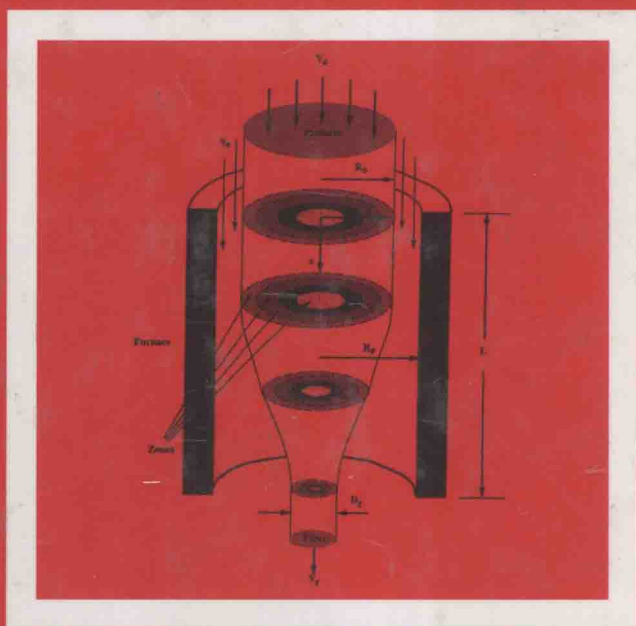


**series in computational
and physical processes
in mechanics and thermal sciences**



Computational Heat Transfer

Second Edition

**Yogesh Jaluria
Kenneth E. Torrance**

COMPUTATIONAL HEAT TRANSFER

SECOND EDITION

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SECOND EDITION

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Shih, Editor, Numerical Properties and Methodologies in Heat Transfer:

Proceedings of the Second National Symposium

PREFACE TO THE SECOND EDITION

This book is a revised, updated and expanded version of the first edition. Though substantial new material is added in response to the changes that have occurred over the past fifteen years in computational heat transfer, the basic approach and philosophy of the first edition have been retained. We had set out to write a concise and comprehensible textbook, which will bridge the gap between numerical methods, on the one hand, and physical aspects of heat transfer and fluid flow processes, on the other. Thus the main thrust was a presentation of the computational approach to the solution of heat transfer and fluid flow problems, rather than of available methods and techniques. Therefore, the basic analysis, numerical modeling, validation and results were all considered in terms of the underlying physical mechanisms that govern the relevant transport processes.

The material presented in the first edition has been thoroughly updated, with additional methods, results, discussion, and references included to bring out advances since the publication of the earlier book. Among the topics included or expanded in coverage are finite element solutions, primitive variables solution of convection transport, radiation in participating media, grid generation, turbulent flow, and recent work on materials processing and on fires. The nomenclature was changed to bring it in line with accepted usage. Additional solved examples and problems have been included to stress the current and emerging trends. Some of the earlier material has been deleted or reduced to reflect the diminished importance and use of certain methods. Though most of the additions and changes have occurred in Chapters 5–7, other chapters have also been updated and revised. As before, solved examples are used at various stages to illustrate the methodology, the important considerations in obtaining an accurate and valid solution, and the interpretation of the results. The large number of problems given at the end of the chapters is used to supplement and strengthen the discussion in the text and to allow the students to try out the ideas presented for themselves. These exercises range from simple mathematical formulation of a numerical scheme to actual computational solution of a given heat transfer or fluid flow problem. The computational problems themselves vary from relatively simple solutions of well-defined heat transfer problems to complicated problems involving combined modes, complex geometry, intricate boundary conditions and other complexities.

Therefore, the book starts with the basic mathematical concepts needed for the understanding, use and development of numerical techniques, and develops the material through the numerical modeling of different heat transfer modes to the solution of complex combined-mode and applied problems. The important features of the book include its systematic approach, consideration of different approaches and methods, comparison of different techniques, selection of an appropriate method, validation, and coupling of the scheme and results with the physical aspects of the process. It presents the use of up-to-date examples from many important areas that involve thermal processes and discusses the state-of-the-art of this important field. Several new problems from areas that have been of particular interest in recent years have been added to those presented in the first edition. A few important computer programs are included in the book to present the basic logic and to provide readily available software for code development. The programs presented in the first edition have been updated and revised, and several new ones have been added. The programs include different methods for the solution of algebraic systems and differential equations.

