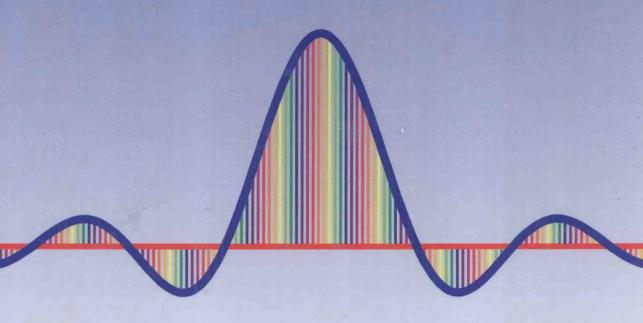
# LINEAR SYSTEMS AND SIGNALS

B.P. LATHI



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### Preface

In this book I have attempted a comprehensive treatment of signals and linear systems at an introductory level. The book emphasizes the physical appreciation of concepts rather than mere mathematical manipulation of symbols. There is a tendency to treat engineering subjects, such as this, as a branch of applied mathematics. Such an approach ignores the physical meaning behind various derivations and deprives a student of intuitive understanding of the subject. Here I have used mathematics not so much to prove axiomatic theory as to enhance physical and intuitive understanding. Wherever possible, theoretical results are interpreted heuristically and are supported by carefully chosen examples and analogies.

#### Organization

For convenience, the book is divided into five parts:

- 1. Introduction.
- 2. Time-domain analysis of linear time-invariant (LTI) systems.
- 3. Frequency-domain (transform) analysis of LTI systems.
- 4. Signal Analysis.
- 5. State-space analysis of LTI systems.

The organization of the book permits great deal of flexibility in teaching the continuous-time and discrete-time concepts. The natural sequence of chapters is meant to integrate continuous-time and discrete-time analysis. It is also possible to use a sequential approach in which all the continuous-time analysis is covered first, followed by discrete-time analysis.

#### **Notable Features**

The notable features of the book include the following:

- 1. Intuitive and heuristic understanding of the concepts and physical meaning of mathematical results are emphasized throughout the book. Such an approach leads to deeper appreciation and easier comprehension of the concepts. Most reviewers of the book have noted the unusual clarity of presentation. As one reviewer put it, "I believe the strongest point of this book is the author's ability to describe very difficult material in a very clear and simple way."
- Many students are handicapped by an inadequate background in basic material such as complex numbers, sinusoids, sketching and mathematically describing functions, Cramer's rule, partial fraction expansion, and matrix algebra. I have

- added a chapter that addresses these basic and pervasive topics in electrical engineering. Response by student has been unanimously enthusiastic.
- 3. There are over 200 worked examples along with exercises (with answers) for students to test their understanding. There is also a large number of selected problems of varying difficulty at the end of each chapter. Many problems are provided with hints to steer a student in the proper direction.
- 4. For those who like to get students involved with computers, computer solutions of several examples are provided using Matlab, which is becoming a standard software package in an electrical engineering curriculum. The problem set also contains several computer problems. Use of computer examples or problems, however, is not essential for the use of this book.
- 5. The discrete-time and continuous-time systems may be treated in sequence, or they may be integrated by using a parallel approach.
- 6. The summary at the end of each chapter proves helpful to students in summing up essential developments in the chapter.
- 7. There are several historical notes to enhance student's interest in the subject. These facts introduce students to historical background that influenced the development of electrical engineering.

#### Suggestions for Using This Book

The book can be readily tailored for a variety of courses of 30 to 60 lecture hours or more. Most of the material in the first eight chapters can be covered in about 43 hours (including tests). To treat continuous and discrete-time systems by using an integrated (or parallel) approach, the appropriate sequence of chapters is 1, 2, 3, 4, 5, 6, 7, and 8. For a sequential approach, where the continuous-time analysis is followed by discrete-time analysis, the proper chapter sequence is 1, 2, 4, 3, 5, 6, 7, and 8.

Logically, the Fourier transform should precede the Laplace transform. I tried such an order in my previous books. In this book, however, I have reversed this order because experience shows that "Laplace before Fourier" goes better for most of the students. The order can be reversed, however, if the instructor so desires.

#### A Typical Course in Signals and Systems

A typical course in signals and systems may be divided into three roughly equal parts:

- 1. Time-domain analysis of LTI systems (Chapters 1, 2, 3).
- 2. Frequency-domain (Transform) analysis of LTI systems (Chapters 4, 5).
- 3. Signal analysis [Chapters 6, 7, 8, and 9 (optional)].

