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Nonlinear Model Predictive Control

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Preface

This volume is based on the contributions, after a rigorous peer review process, for a workshop on Nonlinear Model Predictive Control that took place on June 2–6, 1998, in Ascona, Switzerland. The workshop was the first international conference solely devoted to Nonlinear Model Predictive Control. Nonlinear Model Predictive Control is presently viewed as one of the most promising areas in automatic control. This is partly due to the increasing industrial need for advanced control techniques, that address explicitly the process nonlinearity and operating constraints, and the ever-demanding control performance requirement. Since the research on Nonlinear Model Predictive Control is at its early stage, many theoretical and implementation issues remain open and very few industrial applications have been reported. With this workshop we wanted to bring together internationally recognized researchers to assess the current status and to discuss future research directions. A wide range of important topics, from problem formulation, computations, and algorithms to estimation, modelling, and identification to closed-loop stability and robustness to applications, is covered. We trust that this volume will contribute to shaping the future research on Nonlinear Model Predictive Control.

This workshop was made possible by significant financial contributions from a number of sponsors. We would like to especially thank the Centro Stefano Franscini of ETH, the Swiss National Science Foundation (Schweizerischer Nationalfonds), the US National Science Foundation (NSF), and the Swiss Society for Automatic Control (SGA) for their generous support. We are also indebted to Rolf Findeisen and Alberto Bemporad of the Automatic Control Lab at ETH for the boundless energy they have put into preparing the workshop. Last but not the least we want to express our gratitude to the participants of the workshop for a most stimulating event and especially to the authors of this volume for allowing us to put together, as we hope, a high quality book.

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Stability and Robustness of Nonlinear Receding Horizon Control

G. De Nicolao, L. Magni and R. Scattolini

Abstract. The main design strategies for ensuring stability and robustness of nonlinear *RH* (Receding-Horizon) control systems are critically surveyed. In particular, the following algorithms with guaranteed closed-loop stability of the equilibrium are considered: the zero-state terminal constraint, the dual-mode *RH* controller, the infinite-horizon closed-loop costing, the quasi-infinite method, and the contractive constraint. For each algorithm, we analyse and compare feasibility, performance, and implementation issues. For what concerns robustness analysis and design, we consider: monotonicity-based robustness, inverse optimality robustness margins, nonlinear H_∞ *RH* design, and a new nonlinear *RH* design with local H_∞ recovery.

1. Introduction

Receding-Horizon (*RH*) control, also known as model predictive control, owes its success to the simplicity of its rationale which is well suited to deal with nonlinearities and constraints. In discrete-time, the basic idea is to determine the current control $u(t)$ by solving a finite-horizon optimization problem over the interval $[t, t+N]$. At the next time instant $t+1$, the new control $u(t+1)$ is found by translating the optimization horizon and solving a new problem over $[t+1, t+N+1]$. Being optimization-based, the *RH* scheme can allow for nonlinearities and constraints much more straightforwardly than other methods. In view of the increased efficiency of the hardware, there are more and more plants on which *RH* control can be implemented by solving the finite-horizon optimization on-line.

An important caveat of *RH* control is that closed-loop stability is not guaranteed for a generic finite-horizon cost function, as it was well illustrated by Bitmead, Gevers and Wertz [3]. In the linear case, the first stability result was obtained by complementing the cost function with a terminal zero-state constraint, see e.g. the work by Kwon and Pearson [22]. Rawlings and Muske removed, at least for the stable modes, the need of equality constraints, by introducing a terminal penalty equal to the infinite-horizon cost due to zero control [34]. It is interesting to note that a fairly general stability theory for linear *RH* control can be developed by referring to the monotonicity properties of a suitable difference Riccati equation initialized with the terminal penalty matrix [3].

Although there is a well-established theory in the linear case, its importance is diminished by the fact that, in absence of nonlinearities and constraints, RH control is hardly better than infinite-horizon (IH) linear quadratic (LQ) control. This does not happen in the nonlinear case, because IH optimization becomes computationally intractable.

