SUPPLEMENT TO

A PROGRAMED COURSE IN CALCULUS

David M. Merriell

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The Committee on Educational Media of The Mathematical Association of America

I: Functions, Limits, and the Derivative
II: The Definite Integral
III: Transcendental Functions
IV: Applications and Techniques of Integration
V: Infinite Sequences and Series

PREFACE

In recent years, much attention has been given to the question of the mathematics curriculum in the schools and colleges. At least one major project has concentrated on new teaching methods for school mathematics. However, no comparable effort has been made in college mathematics. The pattern of teaching is remarkably uniform. The instructor lectures on subject matter which is more or less fully developed in a textbook, assigns exercises for the student to work, and gives examinations consisting largely of questions similar to the exercises. The instructor usually assumes that the student will not, and indeed cannot, read the textbook with understanding. The student comes to view the course as a collection of exercise-solving techniques. To that end, the textbook is of partial usefulness, particularly in so far as worked examples provide patterns of solution.

If some teachers of mathematics experienced a mild uneasiness about this state of the art, it was compounded by the post-World War II college population explosion. Faced with a relatively inelastic supply of teachers, together with the trend towards decreasing teaching loads in order to compete for the limited supply, many colleges resorted to large lecture classes. Formerly, the gap between instructor and student was often closed by out-of-class consultation or in-class questioning and cross-examination. The large lectures, with their depersonalization of the instructor-student relation, inhibited this type of communication.

Not only did the new generation of students arrive in increasing numbers, but it was also the first generation to be reared on television, conditioned to a predominantly passive and visual form of learning about the world. The passive mode was extended into college education by the large lecture system. However, the mathematics teachers continued to demand an active, individualistic, and intellectual style of behavior. At a time when even more interaction between teacher and student was needed in order to combat the passive conditioning, the bonds joining the two had become weaker.

The Committee on Educational Media of the Mathematical Association of America (CEM) was formed in recognition of advances in technology and the new set of conditions, requiring new teaching methods, which had arisen in the colleges. The charge to CEM was to conduct and encourage experimentation in this area. Programed learning was to be explored as one promising avenue for such experimentation.

A major contribution of programed learning lies in the active involvement of the student at every point in the learning process. In the programed course the student does not listen passively. He reads and studies the text and, because it is a programed text, his comprehension is continually checked. The need for the instructor is not removed, but his role changes. He is available for answering questions, made more meaningful by the prior participation of the student, and he helps the student to get the overall picture. In fact, the teacher should have firmly under his control the organization of the course. It is up to him to program the presentation of the course itself and to synthesize its details.

The objective of A Programed Course in Calculus is to present a course which is completely modern in its treatment; which does not evade the pedagogical difficulties but faces them squarely; which capitalizes on the gains which have been made in the revision of school mathematics; and which brings a new technique of instruction to a new era of education.

David M. Merriell

Santa Barbara, California
August 1968

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PART I
INTRODUCTION

INTRODUCTION

1. Who should use these books?

The Programed Course in Calculus is adaptable to a wide variety of situations. Several are listed here.

One use is in large universities with large lecture classes. In a situation in which three lectures are given weekly plus one discussion section, the programed course can be conducted with three discussion meetings and no lectures. In order to conserve staff, the discussion sections may be larger than at present. However, junior instructors can handle these sections so that no overall increase in staff is necessary. A department which wishes to experiment with such a system can begin with a small pilot class.

Another situation in which the programed course can be used to advantage is in junior colleges and small colleges with limited teaching resources. Since the program replaces the lecture, the instructor's role approaches that of a tutor. The course can be conducted by an apprentice instructor under the supervision of a senior faculty member.

The programed course is also well suited for use in Advanced Placement mathematics courses. All topics of the Calculus AB course are included in these volumes. The only topics of the Calculus BC course which are not included are vectors, parametric representation of curves, curvature, arc length, areas of surfaces of revolution, and elementary differential equations.