The book is appropriate as a one-semester textbook for a senior undergraduate course or a first-year graduate course. It is assumed that the reader has a basic undergraduate background in heat transfer and in computer programming. The book can also be used as a reference book in engineering and scientific applications and for courses that deal with heat transfer, fluid flow, and thermal systems. Applied areas such as manufacturing, energy, environment and transportation can also use this book as reference in numerical modeling and simulation.

The first edition of this book has been used in courses taught to graduate and undergraduate students at various universities around the world. The inputs obtained from the instructors of these courses have been very useful in the revision of the book. Even though my coauthor, Ken Torrance, has not contributed to the revisions for this edition of the book, the basic treatment and approach are based on the many discussions we have had over the years. Many colleagues and friends have contributed in many significant ways to my understanding of the exciting field of computational heat transfer and to the development of the material in this book. Of particular help has been the interaction with many brilliant and dedicated students I have had the pleasure of guiding and teaching. Professor Wilson Chiu has been of direct help in some of the material presented in this book. The comments of the reviewers of the first edition of this book and of Professor W. J. Minkowycz, the Editor of this series, have been very valuable and helpful in organizing the material and the presentation. The strong support and help provided by the staff of Taylor and Francis Publishers have also contributed very significantly to the book.

Certainly, the greatest contribution to this work has been the unqualified support and encouragement of my wife, Anuradha, the patience and understanding of our children, Pratik, Aseem and Ankur, and the encouragement and inspiration of my parents. Without their help and support, it would not have been possible for me to meet the stringent demands placed on my effort and time by this book.

Yogesh Jaluria
August 2002

PREFACE TO THE FIRST EDITION

There has been a phenomenal increase in the use of computational methods for engineering applications in recent years. This is particularly true for problems in heat transfer and fluid flow, since the complexity of the governing equations generally allows analytical solutions to be obtained only for very simple cases, making it necessary to use numerical techniques for most problems of practical interest. The growing need to optimize thermal processes and systems has made it imperative to numerically simulate the relevant transport phenomena, since experimentation is usually too involved and expensive. However, available analytical and experimental results are of considerable importance in checking the accuracy and validity of numerical results. The availability of large computers has made it possible to solve diverse and complex heat transfer and fluid flow problems that arise in a wide variety of applications.

Along with the growth in the hardware capabilities of computers, substantial progress has been made, particularly in the last two decades, in the development and application of numerical techniques to heat transfer problems. There is every indication that these trends will continue. It is, therefore, important that the basic numerical techniques, as applied to heat transfer, be made available to engineering students, researchers, and professional engineers. However, much of this information can often only be obtained, rather sketchily, from research publications. Also, the textbooks on numerical analysis usually emphasize the methods, without getting into the details of the physical aspects of the problem that are often crucially important to the success of a particular numerical scheme.

This book attempts to bridge the gap between numerical methods and procedures, on the one hand, and the physical characteristics of thermal transport, on the other. The subject is developed by relating various available computational techniques to the basic heat transfer processes, and the text emphasizes the importance of the physical aspects of the problem in the selection of the most appropriate method, in the imposition of the boundary conditions, and in the verification of the numerical results. The intricacies of computational heat transfer are brought out by considering different types of basic and applied problems and the corresponding effect on the solution procedure.

To provide the mathematical background for the subject, the book first discusses, in Part 1, the nature of the equations and the relevant boundary conditions that arise in heat transfer and in the associated fluid flow. The two basic techniques of finite difference and finite element methods are discussed, and the important considerations of accuracy, stability, and convergence are outlined. This background is then employed in Part 2 to consider problems that involve the three basic transport modes of conduction, convection, and radiation. The complexities that arise in these areas are discussed, along with the relevant numerical procedures. Finally, in Part 3, a few practical circumstances are considered to demonstrate the application of computational techniques to physical processes of engineering interest. The numerical simulation of systems is also outlined. This section indicates the concerns and solutions applicable to selected practical processes that involve combined modes and complicated boundary conditions. Examples are given at various stages in the book, particularly in Part 2, to demonstrate the use of numerical methods in actual physical problems.