An important exception is given by LQ RH control with constraints. In such a context, Sznaier and Damborg [40] showed that the IH optimal control law can be found as the solution of a finite-horizon problem where the terminal penalty is equal to the IH cost of the unconstrained LQ problem. Other contributions along this line are due to Scokaert and Rawlings [35] and Chmielewski and Manousiouthakis [11].

For nonlinear systems Chen and Shaw [4], Keerthi and Gilbert [21], and Mayne and Michalska [27] showed for discrete and continuous systems that the terminal zero-state constraint guarantees closed-loop stability also in the nonlinear case. However, the presence of the terminal equality constraint places an heavy requirement on the on-line optimizing controller. This motivated the development by Michalska and Mayne of the dual-mode RH controller, which replaces the equality constraint with an inequality one, namely that $x(t+N)$ belongs to a suitable neighbourhood of the origin where the nonlinear system is stabilized by a linear control law. The scheme is called "dual-mode" because, when such a neighbourhood is eventually reached, the RH controller switches to the linear one [29]. Another stabilization method worked out by Yang and Polak is based on a terminal contractive constraint requiring that the norm of the terminal state $x(t+N)$ is sufficiently smaller than the norm of $x(t)$ [43].

More recently, schemes have been proposed that combine a terminal penalty with a terminal inequality constraint. In particular, Parisini and Zoppoli, and Chen and Allgöwer have shown that stabilization can be enforced by a suitable quadratic terminal penalty [32], [5]. On the other hand, De Nicolao, Magni and Scattolini established closed-loop stability of the equilibrium using a (nonquadratic) terminal penalty equal to the infinite cost due to a locally stabilizing linear control law [17]. The most recent developments concern the design of RH controllers of H_∞ type that are capable of achieving guaranteed robustness margins [7], [25].

The main purpose of the present contribution is to provide a critical survey of the existing literature on stability and robustness of (state-feedback) RH control schemes. The emphasis is on the key ideas rather than on mathematical technicalities. This justifies some simplifications, the use of conservative assumptions, and the lack of formal proofs.

The paper is organized as follows. In Section 2, some preliminary definitions and notions are introduced. Section 3 is devoted to the analysis of the alternative RH stabilization schemes. The results concerning robustness analysis and synthesis are reported in Section 4. Some concluding remarks end the paper.

2. Preliminaries

Consider the time-invariant nonlinear discrete-time system

$$x(k+1) = f(x(k), u(k)), \quad x(t) = \bar{x}, \quad k \geq t \tag{1}$$

where $x(k) \in R^n$ is the state, $u(k) \in R^m$ is the input, $f(\cdot,\cdot)$ is a C^2 function of its arguments, and $f(0,0) = 0$ (the origin is an equilibrium point). The state and input vectors are subject to the constraints

$$x(k) \in X, \quad u(k) \in U, \quad k \geq t \tag{2}$$

where X and U are compact sets of R^n and R^m respectively, both containing the origin as an interior point.

Consider now a control law $u = \kappa(x)$ with $\kappa(0) = 0$. The associated closed-loop system is

$$x(k+1) = F(x(k)) \quad x(t) = \bar{x}, \quad k \geq t \tag{3}$$

where $F(x) := f(x, \kappa(x))$. In the following, $\kappa(\cdot)$ will be said to be *stabilizing* if the origin of (3) is an asymptotically stable equilibrium point (for every $\varepsilon > 0$, there exists δ_ε such that, if $\|\bar{x}\| < \delta_\varepsilon$, then $\|x(k)\| < \varepsilon$ and $\lim_{k\to\infty} \|x(k)\| = 0$, where $x(k)$ is the solution of (3)). Hereafter, with some abuse of terminology, the term "stable" will be used for short in place of the more correct "asymptotically stable". The origin is said to be an exponentially stable equilibrium of (3) if there exist constants r, a, $b > 0$ such that, whenever $\|\bar{x}\| < r$, it results that $\|x(k)\| \leq ae^{-b(k-t)} \|\bar{x}\|$, $\forall k \geq t$.

A sufficient condition for the origin to be exponentially stable is that the linearized matrix $dF/dx|_{x=0}$ has all its eigenvalues inside the unit circle. In the following, we will not discuss exponential stability properties explicitly, but this property will implicitly follow whenever the linearized closed-loop is found to be stable.