Each of the three parts can be covered in roughly 14 hours (including tests). I have used this sequence successfully, where the first eight chapters were covered with the following omissions:

- Chapter 1: Sections 1.3-2, 1.3-3, 1.4, 1.5, and 1.6.
- Chapter 2: Section 2.8. The results of Sec. 2.6-1 were stated but not proved. Students were assigned Sec. 2.7 for self study.
- Chapter 3: Section 3.7. The results of section 3.6-1 were stated but not proved.
- Chapter 4: Section 4.8.

Chapter 5: Sections 5.6 and 5.7.

Chapter 6: Section 6.3.

Chapter 7: Nothing omitted.

Chapter 8: Section 8.3.

#### Testing

Because the continuous-time and the discrete-time concepts are so similar, there is an understandable tendency on the part of students to confuse them. To minimize this difficulty, I find it helpful to test students more often. One test is given on each of these chapters: 2, 3, 4, 5, 6, and one test is given on Chapters 7 and 8 (six tests in all). The test for Chapter 6 (Fourier series) is shorter and is weighed at only 50% of the tests on the other tests. The homework is assigned the weight of one full test. Giving six tests may appear too demanding a task for an instructor. However, because of similarity of the continuous-time and discrete-time concepts, it is possible to make tests shorter (quiz) and to cover all the topics in pairs of chapters. For example, the time-domain analysis concepts in Chapters 2 (continuous-time systems) and 3 (discrete-time systems) are almost the same. Hence half the topics may be tested in Chapter 2, and the remaining half in Chapter 3. The case with Chapters 4 and 5 is similar. Thus, the six tests need not make more demands on the instructor than three or four tests given normally.

#### Credits

The photographs of Gauss (p. 5), Laplace (p. 263), Heaviside (p. 263), Fourier (p. 436), Michelson (p. 462) have been reprinted courtesy of the Smithsonian Institution. The photographs of Cardano (p. 5) and Gibbs (p. 462) have been reprinted courtesy of the Library of Congress.

The MATLAB examples and problems were prepared by Prof. Rory Cooper. In the second printing, Prof. O. P. Mandhana of University of Kentucky (Lexington) thoroughly revised or modified these programs partly by fixing the bugs, or adding some extra parts, or by a better usage of MATLAB syntax or programming. Some of the examples have been changed completely, and some examples are implemented using an alternate (simpler) method.

#### Acknowledgments

Several individuals have helped me in the preparation of this book. I am grateful for the helpful suggestions of the reviewers Professors. K.S. Arun (University of Illinois, Urbana), J.B. Cruz (University of California, Riverside), Kevin Donahue (Kentucky), Gary Ford (University of California, Davis), K.S.P. Kumar (Minnesota), Amy Reibman (Princeton), James Thorpe (Cornell), H. Valenzuela (Penn State), R. Yantorno (Temple). Prof. D. Arvind made many important and useful suggestions to improve the pedagogy of the book. Special thanks go to Paul Hinz, Professor Anne-Louise Radimsky, and Professor Isaac Ghansah (computer assistance), Professor Armand Seri (computer plots), and Joseph Coniglio (cartoons) for their generous help. The beautiful illustrations were prepared by Robert Griswold, James Roberts, Benjamin Park, and Mike Alie. The cover and graphic design is by Mike Alie. Typesetting is by Qin He. Other individuals who helped in a variety of ways include Tej Pandey, Shafique Rahman, Bharat Singh, Ernie Claussen,

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B. P. Lathi

#### Matlab

Throughout this book, examples have been provided to familiarize the reader with computer tools for systems design and analysis using the powerful and versatile software package MATLAB. Much of the time and cost associated with the analysis and design of systems can be reduced by using computer software packages for simulation. Many corporations will no longer support the development systems without prior computer simulation, and numerical results which suggest a design will work. The examples and problems in this book will assist the reader in learning the value of computer packages for systems design and simulation.

MATLAB is the software package used throughout this book. MATLAB is a powerful package developed to perform matrix manipulations for system designers. MATLAB is easily expandable, and uses its own high level language. This makes developing sophisticated systems easier. In addition, MATLAB has been carefully written to yield numerically stable results to produce reliable simulations.

All the computer examples in this book are verified to work with the student edition of the MATLAB when used according to the instructions given in its manual. Make sure that \MATLAB\BIN is added in the DOS search path. MATLAB can be invoked by executing the command MATLAB. The MATLAB banner will appear after a moment with the prompt '>>'. MATLAB has a useful on-line help. To get help on a specific command, type HELP COMMAND NAME and then press the ENTER key. DIARY FILE is a command to record all the important keyboard inputs to a file and the resulting output of your MATLAB session to be written on the named file. MATLAB can be used interactively, or by writing functions (subroutines) often called M files because of the .M extension used for these files. Once familiar with the basics of MATLAB, it is easy to learn how to write functions and to use MATLAB's existing functions.

Rory Cooper O. P. Mandhana

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