FUNDAMENTAL MECHANICS OF FLUIDS

I. G. Currie

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FUNDAMENTAL MECHANICS OF FLUIDS

to my wife CATHIE and our sons DAVID and BRIAN

PREFACE

This book covers the fundamental mechanics of fluids as it is treated at the senior level or in first graduate courses. Many excellent books exist which treat special areas of fluid mechanics such as ideal-fluid flow or boundary-layer theory. However, there are very few books indeed at this level which sacrifice an in-depth study of one of these special areas of fluid mechanics for a briefer treatment of a broader area of the fundamentals of fluid mechanics. This situation exists despite the fact that many institutions of higher learning offer a broad, fundamental course to a wide spectrum of their students before offering more advanced specialized courses to those who are specializing in fluid mechanics. Recognition of this situation is the prime motivation for introducing this book.

The book is divided into four parts, each of which contains three chapters. Part I is entitled "Governing Equations," and it deals with the derivation of the basic conservation laws, flow kinematics, and some basic theorems of fluid mechanics. Part II is entitled "Ideal-Fluid Flow," and it covers two-dimensional potential flows, three-dimensional potential flows, and surface waves. Part III deals with "Viscous Flows of Incompressible Fluids," and it contains chapters on exact solutions, low-Reynolds-number approximations, and boundary-layer theory. The final part of the book is entitled "Compressible Flow of Inviscid Fluids," and this part contains chapters which deal with shock waves, one-dimensional flows, and multidimensional flows. Appendixes are also included which summarize vectors, tensors, the governing equations in the common coordinate systems, complex variables, and thermodynamics.

The treatment of the material is such as to emphasize the phenomena which are associated with the various properties of fluids while providing techniques for solving specific classes of fluid-flow problems. The treatment is not geared to any one discipline, and it may readily be studied by physicists and chemists as well as by engineers from various branches. Since the book is intended for teaching purposes, phrases such as "it can be shown that" and similar clichés which cause many hours of effort for many students have been avoided. In order to aid the teaching process, several problems are included at the end of each of the twelve chapters. These problems serve to illustrate points which are brought out in the text and to extend the material covered in the text.

Most of the material contained in this book can be covered in about 50 lecture hours. For more extensive courses the material contained here may be completely covered and even augmented. Parts II, III, and IV are essentially independent so that they may be interchanged or any one or more of them may be omitted. This permits a high degree of teaching flexibility, and permits the instructor to include or substitute material which is not covered in the text. Such additional material may include free convection, density stratification, hydrodynamic stability, and turbulence with applications to pollution, meteorology, etc. These topics are not included here, not because they do not involve fundamentals, but rather because the author set up a priority of what he considers to be the basic fundamentals.

Many people are to be thanked for their direct or indirect contributions to this text. The author had the privilege of taking lectures from F. E. Marble, C. B. Millikan, and P. G. Saffman. Some of the style and methods of these great scholars is probably evident on some of the following pages. The National Research Council of Canada are due thanks for supplying the photographs which appear in this book. My colleagues at the University of Toronto have been a constant source of encouragement, and the staff of the Department of Mechanical Engineering provided excellent typing and drafting services. Finally, sincere appreciation is extended to the many students who have taken my lectures at the University of Toronto and who have pointed out errors and deficiencies in the material content of the draft of this text.

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CONTENTS

PART I

Governing Equations

In this first part of the book a sufficient set of equations will be derived, based on physical laws and postulates, governing the dependent variables of a fluid which is moving. The dependent variables are the fluid-velocity components, pressure, density, temperature, and internal energy, or some similar set of variables. The equations governing these variables will be derived from the principles of mass, momentum, and energy conservation and from equations of state. Having established a sufficient set of governing equations, some purely kinematical aspects of fluid flow are discussed, at which time the concept of vorticity is introduced. The final section of this part of the book introduces certain relationships which can be derived from the governing equations under certain simplifying conditions. These relationships may be used in conjunction with the basic governing equations or as alternatives to them.

Taken as a whole, this part of the book establishes the mathematical equations which result from invoking certain physical laws which are postulated to be valid for a moving fluid. These equations may assume different forms, depending upon which variables are chosen and upon which simplifying assumptions are made. The remaining parts of the book are devoted to solving these governing equations for different classes of fluid flows and thereby explaining quantitatively some of the phenomena which are observed in fluid flow.

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BASIC CONSERVATION LAWS

The essential purpose of this chapter is to derive the set of equations which results from invoking the physical laws of conservation of mass, momentum, and energy. In order to realize this objective, it is necessary to discuss certain preliminary topics. The first topic of discussion is the two basic ways in which the conservation equations may be derived, the statistical method and the continuum method. Having selected the basic method to be used in deriving the equations, one is then faced with the choice of reference frame to be employed, eulerian or lagrangian. Next, a general theorem, called Reynolds' transport theorem, is derived, since this theorem relates derivatives in the lagrangian framework to derivatives in the eulerian framework.

Having established the basic method to be employed and the tools to be used, the basic conservation laws are then derived. The conservation of mass yields the so-called continuity equation. The conservation of momentum leads ultimately to the Navier-Stokes equations, while the conservation of thermal energy leads to the energy equation. The derivation is followed by a discussion of the set of equations so obtained, and finally a summary of the basic conservation laws is given.

1.1 STATISTICAL AND CONTINUUM METHODS

There are basically two ways of deriving the equations which govern the motion of a fluid. One of these methods approaches the question from the molecular point of view. That is, this method treats the fluid as consisting of molecules whose motion is governed by the laws of dynamics. The macroscopic phenomena are assumed to arise from the molecular motion of the molecules, and the theory attempts to predict the macroscopic behavior of the fluid from the laws of mechanics and probability theory. For a fluid which is in a state not too far removed from equilibrium, this approach yields the equations of mass, momentum, and energy conservation. The molecular approach also yields expressions for the transport coefficients, such as the coefficient of viscosity and the thermal conductivity, in terms of molecular quantities such as the forces acting between molecules or molecular diameters. The theory is well developed for light gases, but it is incomplete for polyatomic gas molecules and for liquids.

The alternative method which is used to derive the equations which govern the motion of a fluid uses the continuum concept. In the continuum approach, individual molecules are ignored and it is assumed that the fluid consists of continuous matter. At each point of this continuous fluid there is supposed to be a unique value of the velocity, pressure, density, and other so-called "field variables." The continuous matter is then required to obey the conservation laws of mass, momentum, and energy, which give rise to a set of differential equations governing the field variables. The solution to these differential equations then defines the variation of each field variable with space and time which corresponds to the mean value of the molecular magnitude of that field variable at each corresponding position and time.

The statistical method is rather elegant, and it may be used to treat gas flows in situations where the continuum concept is no longer valid. However, as was mentioned before, the theory is incomplete for dense gases and for liquids. The continuum approach requires that the mean free path of the molecules be very small compared with the smallest physical-length scale of the flow field (such as the diameter of a cylinder or other body about which the fluid is flowing). Only in this way can meaningful averages over the molecules at a "point" be made and the molecular structure of the fluid be ignored. However, if this condition is satisfied, there is no distinction amongst light gases, dense gases, or even liquids—the results apply equally to all. Since the vast majority of phenomena encountered in fluid mechanics fall well within the continuum domain and may involve liquids as well as gases, the continuum method will be used in this book. With this background, the meaning and validity of the continuum concept will now be explored in some detail.