studies related either to large perturbations or small perturbations. These were called transient and steady state stability, respectively. As systems became more interconnected, a new phenomenon in the form of spontaneous build-up of low-frequency oscillations began to appear. A well-documented example was the repeated occurrence of oscillation in voltage, frequency, and power in the Saskatchewan-Manitoba-Hydro West interconnection in the period 1962–65. The oscillations at a frequency of approximately 0.35 Hz often increased in size until one of the tie lines tripped due to protective relay action. In 1964-65 a similar phenomena was observed in the Western States Co-ordinating Council (WSCC) system. Since then, there have been reports of similar instability from all parts of the world. To distinguish it from the historical steady state stability, this phenomena is generally referred to as the dynamic stability, or more frequently, as low-frequency oscillations. Linear models around an operating point are adequate to study this phenomena and devise appropriate controls. These models are referred to as the Heffron-Phillips or DeMello-Concordia models. This oscillatory phenomena is generally of the rotor mechanical mode. It is either a local or an inter-area mode, the former in the range of 1–3 Hz, and the latter in the range of 0.3–0.8 Hz.

Chapter 4 discusses the sub-synchronous resonance issue. These are not low frequency oscillations but high frequency ($\approx$ 30 Hz) oscillations due to long shafts and distributed rotor mass on these shafts. They are also called torsional oscillations and may occur due to interaction with the network L-C parameters. Techniques to suppress these are quite critical.

Chapter 5 discusses flexible ac transmission systems devices. Because of the excellent reliability of high-power electronics that can be switched at high speeds, one can have variable compensation from bus to ground or between buses. This increases the stability as well as transmission capability of the network. The emphasis will be on development of block diagrams suitable for systems studies.

Chapter 6 briefly reviews the nonlinear multimachine power system model. The exciter models as well as load representation are discussed. The most popular form of the differential-algebraic equation (DAE) model of a multimachine system is given by

$$\dot{x} = f(x, V, u) \tag{1.3}$$

$$I(x, V) = YV \tag{1.4}$$

In this model, the so-called stator algebraic equations have been eliminated. x is the state vector representing the dynamic variables. I, V are the bus injected currents and bus voltages, respectively, in polar or rectangular form, respectively. u is the input vector typically consisting of the turbine power setting and voltage reference setting of the excitation system. Equation (1.4) can be converted to the power balance form by pre-multiplying (1.4) by $Diag(V_i^*)$ and then equating the real and imaginary parts of the resulting vector elements. Thus, (1.2) can be considered as a generic representation in either current or power balance form for the algebraic part of the DAE model. The reason for having the DAE form is the recognition of the fact that the dynamics of the state variables x is much slower than the dynamics in the network or the machine stators. Strictly, one should have (1.2) in the form

$$\epsilon \dot{y} = g(x, y) \tag{1.5}$$

where ϵ is a small parameter and y is an appropriately defined state vector. For small systems such as the single machine system, (1.1) and (1.5) can be solved using singular perturbation [1.3] or by the asymptotic expansion method. But in most system studies, ϵ is set equal to zero, thereby giving rise to the differential-algebraic system. The DAE model either in (1.1) and (1.2), or (1.3) and (1.4) form is rich in terms of nonlinear phenomena. Symbolically, we have a generic description of the form

$$\dot{x} = f(x, y, u) \tag{1.6}$$

$$0 = g(x, y, p) \tag{1.7}$$

p is a parameter vector in the algebraic part of the system and could represent loads or transmission line parameter. As the parameter vector p is varied, we can examine the eigenvalues of the linearized system. Depending on the modelling, either a real eigenvalue goes to the right half plane or a pair of complex eigenvalues goes to the right half plane. The former is called saddle-node bifurcation and the latter is called Hopf bifurcation. Saddle node bifurcation was first studied using only static models. Since the 1980's there has been a very extensive literature on these topics [1.6], [1.7]. Whereas in the 1960's, the interconnected nature of the system gave rise to low-frequency oscillation. In the 1980's the stressed nature of the system was responsible for the voltage instability phenomena. To study these phenomena, we need appropriate linear models. These are developed including the control devices such as the power system stabilizer (PSS) and FACTS controllers.

In Chapter 7, the general theory underlying selective eigenvalue computation is first discussed. A generalized procedure to compute eigenvalues of interest is explained. It also turns out that this generalized procedure is the starting point of the analysis of essentially spontaneous oscillations in power systems (AESOPS) algorithm and later enhanced as the program for eigenvalue analysis of large systems (PEALS) program. We explain the general Newton-based algorithm and, as a special case, explain the PEALS algorithm. The motivation for selective eigenvalue analysis is that in the case of low-frequency oscillations one would like to associate a particular eigenvalue with a particular machine. Also, this area of interest is closely related to the selective modal analysis (SMA) that is used in obtaining reduced order models [1.4].

The manuscript is organized in three parts.

I.	Chapters 2 & 3	Synchronous machine model and low-frequency oscillations
II.	Chapters 4 & 5	SSR and FACTS controllers
III.	Chapters 6 & 7	Multimachine small signal analysis, dynamic voltage stability, and selective eigenvalue analysis

The notation in each of the three parts may not be consistent with the other parts. Hence, the three parts can be read independently.

1.3 References

[1.1] P. M. Anderson and A. A. Found, *Power System Control and Stability*, Iowa State University Press, 1977, and IEEE Press, 2002.

[1.2] K. R. Padiyar, *Power System Dynamics.* (Second edition) B. S. Publications, Hyderabad, India, 2002. (First edition) John Wiley & Sons, Singapore, 1996.

[1.3] P. W. Sauer and M. A. Pai, *Power System Dynamics and Stability*, Prentice Hall, NJ, 1998, and Pearson Education, India, 2001.

[1.4] P. Kundur, *Power System Stability and Control*, EPRI Power System Engineering Series, McGraw Hill, New York, 1994.

[1.5] J. Machowski, J. W. Bialek, and J. R. Bumby, *Power System Dynamics and Stability*, John Wiley & Sons, England, 1997.

[1.6] C. W. Taylor, *Power System Voltage Stability*, EPRI Power System Engineering Series, McGraw Hill, New York, 1994.

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Chapter 2

OVERVIEW OF SYNCHRONOUS MACHINE MODELS: A Physical Interpretation

2.1 Introduction

Synchronous machine models have many versions. Almost all of these are based on Park's model first proposed in 1929 [2.1]. The variations are mostly in details, mainly in sign conventions and in the representation of damper windings. Blondel's two-reaction theory [2.2] was indeed a breakthrough in the representation of a synchronous machine, and Park's transformation owes its origin to the two-reaction theory. The main problem in the transient analysis of an electrical machine is that the inductances of the coils are functions of rotor position.

In an alternator, for example, Figure 2.1, we may describe the transient voltage/current relationship of the three stator coils a, b, and c and the field coil f as:

$$\begin{aligned}
V_a &= R_a i_a + p(L_a i_a) + p(M_{ab} i_b) + p(M_{ac} i_c) + p(M_{bf} i_f) \\
V_b &= R_b i_b + p(L_b i_b) + p(M_{ba} i_a) + p(M_{bc} i_c) + p(M_{bf} i_f) \\
V_c &= R_c i_c + p(L_c i_c) + p(M_{ca} i_a) + p(M_{cb} i_b) + p(M_{cf} i_f) \\
V_f &= R_f i_f + p(L_f i_f) + p(M_{fa} i_a) + p(M_{fb} i_b) + p(M_{fc} i_c) \qquad (2.1)
\end{aligned}$$

($p = d/dt$, L stands for self-inductance of a coil and M stands for the mutual