Throughout this chapter, our aim is to design a state-feedback controller that stabilizes (1) around the origin complying with the constraints (2).

2.1. Linear quadratic control (LQ)

The easiest (and most used) way to stabilize (1) is by means of a linear control law designed on the base of the linearization of (1) around the equilibrium. To this purpose, let

$$A = \left.\frac{\partial f}{\partial x}\right|_{x=0,u=0} \qquad B = \left.\frac{\partial f}{\partial u}\right|_{x=0,u=0}$$

Assumption: In order to design the linear controller, it is assumed that the pair (A, B) is stabilizable. □

The idea is to find some K such that $A + BK$ is stable and then apply the control law $u = Kx$ to the nonlinear system (1). In particular, K can be found by minimizing the infinite-horizon quadratic cost function

$$J_{IH}(\bar{x}, u(\cdot)) = \sum_{k=t}^{\infty} \{x(k)'Qx(k) + u(k)'Ru(k)\} \tag{4}$$

where $Q > 0$, $R > 0$, subject to the linearized dynamics

$$x(k+1) = Ax(k) + Bu(k), \quad x(t) = \bar{x}, \quad k \geq t \tag{5}$$

As is well known, the solution to the above LQ problem is given by the linear state-feedback

$$u(k) = K^{LQ}x(k), \quad k \geq t \tag{6}$$

where

$$K^{LQ} = -(R + B'P_\infty B)^{-1}B'P_\infty A$$

and P_∞ is the (unique) nonnegative definite solution of the ARE (Algebraic Riccati Equation)

$$P = A'PA + Q - A'PB\left(R + B'PB\right)^{-1}B'PA \tag{7}$$

When (6) is applied to (1), we obtain the closed-loop system

$$x(k+1) = f(x(k), K^{LQ}x(k)) \quad x(t) = \bar{x}, \quad k \geq t \tag{8}$$

Proof of stability: As is well known, the stability of the origin of (8) is proven by using $V(x) = x'P_\infty x$ as a Lyapunov function [42].

Output admissible set: Due to the stabilizability assumption, the origin is a stable equilibrium point of (8) with a nonzero-measure domain of attraction. However, the controller design completely ignores the constraints (2). Therefore, for a given $\bar{x}$, there is no guarantee that $x(k)$ and $u(k)$ defined by (8), (6) will satisfy (2). Hereafter, the term *output admissible set* [19], referred to the closed-loop formed by (1) joined with the state feedback

$$u(k) = \kappa(x(k)) \tag{9}$$

will denote an invariant set $\tilde{X}$ which is a domain of attraction of the origin and such that $\bar{x} \in \tilde{X}$ implies that $x(k)$, $u(k)$, $k \geq t$ defined by (1), (9) satisfy (2). Moreover $u(k)$, $k \geq t$, is called a *feasible control sequence.* In particular, the maximal output admissible set of the controller (6) will be indicated by $X(K^{LQ})$. Needless to say, $X(K^{LQ})$ may well be unsatisfactorily small, since both the nonlinear nature of the system and the presence of constraints have been neglected.

Under a computational viewpoint, the evaluation of $X(K^{LQ})$ for a nonlinear system may be difficult (or even impossible). It is however possible to obtain an inner bound by computing an output admissible set given by a suitable level set of a local Lyapunov function for the linearized system, see [29] and also [6], [5].

Performance: It is apparent that the linear design optimizes performance only close to the equilibrium point. The acceptability of the performance clearly depends on how well the system (1) can be approximated by its linearization (5) in the considered region.

Implementation: The calculation of the gain K^{LQ} requires only the (off line) solution of the ARE (7), which is a standard task.

2.2. Infinite Horizon Nonlinear Controller (IH)

The limitations of the LQ controller motivate the search for more sophisticated approaches capable of improving both the output admissible set and the performance. The most direct way is to take into account explicitly both nonlinearity and constraints in the minimization of (4). Along this direction, the IH (Infinite Horizon) nonlinear optimal controller is obtained by minimizing (4) subject to (1) and (2).