Finally, these books may be used for correspondence courses and by individuals for independent study. Experience has shown that few persons have the self-discipline to pursue a complete course in programed form without external supervision. In the correspondence course, such supervision is provided. For a full-time student taking a calculus course from a standard textbook, time is not usually available to pursue the complete programed course simultaneously. However, parts of it can be used as supplements in this situation, with the instructor selecting those sections which in his opinion will contribute to a deeper understanding and greater mastery of skills.

2. Ways of using the books

The Programed Course assumes that the student has studied the elements of analytic geometry and trigonometry. The instructor who uses it as the textbook for a class which has not studied analytic geometry must present supplementary material at certain points. A brief review of trigonometry is given in Chapter E.

When the Programed Course is being used as the textbook for a complete course, a course outline such as the one suggested in this Introduction should be followed. In some cases, and particularly for use in high schools, the students may work on the program during class meetings. There is wide variation in the time which different students use in working programs. A section of a program requiring a class period for an average student may be completed in half the time by some whereas others may not finish it. In college courses, it is assumed that the normal practice will be to have students work on the program outside of class meetings. During the class periods, the instructor may give short written or verbal quizzes, answer questions, supplement the text by additional examples and related topics, and have students work on supplementary problems.

Some of the programed units may be used for a part of a complete course, with the remainder of the course given in the conventional manner. This has the advantage over the traditional course of giving a change of pace and variety to the learning experience. Also, some instructors may wish to experiment with programed materials by using only one part of the course at first so that they can get the feel of this new mode of instruction.

When using the Programed Course to supplement a course given from a standard textbook, the instructor must correlate the two and make it clear to the student which sections of the program would be of the most benefit.

For best results, an individual using the programed course for independent study should follow a course outline systematically.

3. Course outline

The suggested outline is for a two-semester course meeting three hours each week. In this outline, Chapters A through D are covered in the first semester, giving an introduction to differentiation and (more briefly) to integration. Chapters E through H are covered in the second semester. Under the quarter system, Chapters A and B would be covered in the first quarter, C through F.4 in the second quarter, and the remainder of F plus G and H in the third quarter. The outline leaves some unallocated time in the second semester which can be spent on such topics from analytic geometry as conic sections, vectors, and polar coordinates, or on other topics depending on local needs and the wishes of the instructor.

First Semester Outline

1. A.1, Frames 1 – 54
2. Finish A.1
3. A.2; A.3, Frames 1 – 16
4. Finish A.3
5. A.4
6. A.5
7. A.6
8. A.7, Frames 1 – 36
9. Finish A.7
10. A.8
11. A.9
12. A.10
13. Review Chapter A
14. Test
15. B.1
16. B.2; B.3
17. B.4; B.5, Frames 1 – 15
18. Finish B.5
19. B.6
20. B.7
21. B.8; B.9
22. B.10
23. B.11; B.12, Frames 1 – 11
24. Finish B.12
25. B.13
26. Review Chapter B
27. Test
28. C.1; C.2, Frames 1 – 28
29. Finish C.2; C.3, Frames 1 – 28
30. Finish C.3
31. C.4
32. C.5; C.6, Frames 1 – 11
33. Finish C.6; C.7
34. D.1
35. D.2
36. D.3
37. D.4
38. D.5
39. D.6
40. D.7

41. D.8
42. Review Chapters C and D
43. Final Examination

Second Semester Outline

1. E.1; E.2
2. E.3
3. E.4
4. E.5; E.6
5. E.7; E.8
6. E.9
7. E.10
8. Review Chapter E
9. Test
10. F.1, Frames 1 – 26
11. Finish F.1; F.2
12. F.3
13. F.4
14. F.5
15. F.6
16. F.7
17. F.8
18. Review Chapter F
19. Test
20. G. 1; G.2; G.3
21. G.4
22. G.5
23. G.6
24. G.7
25. G.8; G.9, Frames 1 – 6
26. Finish G.9
27. G.10
28. Review Chapter G
29. Test
30. H.1
31. H.2
32. H.3
33. H.4
34. H.5
35. H.6
36. Review Chapter H
37. Final Examination

4. Suggestions to students

In addition to the directions to the student at the beginning of each volume, further advice as to study methods should be given as the student proceeds through the program. Too long a session with a programed text tends to produce boredom and fatigue. For most students, an hour is the longest period which will produce good results. If the assigned section cannot be completed in that time, the student should return to it later.

