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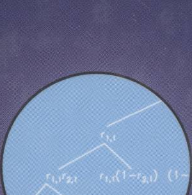
Integration of Chaos and Random Fractal Theory, and Beyond



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JIANBO GAO • YINHE CAO • WEN-WEN TUNG • JING HU

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Integration of Chaos and Random Fractal Theory, and Beyond

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MULTISCALE ANALYSIS OF COMPLEX TIME SERIES

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To our teachers,
Xianyi Zhou of Zhejiang
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Yilong Bai and Zhemin
Zheng of Chinese Academy
of Sciences,
Michio Yanai of UCLA,
and to our families

PREFACE

Complex interconnected systems, including the Internet, stock markets, and human heart or brain, are usually comprised of multiple subsystems that exhibit highly nonlinear deterministic as well as stochastic characteristics and are regulated hierarchically. They generate signals that exhibit complex characteristics such as nonlinearity, sensitive dependence on small disturbances, long memory, extreme variations, and nonstationarity. A complex system usually cannot be studied by decomposing the system into its constituent subsystems, but rather by measuring certain signals generated by the system and analyzing the signals to gain insights into the behavior of the system. In this endeavor, data analysis is a crucial step. Chaos theory and random fractal theory are two of the most important theories developed for data analysis. Unfortunately, no single book has been available to present all the basic concepts necessary for researchers to fully understand the ever-expanding literature and apply novel methods to effectively solve their signal processing problems. This book attempts to meet this pressing need by presenting chaos theory and random fractal theory in a unified way.

Integrating chaos theory and random fractal theory and going beyond them has proven to be much harder than we had thought, because the foundations for chaos theory and random fractal theory are entirely different. Chaos theory is mainly concerned about apparently irregular behaviors in a complex system that are generated by nonlinear deterministic interactions of only a few numbers of degrees of freedom, where noise or intrinsic randomness does not play an important role, while random fractal theory assumes that the dynamics of the system are inherently random. After postponing delivery of the book for more than two and half years, we are finally satisfied. The book now contains many new results in Chapters 8–15 that have not been published elsewhere, culminating in the development of a multiscale complexity measure that is computable from short, noisy time series. As shown in Chapter 15, the measure can readily classify major types of complex motions, effectively deal with nonstationarity, and simultaneously characterize the behaviors of complex signals on a wide range of scales, including complex irregular behaviors on small scales and orderly behaviors, such as oscillatory motions, on large scales.

This book has adopted a data-driven approach. To help readers better understand and appreciate the power of the materials in the book, nearly every significant concept or approach presented is illustrated by applying it to effectively solve real problems, sometimes with unprecedented accuracy. Furthermore, source codes, written in various languages, including Java, Fortran, C, and Matlab, for many methods are provided in a dedicated book website, together with some simulated and experimental data (see Sec. A.4 in Appendix A).

This book contains enough material for a one-year graduate-level course. It is useful for students with various majors, including electrical engineering, computer science, civil and environmental engineering, mechanical engineering, chemical engineering, medicine, chemistry, physics, geophysics, mathematics, finance, and population ecology. It is also useful for researchers working in relevant fields and practitioners who have to solve their own signal processing problems.

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CHAPTER 1

INTRODUCTION

Complex systems are usually comprised of multiple subsystems that exhibit both highly nonlinear deterministic and stochastic characteristics and are regulated hierarchically. These systems generate signals that exhibit complex characteristics such as sensitive dependence on small disturbances, long memory, extreme variations, and nonstationarity. A stock market, for example, is strongly influenced by multilayered decisions made by market makers, as well as by interactions of heterogeneous traders, including intraday traders, short-period traders, and long-period traders, and thus gives rise to highly irregular stock prices. The Internet, as another example, has been designed in a fundamentally decentralized fashion and consists of a complex web of servers and routers that cannot be effectively controlled or analyzed by traditional tools of queuing theory or control theory and give rise to highly bursty and multiscale traffic with extremely high variance, as well as complex dynamics with both deterministic and stochastic components. Similarly, biological systems, being heterogeneous, massively distributed, and highly complicated, often generate nonstationary and multiscale signals. With the rapid accumulation of complex data in health sciences, systems biology, nano-sciences, information systems, and physical sciences, it has become increasingly important to be able to analyze multiscale and nonstationary data.

Multiscale signals behave differently, depending upon the scale at which the data are examined. How can the behaviors of such signals on a wide range of scales be simultaneously characterized? One strategy we envision is to use existing theories synergistically instead of individually. To make this possible, appropriate scale ranges where each theory is most pertinent need to be identified. This is a difficult task, however, since different theories may have entirely different foundations. For example, chaos theory is mainly concerned about apparently irregular behaviors in a complex system that are generated by nonlinear deterministic interactions with only a few degrees of freedom, where noise or intrinsic randomness does not play an important role. Random fractal theory, on the other hand, assumes that the dynamics of the system are inherently random. Therefore, to make this strategy work, different theories need to be integrated and even generalized.