Assumption: As is the case of the LQ controller, we assume that (A, B) is stabilizable. □

The solution of the $IHNOCP$ (IH Nonlinear Optimal Control Problem) is given by a nonlinear state-feedback

$$u(k) = \kappa^{IH}(x(k)) \tag{10}$$

which stabilizes the origin, see e.g. [21]. In the following $\varphi^{IH}(k, \bar{x}) := x(k)$ where $x(k)$ is the solution of (1) subject to (10).

Proof of stability: Closed-loop stability of the origin is proven by using $V(x) = J^o_{IH}(x)$ as a Lyapunov function where

$$J^o_{IH}(x) = J_{IH}(x, \kappa^{IH}(\varphi^{IH}(\cdot, x)))$$

denotes the minimal value of the cost function, see e.g. [21].

Output admissible set: When $\bar{x} \in X(K^{LQ})$ the control sequence $u(k) = K^{LQ}x(k)$, $k \geq t$, is always an admissible solution for the $IHNOCP$. Consequently, the output admissible set of the $IHNOCP$, hereafter denoted by X^{IH}, is nonempty and such that $X^{IH} \supseteq X(K^{LQ})$. In the present context, we can regard X^{IH} as the largest achievable output admissible set.

Linearization: Rather expectedly, it can be shown that the linearization of $\kappa^{IH}(x)$ coincides with K^{LQ}, i.e.

$$\left.\frac{d\kappa^{IH}(x)}{dx}\right|_{x=0} = K^{LQ}.$$

This observation can help in the tuning of the design matrices Q and R . In fact, a reasonable procedure is to adjust Q and R with reference to the LQ problem (4), (5) by means of well-established methods until the (linearized) closed-loop performance becomes satisfactory. This is clearly easier than tuning Q and R by extensive trials on the nonlinear model (1).

Performance: Assuming that the weights Q and R have been properly chosen, the performance of the IH controller is "optimal" by definition.

Implementation: This is the main problem with IH control. For a generic nonlinear system, analytic solutions of the $IHNOCP$ do not exist and the attempt to approximate the IH cost functional by means of a finite-horizon one

(by truncating the series in (4)) leads to a hard optimization problem in a
dimensional space.

3. Nonlinear RH control

The difficulties inherent in the implementation of the IH controller motiva
development of control strategies based on FH (Finite-Horizon) optimizati
particular, letting $u_{t,t+N-1} := [u(t), u(t+1), \ldots u(t+N-1)]$, we will co
cost functions of the type

$$J_{FH}(x, u_{t,t+N-1}, N) = \sum_{k=t}^{t+N-1} \{x(k)'Qx(k) + u(k)'Ru(k)\} + V_f(x(t+N))$$

to be minimized with respect to $u_{t,t+N-1}$, subject to (1), (2) as well as t
terminal constraint

$$x(t+N) \in X_f \subset R^n$$

As discussed below, the different algorithms are characterized by the choices
terminal penalty function $V_f(x)$ and the terminal region X_f. For computa
reasons the optimization horizon N should be as short as possible, compatibl
the desired performance and output admissible set.

Associated with (11), (12) it is possible to define an RH (Receding Ho
control strategy in the usual way: at every time instant t, define $\bar{x} = x(t$
compute the optimal solution $u^o_{t,t+N-1}$ for the FH problem (11) subject t
(2), (12); then apply the control $u(t) = u^o(\bar{x})$ where $u^o(\bar{x})$ is the first colu
$u^o_{t,t+N-1}$. Although the FH minimization of (11) has to be performed at each
instant, this is much more viable than solving an IH problem.

The main challenge is to guarantee closed-loop stability and performanc
for small values of N. Along this direction, we can take advantage of the exper
gained in the RH control of linear systems [22], [2], [3], [34], [11]. In particu
is well known that, if $V_f(x) \equiv 0$ and $X_f = R^n$, for a given N it may well h
that the RH controller yields an unstable closed-loop system. Nevertheless,
proper design it is possible to ensure closed-loop stability with a finite horizo

In the LQ case a fairly complete stability theory is available which is b
on the so-called Fake Riccati analysis. The main point is to choose $V_f(x)$
X_f so as to force the monotonicity of the solution of a relevant difference Ri
equation. Once monotonicity is established, it follows that

$$J^o_{FH}(x, N) = \min_{u_{t,t+N-1}} J_{FH}(x, u_{t,t+N-1}, N)$$

is a Lyapunov function for the closed-loop.

