AN INTRODUCTION TO

CLASSICAL DYNAMICS

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AN INTRODUCTION TO

CLASSICAL DYNAMICS

Mechanics are the Paradise of mathematical science, because here we come to the fruits of mathematics.

LEONARDO DA VINCI

To the memory of
Professor Roy Overstreet

and

To Mary

Preface

This book is intended for use in courses of one semester or two quarters on the dynamics of particles and assemblies of particles, including rigid bodies. Although the book begins with the most basic concepts, a useful assimilation of its contents requires a previous exposure to introductory mechanics based in calculus. No fine details of that exposure must be remembered; it is assumed only that the student has enough familiarity with the fundamental concepts to appreciate the need for careful discussions of the physical and mathematical aspects of classical dynamics. Differential equations and partial derivatives are used throughout but are applied in "cookbook" fashion. Vectors also are essential, beginning with Chapter 4, and a detailed discussion of them is given in the Mathematical Appendix along with material on rotations and curvilinear coordinate systems.

The central theme is that classical dynamics is a self-consistent discipline of physics, with many applications in a number of its modern subfields, which today are of increasing importance. These include geophysics, atmospheric physics, space physics, physical oceanography, and environmental physics, all of which are represented significantly in the textbook and in the problems. Some examples are the pendulum seismograph, atmospheric and oceanic circulation, artificial Earth satellites, and the settling of particulates. I hope that their inclusion will broaden the perspective of the student and stimulate the lectures of the teacher.

The plan of the book is to discuss the physical bases of dynamics—Newton's Laws and the conservation theorems—and to illustrate them extensively through a discussion of one-dimensional motions in the first three chapters. In Chapters 4 and 5 the essential features of motion in space are surveyed, including the use of curvilinear coordinates, the concept of angular momentum, the two-particle problem, the Kepler problem, and the theory of elastic scattering. Chapters 6 through 9 discuss important topics from which the teacher may choose material to complete his lectures: assemblies of particles (including molecular mechanics), rotating frames of reference, special relativity, and an introduction to Lagrangian and Hamiltonian mechanics. To provide the broadest possible choice of classroom subject matter, four "Special Topics" also have been added following Chapters 1 to 3, and 8. Problems relating to these topics have been included in the sets at the ends of those chapters. Answers and hints for *all* of the problems are given at the end of the book.

The manuscript was diligently typed by Ms. Linda G. Smith, to whom I am greatly indebted. I also express my appreciation to Professors Eugen Merzbacher, John McCullen, and Robert Eisberg for their helpful reviews. The responsibility for errors or unclear passages is my own, and I would appreciate students calling them to my attention.

Garrison Sposito

Contents

1 The Foundations of Classical Dynamics

1.1 THE NEWTONIAN PROGRAM FOR DYNAMICS

Classical dynamics began with the publication, in 1687, of Isaac Newton's monumental treatise, *Philosophiae Naturalis Principia Mathematica.*[1] In this book Newton set down three postulates which he believed would make possible the mathematical description of the motion of any single particle or collection of particles. These famous postulates are familiar from introductory physics as Newton's Laws of Motion. We know today that they are only very good approximations to the true axioms of particle dynamics, which must account for the quantization of energy and the Einsteinian principles of relativity. Nonetheless, because the degree to which it differs in its predictions from what is known to be correct theory is in practice often insignificant, and because its conceptual framework is easily related to common experience, we shall be on excellent ground to

[1] *The Mathematical Principles of Natural Philosophy*, revised translation by F. Cajori, University of California Press, Berkeley, 1960.

begin our discussion by considering in detail what we shall call *Newton's program for dynamics.*

In order that Newton's Laws of Motion be understood with the least ambiguity, we shall give a few prefatory definitions of terms that should be recalled from introductory mechanics. These terms are as follows.

Particle A particle is any physical object whose motion can be described fully by its position in space and by its velocity as functions of the time. Particles are the direct concern of Newton's Laws. An assembly of particles, which usually requires more to describe its motion than just a single position and velocity, is an indirect concern of Newton's Laws of Motion in that its behavior may be considered by extending the Laws.[2]

Position We shall consider this quantity to be a continuous function of the time that possesses at least two derivatives. The position of a particle is measurable, in the simplest case, by means of a ruler and a clock and may be represented geometrically by a succession of points in a suitably chosen reference frame. Ordinarily each of these points is specified by a set of three real numbers called *coordinates*. However, we shall suppose initially, for the sake of mathematical simplicity, that just one spatial coordinate is enough to determine the position of a particle. In this way the position becomes what is called a "scalar function" and the rules of conventional algebra apply. Whenever the more general situation obtains, the position becomes a "vector function" and the rules of vector algebra must be used.[3] We shall consider this mathematical complication in Chapter 4.

