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CONCEPTS AND METHODS IN DISCRETE EVENT DIGITAL SIMULATION

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PREFACE

Although discrete event digital computer simulation has been with us as a research tool for almost two decades, the absence of a balanced and comprehensive account of its essential features has made the teaching of this form of simulation almost entirely a matter of the teacher's choice. No doubt the diversity of topics which discrete event simulation comprises has contributed to this situation. For example, modeling, programming languages, and statistical considerations all represent essential features of discrete event simulation; however, who can claim both equal expertise and equal interest in each of these areas?

This variety of choices has led to a situation in which one cannot tell what a student who claims to have taken a course in simulation knows about the subject. Does he know the alternative ways of modeling a queueing environment? Does he know the differences between GPSS and SIMSCRIPT? Is he familiar with the problem of statistical reliability in evaluating a simulation experiment?

One way to reduce this variability in course content is to provide a textbook that deals with modeling, programming languages, and statistical considerations in some depth. This I have attempted to do in this volume. However, I must quickly add that, although the book includes topics in each area that are necessary to a useful knowledge of simulation methodology, my own interest in statistical considerations has led to a considerably deeper discussion in this area than in the other two. Moreover, I believe that this imbalance in favor of statistics is justified on the grounds that the ability of the average user of simulation to make statistically valid statements about his simulation result is considerably more in doubt than are his abilities to model and program his simulation.

In writing this book I have, wherever possible, described alternative techniques for accomplishing a particular objective. The importance of having alternative methods available cannot be underestimated in simulation work,

where the nature of the situation often dictates the methods that can be employed. In addition, I have included flowcharts for modeling and for computational algorithms to speed the transition from theoretical ideas to practical use. The reader should realize that these algorithms serve a descriptive purpose and that he is free to alter them to accomplish the same objectives in more efficient ways.

The material in this book has been used in both undergraduate and graduate semester courses at Yale. The undergraduate course concentrated on modeling and programming considerations along with selected topics in the statistical area. For an undergraduate course with a prerequisite knowledge of a programming language and an introductory course in statistics, Chapters 1 through 5, Sections 7.1 through 7.6, 8.1 through 8.5, 10.1 through 10.8, and Chapter 12 are recommended. For a graduate course with the same programming requirement but a comprehensive probability and statistics prerequisite, the entire book should be used. Because of the volume of material, an instructor may wish to make Chapters 1 and 2 a reading assignment and begin his lectures with Chapter 3.

I began this book in September 1970 and completed the present draft in August 1972. I am grateful to Professor Robert Fetter, Mr. Philip Kiviat, Mr. Arnold Ockene, and Professor Alan Pritsker, who kindly read a preliminary draft and provided me with their critical comments. The book has benefited substantively from their contributions. My thanks go also to Mr. Joseph Faulkner of UNIVAC, who provided me with the correct coding of the SIMULA example in Chapter 5 and to Mr. William Eddy, who wrote the FORTRAN ANALYS subroutine in Appendix B.

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CHAPTER 1

INTRODUCTION

Discrete event digital computer simulation is now moving into its third decade. As a tool of analysis, it has been used to study many problems. Although queueing-oriented problems account for a considerable share of these, simulation has been used to study subjects as diverse as the turtle population on the Australian Great Barrier Reef [5] and the resolution of conflicts between nations [4]. A review of problems to which simulation has been applied indicates that success in applying it as a research technique depends, to a great extent, on the user's familiarity with simulation modeling concepts, programming language options open to him, and statistical techniques needed to produce desired input behavior and to analyze output behavior. Although the relative importance of each consideration varies with the problem to be studied, experience has shown that all three play important roles.

The purpose of this book is to introduce readers to the modeling, programming language, and statistical aspects of discrete event digital computer simulation as it applies to the study of systems. The book emphasizes methods of accomplishing individual steps in designing, running, and evaluating a simulation. The need for a discussion of simulation methodology was first noted by Conway, Johnson, and Maxwell in their 1959 paper [1]. However, efforts to provide such a discussion have usually been descriptive in character with a relatively low content on the evaluation of alternative methods. The papers by Kiviat on modeling [2] and simulation programming languages [3] are exceptions, being unusually helpful to both the experienced simulation user and the novice.