An analogous rationale can be extended to the nonlinear case. In fact,
closed-loop stability of most RH schemes is proven by showing that $J^o_{FH}(x, N)$
a Lyapunov function. For this purpose, the main point is to demonstrate tha

$$J^o_{FH}(f(x, u^o(x)), N) < J^o_{FH}(x, N)$$

can be done in two steps. First, by optimality it always holds that

$$\begin{aligned} & J^o_{FH}(f(x,u^o(x)),N-1) \\ = \; & J^o_{FH}(x,N) - x'Qx - u^o(x)'Ru^o(x) < J^o_{FH}(x,N) \end{aligned} \quad (14)$$

cond step is to show that

$$J^o_{FH}(\xi,N) \le J^o_{FH}(\xi,N-1), \quad \forall \xi \in \Xi \quad (15)$$

Ξ is a neighbourhood of the origin in R^n. From (15), letting $\xi = f(x,u^o(x))$ calling (14), one sees that (13) follows. Now, the inequality (15), which will eafter termed as *monotonicity property*, is fulfilled only for a suitable choice $x)$ and X_f. In the remaining part of this section we will examine different s of $V_f(x)$ and X_f that enforce stability.

he zero-state terminal constraint (ZS)

lea is to let $V_f(x) \equiv 0$ and $X_f = \{0\}$. In this way, the FH minimization m becomes equivalent to the minimization of (11) subject to (1), (2) and *STC* (*Zero-State Terminal Constraint*) $x(t+N) = 0$. The corresponding ontrol law will be indicated as $u(k) = \kappa^{ZS}(x(k))$. Historically, this was the H method with guaranteed stability, see [22] for linear systems and [4], [21], 28], [36] for nonlinear systems.

Assumption: For a given integer $N > 0$, there exists a nonempty neighbour- $X^c(N)$ of the origin such that $\forall \bar{x} \in X^c(N)$ one can find a control sequence $t \le k \le t+N-1$ driving state of (1) to the origin in N steps, i.e. $x(t+N) = 0$, uch that the constraints (2) are satisfied $\forall k \in [t, t+N-1]$.

Proof of stability: The keystone of the proof is establishing the monotonic- operty (15). To this purpose, let $u^o_{t,t+N-2}$ be the optimal solution mini- g $J_{FH}(\bar{x}, u_{t,t+N-2}, N-1)$ subject to (1), (2) and $x(t+N-1) = 0$. Con- now the problem of minimizing $J_{FH}(\bar{x}, u_{t,t+N-1}, N)$ subject to (1), (2) and $N) = 0$. It is clear that $\tilde{u}_{t,t+N-1} = [u^o_{t,t+N-2}, \; 0]$ is an admissible so- for the new problem and moreover (recalling that $x(t+N-1) = 0$) we $J_{FH}(\bar{x}, \tilde{u}_{t,t+N-1}, N) = J_{FH}(\bar{x}, u^o_{t,t+N-1}, N-1) = J^o_{FH}(\bar{x}, N-1)$. Since opti- y implies that $J^o_{FH}(\bar{x},N) \le J_{FH}(\bar{x}, \tilde{u}_{t,t+N-1}, N)$, the monotonicity property $\bar{x}, N) \le J^o_{FH}(\bar{x}, N-1)$ follows.

Output admissible set: The output admissible set coincides with the (con- ned) controllability region $X^c(N)$. Note that $X^c(N)$ may be "small". In par- ar, there is no guarantee that $X^c(N)$ is larger than the output admissible set $_{LQ})$ of the LQ controller. Of course, $X^c(N)$ grows with N, but increasing the nization horizon has computational drawbacks.

Linearization: The linearization

$$K^{ZS} = \left. \frac{d\kappa^{ZS}(x)}{dx} \right|_{x=0}$$

ides with the gain matrix of the linear RH controller associated with the mization of the FH cost function (11) (with $V_f(x) \equiv 0$) subject to (5) (the