Students should make a note of their erroneous responses so as to avoid repeating their errors. This practice will pinpoint the parts of the program which caused difficulty for purposes of review. If the point is still unclear after studying the correct response and rereading the frame, it should be brought up in class. Students should write out problems and exercises in presentable form in order to develop good mathematical style. At the end of a proof in the program, the student should review the proof, locate its central idea, and try to reproduce the proof in writing.

When references to earlier frames are given, it is usually a good idea to look up the reference. At the end of each chapter, the student should make a detailed outline of the contents of the chapter. He should practice writing out the definitions and the statement of theorems.

5. References

Eight calculus textbooks, selected from the many excellent books on the market, are listed here. There follows a table correlating A Programed

Course in Calculus with each one of these. References are to chapter and section; for example, 6.7 indicates Chapter 6, Section 7. The listing does not imply a close correspondence between the Programed Course and the reference book, but only that the same topic occurs at the cited places.

As the table indicates, the Programed Course deviates from the standard texts for first-year calculus in two principal places, namely in Chapters C and H. Whereas standard practice is merely to state the fundamental theorems about continuous functions, this course develops the proofs from the properties of the real numbers. Chapter H presents the properties of sequences of real numbers much more thoroughly than do most calculus texts. The subject of sequences and series is generally postponed to the sophomore course.

Apostol, T. M., Calculus, Vol. I, 2nd ed. New York: Blaisdell, 1967.

De Leeuw, K., Calculus. New York: Harcourt, Brace and World, 1966.

Fisher, R. C., and Ziebur, A. D., Calculus and Analytic Geometry, 2nd ed. Englewood Cliffs, New Jersey: Prentice-Hall, 1965.

Johnson, R. E., and Kiokemeister, F. L., Calculus with Analytic Geometry, 3rd ed. Boston: Allyn and Bacon, 1964.

Protter, M. H., and Morrey, C. B., Calculus with Analytic Geometry: A First Course. Reading, Massachusetts: Addison-Wesley, 1963.

Schwartz, A., Calculus and Analytic Geometry, 2nd ed. New York: Holt, Rinehart and Winston, 1967.

Spivak, M., Calculus. New York: W. A. Benjamin, 1967.

Thomas, G. B., Calculus and Analytic Geometry, 4th ed. Reading, Massachusetts: Addison-Wesley, 1968.