Velocity This quantity is the first derivative of the position with respect to the time. In the special case of a scalar position function we have, therefore,

$$v(t) = \lim_{\Delta t \to 0} \left[\frac{x(t + \Delta t) - x(t)}{\Delta t} \right] \equiv \frac{dx}{dt} \tag{1.1}$$

in a conventional calculus notation, where $x(t)$ is the position. The velocity $v(t)$ is a continuous function of the time t and possesses at least one derivative. A sense of direction is always associated with it and, in the case of $v(t)$ given by Equation 1.1, that direction is designated by an implicit plus or an explicit minus sign appearing before the numerical value. The plus sign will be taken generally to mean either "up" or "to the right," depending on the orientation of the frame of reference, and the minus sign will mean either of the opposite directions relative to the origin of the coordinate $x(t)$. The velocity can be measured by assigning its direction to be

[2] For example, Newton himself showed (*Principia*, pp. 193–195, Cajori translation) that it is possible to regard a planet as a particle when we consider its orbit about the sun, if the planet is a perfect sphere.

[3] These rules are discussed in section A.1 of the Appendix.

the direction of motion of the particle and by calculating the slopes of the lines tangent to the graph of $x(t)$, in keeping with the usual geometric interpretation of a derivative, in order to get its numerical values (see Figure 1.1).

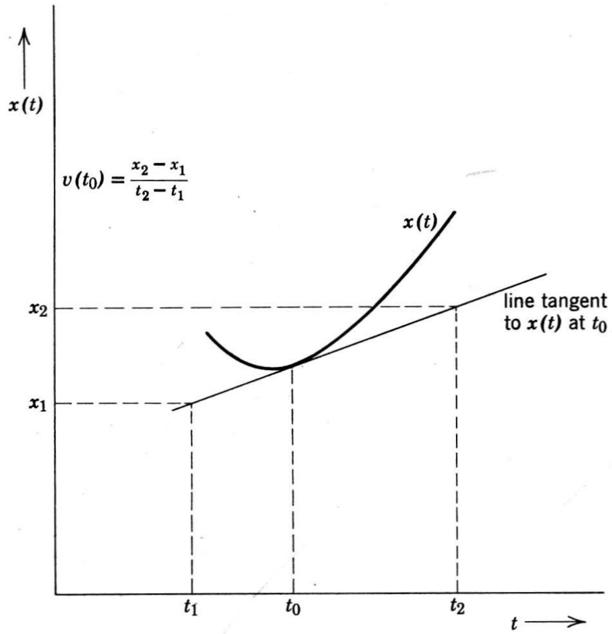

FIGURE 1.1. The velocity $v(t_0)$ is the slope of the line tangent to $x(t)$ at $t = t_0$.

Acceleration Acceleration is the first derivative of the velocity with respect to time and is also a continuous function of the latter variable. Conventionally we have

$$a(t) = \lim_{\Delta t \to 0} \left[\frac{v(t + \Delta t) - v(t)}{\Delta t} \right] \equiv \frac{dv}{dt} \tag{1.2}$$

for the acceleration $a(t)$. As with the velocity, acceleration has both a numerical magnitude and an associated direction. The magnitude is measured in a way analogous to that for determining the velocity. The direction assigned is that direction in which the velocity is *changing*. The direction of the acceleration, therefore, need not have anything to do with the observed direction of motion of the particle.

The quantities position, velocity, and acceleration are referred to as *kinematical quantities* because their definitions involve only the fundamental ingredients of motion: length and time. In Table 1.1 the kinematical quantities are listed along with a recounting of their dimensions and their units of measure in the three most commonly used systems. The definitions of the systems of units and the relations among them are assumed to be known from introductory physics.

