

AUTOMATION AND CONTROL ENGINEERING SERIES

Linear Control Theory

Structure, Robustness,
and Optimization

Shankar P. Bhattacharyya
Aniruddha Datta
L. H. Keel



CRC Press
Taylor & Francis Group

TP273
B575

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and Optimization**



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E2009002639



CRC Press

Taylor & Francis Group

Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **Informa** business

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

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CRC Press is an imprint of Taylor & Francis Group, an Informa business

No claim to original U.S. Government works
Printed in the United States of America on acid-free paper
10 9 8 7 6 5 4 3 2 1

International Standard Book Number-13: 978-0-8493-4063-5 (Hardcover)

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Library of Congress Cataloging-in-Publication Data

Datta, Aniruddha, 1963-

Linear control theory : structure, robustness, and optimization / authors,
Aniruddha Datta, Lee H. Keel, Shankar P. Bhattacharyya.

p. cm. -- (Automation and control engineering)

A CRC title.

Includes bibliographical references and index.

ISBN 978-0-8493-4063-5 (hardcover : alk. paper)

I. Linear control systems. I. Keel, L. H. (Lee H.) II. Bhattacharyya, S. P.
(Shankar P.), 1946- III. Title. IV. Series.

TJ220.D38 2009

629.8'32--dc22

2008052001

Visit the Taylor & Francis Web site at
<http://www.taylorandfrancis.com>

and the CRC Press Web site at
<http://www.crcpress.com>

Linear Control Theory

**Structure, Robustness,
and Optimization**

AUTOMATION AND CONTROL ENGINEERING

A Series of Reference Books and Textbooks

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DEDICATION

To my Guru, Ustad (Baba) Ali Akbar Khan, the greatest musician in the world. Baba opened my eye to Nada Brahma through the divine music of the Seni Maihar Gharana.

S.P. Bhattacharyya

To My Beloved Wife Anindita

A. Datta

To My Beloved Wife Kuisook

L.H. Keel

PREFACE

This book describes three major areas of Control Engineering Theory. In Part I we develop results directed at the design of PID and first order controllers for continuous and discrete time linear systems possibly containing delays. This class of problems is crucially important in applications. The main features of our results are the computation of complete sets of controllers achieving stability and performance. They are developed for model based as well as measurement based approaches. In the latter case controller synthesis is based on measured responses only and no identification is required. The results of Part I constitute a modernized version of Classical Control Theory appropriate to the computer-aided design environment of the 21st century.

In Part II we deal with the Robust Stability and Performance of systems under parametric as well as unstructured uncertainty. Several elegant and sharp results such as Kharitonov's Theorem and its extensions, the Edge Theorem and the Mapping Theorem are described. The main thrust of the results is to reduce the verification of stability and performance over the entire uncertainty set to certain extremal test sets, which are points or lines. These results are useful to engineers as aids to robustness analysis and synthesis of control systems.

Part III deals with Optimal Control of linear systems. We develop the standard theories of the Linear Quadratic Regulator (LQR), H_∞ and ℓ^1 optimal control, and associated results. In the LQR chapter we include results on the servomechanism problem.

We have been using this material successfully in a second graduate level course in Control Systems for some time. It is our opinion that it gives a balanced coverage of elegant mathematical theory and useful engineering oriented results that can serve the needs of a diverse group of students from Electrical, Mechanical, Chemical, Aerospace, and Civil Engineering as well as Computer Science and Mathematics. It is possible to cover the entire book in a 14-week semester with a judicious choice of reading assignments.

Many of the results described in the book were obtained in collaboration with our graduate students and it is a pleasure to acknowledge the many contributions of P.M.G. Ferreira, Herve Chapellat, Ming-Tzu Ho, Guillermo J. Silva, Hao Xu, Sandipan Mitra, and Richard Tantarís.

Part I contains material published in the earlier monograph *PID Controllers for Time-Delay Systems* by Guillermo J. Silva, A. Datta, and S.P. Bhattacharyya, Birkhaueser, 2005. Much of the material of Part II appeared in the earlier book *Robust Control: The Parametric Approach* by S.P. Bhat-

tacharyya, H. Chapellat and L.H. Keel, Prentice Hall, 1995. A.D. would like to thank Professor M. G. Safonov of the University of Southern California for teaching him the basics of H_∞ control theory almost two decades ago. Indeed, a lot of the material in Part III of this book is based on a Special Topics course taught by Professor Safonov at USC in the Spring of 1990. The authors would also like to thank Dr. Nripendra Sarkar, Dr. Ranadip Pal, Ms. Rouella Mendonca, and Mr. Ritwik Layek for assistance with Latex and figures on several occasions.

A book of this size and scope inevitably has errors and we welcome corrective feedback from the readers. We also apologize in advance for any omissions or inaccuracies in referencing and would want to compensate for this in future editions.

S. P. Bhattacharyya
A. Datta
L. H. Keel

June 23, 2008
College Station, Texas
USA

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