Programed Course	Apostol	De Leeuw	Fisher & Ziebur	Johnson & Kiokemeister	Protter & Morrey	Schwartz	Spivak	Thomas
A.1	I2, 1.8	–	1.2	1.2	–	–	3	–
A.2	1.2, 1.3	1.2	1.5	3.1	2.1, 2.2	1.1, 1.2	3	1.6
A.3	1.3	1.2	1.4	3.3	3.2, 3.4	–	4	1.6
A.4	–	1.4	–	3.4	–	–	3	1.7
A.5	3.7	1.4	1.7	3.4	–	1.7	3	3.5
A.6	3.12	5.2	6.45	10.6	11.4	4.4	12	3.3
A.7	3.2	1.5, 9.2	1.10	4.1, 4.2	4.1, 5.1	7.1	5	2.1
A.8	3.3	1.5, 9.3	1.10	4.4	5.3	7.3	6	3.9
A.9	3.4	9.2	1.11	4.7	5.2	7.2	5	2.2, 2.3
A.10	7.14, 7.15	–	13.109	4.5, 4.6	5.4	7.2	5	2.4
B.1	4.2	2.2	2.14	6.7	4.5	1.4	9	1.11
B.2	4.7	2.1	2.12	5.2	4.4	1.3	9	1.9
B.3	–	–	–	–	–	–	–	–
B.4	4.3	2.3	2.13	5.1	4.3	1.5	9	1.10
B.5	4.5	2.6	2.17	5.5	6.1	1.6, 1.8	10	3.1
B.6	4.5	2.6, 9.3	2.17	5.3	6.1	1.8, 7.3	10	3.2
B.7	4.10	2.7	2.16	5.6	6.2	1.7	10	3.6
B.8	4.8	–	–	5.10	–	–	9	3.8
B.9	–	4.6	3.25	5.2, 5.10	7.9	8.3	–	3.7
B.10	4.13	4.1	2.22	6.1, 6.3, 6.5	7.5	3.18	11	4.5
B.11	4.14	9.4	2.19	6.1	7.2	7.5	11	4.7, 4.8
B.12	4.16-4.18	4.2, 4.3	2.21	6.2, 6.4	7.3	3.19	11	4.4
B.13	4.11, 4.20	4.4, 4.5	2.23, 2.24	6.6	7.6	8.1, 8.2	–	4.2, 4.6
C.1	–	–	–	–	–	–	–	–
C.2	I3.8	–	–	–	–	–	–	–
C.3	I3.9	9.1	–	7.1	–	–	8	–
C.4	1.16	–	–	7.4	–	–	8	–
C.5	3.10	9.3	–	7.7	8.6	7.4	7	3.9
C.6	3.17	–	–	13.8	8.3	7.4	–	3.9
C.7	3.16	9.3	2.19	13.1	–	7.4	7	3.9
D.1	1.6	3.1	5.32	7.3	4.6, 8.1	2.4	13	2.5
D.2	1.16, 1.17	–	–	7.4	8.2	2.4	13	5.6
D.3	1.16	3.3, 9.5	5.33	7.5	8.3	2.4	13	5.9
D.4	1.21, 3.18	–	5.34	7.5	8.3	7.6	13	5.12
D.5	3.19	–	5.34	7.7	8.4	7.7	13	5.8
D.6	5.3	3.4, 3.5	5.37	7.6	8.5	2.6, 7.7	14	5.9

Programed Course	Apostol	De Leeuw	Fisher & Ziebur	Johnson & Kiokemeister	Protter & Morrey	Schwartz	Spivak	Thomas
D.7	–	–	–	–	–	–	–	6.5
D.8	10.23	–	13.110	13.5	16.5	8.13	14	9.10
E.1	–	–	–	–	–	–	–	–
E.2	6.2, 6.3	5.4	6.44	9.6	11.7	5.2, 5.6	17	7.4, 7.6, 7.7
E.3	6.3, 6.7, 6.8	5.5	6.44	9.7	11.9	5.3	17	7.5
E.4	6.12–6.14	5.4	6.46	9.1, 9.4	11.8	5.4	17	7.8
E.5	6.15	5.4	6.48	9.1, 9.2	11.9	–	17	7.9
E.6	6.6	5.4	6.48	9.8	11.7	5.3	–	7.9
E.7	2.7	1.3	1.6	–	9.5, 9.6	4.1	15	5.4
E.8	4.4	5.1	2.18	10.2, 10.3	11.2	4.2, 4.3	15	5.5, 7.1
E.9	6.21	5.3	6.49, 6.50	10.4, 10.5	11.5	4.4	15	7.2, 7.3
E.10	6.18	–	6.52	9.5	11.12, 11.13	5.5	17	8.2–8.5
F.1	2.2	3.1	5.39	6.10	8.8	2.7	–	5.8, 5.9, 6.2
F.2	–	3.1	5.36	8.7, 8.8	16.14	6.8	–	5.10
F.3	2.14	3.2	5.42	8.5	8.9	2.8	–	6.13
F.4	–	–	–	–	8.9	–	–	6.13
F.5	2.12	3.6	5.41	8.4	16.3	12.2	–	6.4
F.6	–	–	–	–	16.4	12.3	–	6.4
F.7	–	–	12.101	12.4, 12.5	16.9, 16.10	8.11	–	6.9, 6.10
F.8	–	–	–	–	–	–	–	–
G.1	–	–	–	–	–	–	–	–
G.2	5.3	6.1	5.37	6.8	4.6	2.1	18	5.2
G.3	5.6	6.1	–	6.8	8.7	2.6	18	5.2
G.4	5.7	6.2	7.54	6.9	15.1	2.6	18	9.1
G.5	5.9	6.3	7.59	11.2	15.6	6.6	18	9.7
G.6	–	–	7.57	–	15.3	6.2	18	9.2, 9.3
G.7	–	–	–	11.3	15.4	6.3	18	9.4
G.8	5.7	–	7.55	–	–	–	–	9.9
G.9	6.23	6.4	7.58	11.4	15.7	6.4	18	9.6
G.10	–	–	–	–	–	6.7	–	–
H.1	10.2	–	14.113	8.1	5.5	13.1	21	18.1
H.2	10.2	–	14.113	8.1	5.5	13.2	21	18.1
H.3	–	–	–	–	–	–	–	–
H.4	10.3	–	14.113	–	–	13.1	21	–
H.5	10.3	–	14.113	–	–	13.1	21	–
H.6	10.5	–	14.114	–	–	13.3	21	–