The second vital strategy we envision is to develop measures that explicitly incorporate the concept of scale so that different behaviors of the data on varying scales can be simultaneously characterized by the same scale-dependent measure. In the most ideal scenario, a scale-dependent measure can readily classify different types of motions based on analysis of short, noisy data. In this case, one can readily see that the measure will be able not only to identify appropriate scale ranges where different theories, including information theory, chaos theory, and random fractal theory, are applicable, but also to automatically characterize the behaviors of the data on those scale ranges.

The vision presented above dictates the style and the scope of this book, as depicted in Fig. 1.1. Specifically, we aim to build an effective arsenal by synergistically integrating approaches based on chaos and random fractal theory, and going beyond this, to complement conventional approaches such as spectral analysis and machine learning techniques. To make such an integration possible, four important efforts are made:

1. Wavelet representation of fractal models as well as wavelet estimation of fractal scaling parameters will be carefully developed. Furthermore, a new fractal model will be developed. The model provides a new means of characterizing long-range correlations in time series and a convenient way of modeling non-Gaussian statistics. More importantly, it ties together different approaches in the vast field of random fractal theory (represented by the four small boxes under the "Random Fractal" box in Fig. 1.1).

2. Fractal scaling break and truncation of power-law behavior are related to specific features of real data so that scale-free fractal behavior as well as structures defined by specific scales can be simultaneously characterized.

3. A new theoretical framework for signal processing — power-law sensitivity to initial conditions (PSIC) — will be developed, to provide chaos and random fractal theory a common foundation so that they can be better integrated.

4. The scale-dependent Lyapunov exponent (SDLE), which is a variant of the finite-size Lyapunov exponent (FSLE), is an excellent multiscale measure. We shall develop a highly efficient algorithm for calculating it and show that it can readily classify different types of motions, aptly characterize complex behaviors of real-world multiscale signals on a wide range of scales, and, therefore, naturally solve the classic problem of distinguishing low-dimensional chaos from noise. Furthermore, we shall show that the SDLE can effectively deal with nonstationarity and that existing complexity measures can be related to the value of the SDLE on specific scales.

To help readers better understand and appreciate the power of the materials in this book, nearly every significant concept or approach presented will be illustrated by applying it to effectively solve real problems, sometimes with unprecedented accuracy. Furthermore, source codes, written in various languages, including Fortran, C, and Matlab, for many methods are provided together with some simulated and experimental data.

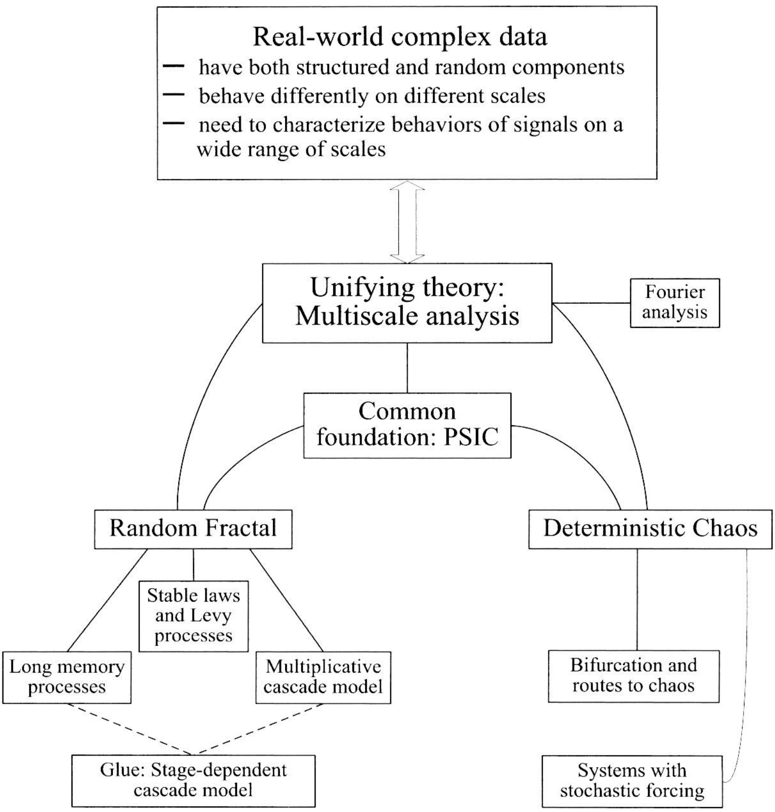

Figure 1.1. Structure of the book.