The material in this book has been used in several courses taught at the senior/graduate level at Cornell University; the Indian Institute of Technology, Kanpur (India); and Rutgers University. It is assumed that the reader has taken an undergraduate heat transfer course and has a background in computer programming. The book is suitable as a text for a one-semester course at the senior or graduate level. Because of the nature of the subject, the material will be useful for workers in diverse fields, such as energy systems, environmental sciences, aerospace, meteorology, chemical engineering, and mechanical engineering. Though personal preference is inevitable in the topics covered and in the depth of coverage, a fairly wide spectrum of applicable methods is discussed, and comparisons between these are made wherever possible.

Several problems are given at the end of each chapter, except Chapter 1. These exercises serve largely to clarify the material covered in the text and to allow the reader to gain experience on the various stages in the numerical solution of a physical problem. There are many problems directed at the formulation of the numerical procedure and at obtaining the relevant equations for computation. Also included are problems that need to be solved on the computer. In the various courses we have taught, we have felt that the actual solution of problems on the computer is essential to the learning process in this area. A few important computer programs are given in Appendix A, particularly for the solution of algebraic systems and ordinary differential equations. Programs have not been given for all the aspects covered, in order to keep the discussion general. Also, it is felt that the readers of this book would have an adequate background in computer programming.

We would like to acknowledge the contribution of several people in the development of the material covered in this book. Of particular importance has been the interaction with several perceptive students who took our courses and made the experience of teaching the subject very worthwhile by their appreciation and enthusiasm. Their suggestions and comments, over the years, have been useful in this presentation. We have also profited from discussions with

several of our colleagues. In particular, the first author would like to acknowledge the several stimulating discussions with Professors J. Srinivasan and D. G. Briggs. We also wish to acknowledge the support and encouragement provided by the staff of Hemisphere Publishing Corporation, particularly William Begell, and by the editors of this book series, Professors W. J. Minkowycz and E. M. Sparrow. The typing was done with great patience and competence by Barbara Benedict and Lynn Ruggiero. Certainly the greatest contribution to this effort has been the patience and encouragement of our families, particularly our wives Anuradha and Marcia.

Yogesh Jaluria
Kenneth E. Torrance

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INTRODUCTION

The study of heat transfer has, in the recent past, become a matter of considerable importance and interest because of the growing demand for energy, environmental comfort, safety, rapid transportation, and advanced materials, and the need to optimize processes that involve energy transport. Heat transfer is concerned with the physical processes underlying the transport of thermal energy due to a temperature difference or gradient. It is of importance in a wide variety of problems, ranging from heat transfer problems in manufacturing processes to those in aerospace systems, engines, heating/cooling systems, and the environment. Interest lies in determining the rate of heat transfer, the temperature field that results, and the associated fluid flow that arises in many of these processes.

Frequently, the characteristics of the product, such as a semiconductor device, are strongly influenced by the thermal gradients and temperature levels encountered by the material during the manufacturing process. Similarly, the temperature and flow fields in the vicinity of a power plant indicate its environmental impact and may also be used to devise possible means to reduce the negative effects of pollution. The efficiency of systems, such as electronic equipment, gas turbines and automobile engines, is also a strong function of the heat transfer to and from various components of the system. All these considerations make it imperative to obtain accurate, consistent, and physically valid results on heat transfer and on the associated temperature and velocity distributions and to relate these to the design, control, and optimization of the system.

1.1 THERMAL TRANSPORT

There are two basic processes by which heat transfer occurs: conduction and radiation. Conduction occurs if a temperature gradient exists in a material and is due to the thermal motion of the microscopic particles that constitute the material. Radiative heat transfer occurs between materials at different temperatures in the form of electromagnetic waves, transmitted through vacuum or an intervening medium between two bodies. When heat transfer occurs in a