TABLE 1.1 The Kinematical Quantities Important in Classical Mechanics

		Unit		
Quantity	Dimensions	SI	CGS	American Engineering
Length	L	meter (m)	centimeter (cm)	foot (ft)
Time	T	second	second (s)	second
Position	L	m	cm	ft
Velocity	LT^{-1}	m/s	cm/s	ft/s
Acceleration	LT^{-2}	m/s^2	cm/s^2	ft/s^2

1 foot = 0.3048 meter (exactly). 1 centimeter = 0.01 meter.

Now we are in a position to state and explain Newton's Laws of Motion. We shall not be presenting these laws exactly as they were given in the *Principia*, but instead we shall express them in a form that reflects the contemporary status of classical dynamics. A discussion of Newton's original conceptions regarding the Laws of Motion will be found in Special Topic 1 at the end of this chapter.

THE FIRST LAW *It is always possible to find a frame of reference in which a particle, free of influence from matter and radiation, is moving with a constant velocity.*

A glance at the First Law in this form shows that it deals with an ideal situation, since it is not possible to isolate completely any physical object from the rest of the universe. However, no difficulty in interpretation should develop as long as it is remembered that every physical law or postulate has validity only insofar as it agrees with experience within some predetermined level of tolerance for error. In practice the influence of surrounding matter and radiation can be minimized to whatever extent is desired and the First Law can be understood as an extrapolation to vanishing influence. Then it states that we can always find a "preferred" frame of reference in which the particle under consideration will be moving with a constant velocity (i.e., with a constant speed in a fixed direction). Evidently, if one such frame of reference exists, an infinite number of them exists. This is because all frames moving at constant velocity with respect to *one another* would yield an observation of constant velocity (including, possibly, zero velocity) for a particle moving that way in any one of them. Newton (and before him Galileo) believed that this uniform motion was the "natural" or "equilibrium" motion of a free particle and that it would be observed at least in a limiting sense within a certain class of frames of reference. These special frames of reference are called *inertial frames. Inertia* is the name given to the "inherent tendency" of a free particle to move with constant velocity.

It should be clear, even from these few remarks, that the First Law has much the same character as a definition. When a free particle is observed to move uniformly, that motion is defined as the natural one and the frame of reference in which it is observed is deemed a member of the preferred inertial class. What remains quite vague about the First Law is the reason the inertial frames are preferred and the precise meaning of the phrase "free of influence." These questions are, in fact, only two faces of the same problem and are answered admirably by the Second Law.

THE SECOND LAW *The influence of matter and radiation on a particle is manifest in the form of a force. A force causes the velocity of the particle to change in such a way that the time rate of change of the velocity is in the direction of and is proportional to the force.*

The Second Law tells us that forces are the purveyors of all the interactions in the universe. These quantities possess both magnitude and direction. They bring about *changes in the velocity* of a particle, that is, they cause a particle to *accelerate*. It is now clear why inertial frames of reference are the preferred ones. Relative to those frames, a particle unaffected by forces moves with constant velocity. If a force should act on the particle, its velocity changes and we may attribute all of that change to the force, according to the Second Law. In some other, noninertial frame of reference, the free particle would not move with constant velocity and we could not unambiguously determine the influence of matter and radiation on it by invoking the concept of force.

The force acting on a particle generally may be represented as a function of the position, velocity, and time. In the special case we now are considering, this function is a scalar one, but it is associated with a direction according to the sign convention that we previously discussed for acceleration. With this stipulation we can write the Second Law in the mathematical form

$$F(x, v, t) = ma(t) \tag{1.3}$$

or, by Equation 1.2,

$$F(x, v, t) = m\frac{dv}{dt} \tag{1.4}$$

where m is simply a constant of proportionality. As is well known, the constant m is supposed to be a number characteristic of the particle itself and is called the *mass*. Since the force $F(x, v, t)$ and the acceleration $a(t)$ always have the same direction the mass is always a positive number. Moreover, the algebraic form of Equation 1.3 suggests that, for a given, fixed magnitude of the force, small accelerations are associated with large masses, and vice versa. It follows that the mass must represent the relative degree to which a particle can resist force, a large mass being characteristic of a particle difficult to accelerate. But unaccelerated motion reflects the inertia of a particle, according to what was stated previously. Therefore, *the mass of a particle should be a numerical measure of its inertia.* Newton was aware of the common experience that bulky objects were difficult to accelerate and was led to equate mass with the quantity of matter in an object. But the better definition is the one related to inertia, since it develops in a straightforward way from the first two Laws of Motion and is independent of whatever range of experience with