Before one can understand the meaning of system simulation in perspective, he must be familiar with the formal concepts of a system and a model. Chapter 2 begins with a description of these concepts, followed by a discussion of the alternative forms of system simulation, with special emphasis on computer system simulation.

Chapter 3 discusses system modeling, especially queueing-oriented systems. It describes three alternative simulation modeling methods [3]; the event

2 INTRODUCTION

scheduling approach, the activity approach, and the process interaction approach. Although the first and third of these are more commonly employed in practice, a comparison of the features of all three provides useful insight into discrete event modeling.

While the modeling of a system provides a major step toward organizing one's understanding of it, the actual simulation of the system, using the model on the computer, is the major focus of attention in a simulation study. Many programming considerations must be borne in mind, to facilitate both the programming of the model and its execution on a computer. Chapter 4 describes these programming considerations, and Chapter 5 presents limited discussions of GPSS/360, SIMSCRIPT II, and SIMULA, three commonly employed simulation programming languages. In addition to describing statements in the three languages, Chapter 5 illustrates their use.

Statistical considerations pervade almost all aspects of a system simulation. Since the passage of time also plays an integral role in system simulation, one cannot hope to view most of the statistical aspects of a simulation unless these aspects are cast in appropriate perspective with regard to time. To provide this perspective, Chapter 6 gives an introductory discussion of stochastic sequences. In particular, it describes the concepts of the autocorrelation function and spectrum and illustrates their value for understanding statistical phenomena evolving through time.

Chapter 7 describes the generation of pseudorandom numbers, which form the basis for producing random variates from different distributions during a simulation. The history of pseudorandom number generation is reviewed, presently employed methods are discussed, and testing procedures to reduce the possibilities of nonrandomness and dependence are described. The chapter also suggests procedures to follow, when using pseudorandom numbers, that reduce the chance of their misuse.

Algorithms for generating random variates from a variety of continuous and discrete probability distributions are presented in Chapter 8. Flowcharts are included for most of these. The chapter also gives an account of how to generate autocorrelated time series from moving average and autoregressive representations of stochastic sequences. These time series generation techniques are especially useful in econometric simulations.

Every simulation has input parameters. Sometimes these parameters assume hypothetical values because few, if any, data are available about their true values in the real system. Whenever data are available, however, it behooves the investigator to use them to estimate the parameter values. Naturally some methods of estimation provide more accurate estimates than others do, and consequently we would like to use the "better" methods. Chapter 9 describes methods of deriving maximum likelihood estimators of the parameters of most probability distributions for which generation

algorithms are given in the previous chapter. These estimators have many desirable statistical properties. However, they are sometimes difficult to compute. Accordingly, the chapter describes computational algorithms and gives references to many useful tables, some of which are contained in Appendix A. Chapter 9 also describes estimation methods for autoregressive parameters for use in generating time series, as described in Chapter 8, and in evaluating simulation output, as described in Chapter 10.

Among the statistical topics relevant to computer simulation, the analysis of simulation results has traditionally been one of the least organized. No doubt, this situation has occurred partly because of the diversity of statistical measures that are encountered in simulation. Chapter 10 reviews many of these measures and describes inferential methods applicable to them. In particular, the chapter devotes special attention to estimating how accurate a measure the sample mean of an autocorrelated stochastic sequence is of the true mean. This constantly recurring problem in simulation can be solved in one of the several ways that Chapter 10 describes.

Whether we realize it or not, every study that we undertake has an implicit experimental design associated with it. This design relates to how the study is conducted, and it is not difficult in most instances to identify one experimental design that meets the requirements of the study better than others do. Chapter 11 discusses design considerations in the peculiar environment of computer simulation. In particular, it dwells on methods of making individual experiments efficient with regard to providing useful results and also stresses the need to select the experiments to be performed with an eye toward the same efficiency considerations. Chapter 12 provides a guide of important steps for the conscientious simulation user to follow.

As the reader may perceive by this time, discrete event digital computer simulation of systems covers many topics. For each of these an established literature exists. By including these topics here we hope to give the investigator wishing to use simulation a background that has both breadth and depth.