6. Supplementary list of calculus textbooks

Ayres, F., Theory and Problems of Differential and Integral Calculus, 2nd ed. New York: Schaum, 1964.

Begle, E. G., Introductory Calculus with Analytic Geometry. New York: Holt, Rinehart & Winston, 1954.

Breusch, R., Introduction to Calculus and Analytic Geometry. Boston: Prindle, Weber and Schmidt, 1966.

Courant, R., and John, F., Introduction to Calculus and Analysis, Vol. I. New York: Interscience, 1965.

Lang, S., A First Course in Calculus. Reading, Massachusetts: Addison-Wesley, 1964.

Moise, E. E., Calculus. Reading, Massachusetts: Addison-Wesley, 1967.

Olmsted, J. M. H., Calculus with Analytic Geometry, Vol. I. New York: Appleton-Century-Crofts, 1966.

Ostrowski, A., Differential and Integral Calculus. Glenview, Illinois: Scott, Foresman, 1968.

Stein, S., Calculus in the First Three Dimensions. New York: McGraw-Hill, 1967.

Taylor, H. E., and Wade, T. L., University Calculus. New York: Wiley, 1962.

Wilf, H. S., Calculus and Linear Algebra. New York: Harcourt, Brace and World, 1967.

7. Bibliography for programed learning

Teachers who are unfamiliar with programed learning or those who wish to know more about its development and the results of research may benefit by reading the following pamphlets, each of which contains a bibliography.

May, K., Programed Learning and Mathematical Education. San Francisco: Committee on Educational Media, 1964.

Silverman, R. E., How to Use Programed Instruction in the Classroom. Cambridge, Massachusetts: Honor Products Co., 1967.

The first of these is frank in its criticism of programs in mathematics available at the time of writing and in discussing the limitations of programed instruction. Although the author is skeptical of the possibility of complete college courses appearing in programed form, he urges further experimentation by mathematicians. In his terminology, A Programed Course in Calculus is written in hybrid style.

Although the second pamphlet is oriented towards the elementary and secondary schools, it deals with some of the practical questions which arise when using programed instruction at any level.

In addition to these pamphlets, the following references on programed learning bear directly on mathematical education.

Heimer, R. T., Designs for Future Explorations in Programed Instruction. Math. Teacher, 1966, 59, 110 – 114.

Kalin, R., Some Guidelines for Selecting a Programed Text in Mathematics. Math. Teacher, 1966, 59, 14 – 23.