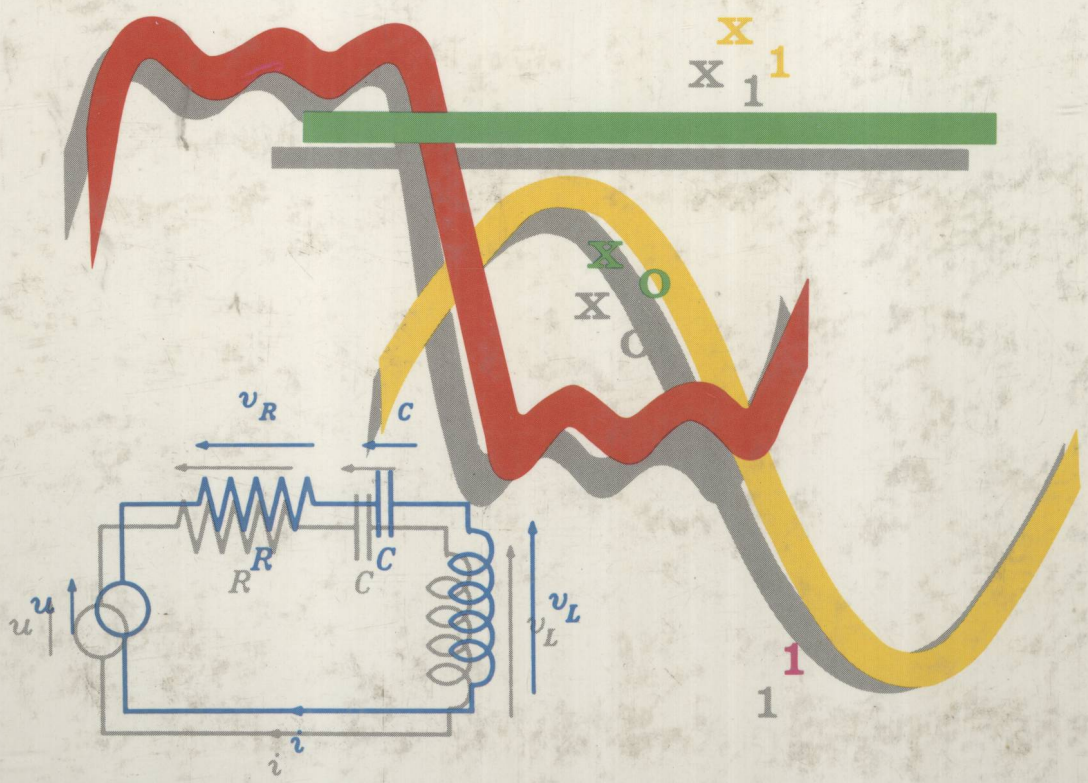


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MODERN SIGNALS AND SYSTEMS



HUIBERT KWAKERNAAK
RAPHAEL SIVAN

PRENTICE HALL INFORMATION AND SYSTEM SCIENCES SERIES
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MODERN SIGNALS AND SYSTEMS



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with software by
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To my parents
H. K.

To Ilana, Ori, Ayelet, Keren, and Yael
R. S.

Preface

Almost twenty years ago, as young and inexperienced scientists, we dared (where did we take the chutzpah?) to write an advanced graduate textbook, *Linear Optimal Control Systems* (New York: John Wiley, 1972). We found the collaboration enjoyable and rewarding. A few years ago, being not so young anymore, and possibly more mature, we decided to join hands again, this time in what we thought to be a more modest endeavor, to write an undergraduate textbook on modern signals and systems.

Having taught this material over and over again, we had little doubt that it would not be a too difficult or lengthy task to organize what we thought to be the Signals and Systems material. How wrong we were! We ignored the fact that time and again, while standing in front of our classes, we experienced a moment of hesitation, realizing that not all the material we were teaching was totally consistent and completely clear.

It took us nearly four years of work, and almost endless brain racking and soul searching, to integrate the signals and systems material into a unified framework and to produce a book we are satisfied with. We hope that the meticulous care we took in creating the text shows. We also hope that teachers and students alike appreciate the concise and precise style to which we aspired without wanting to compromise the intuitive appeal.

Our *Modern Signals and Systems* is a textbook for a one- or two-semester junior course, which often goes by the name “Signals and Systems”, or sometimes by names such as “Linear Systems” or “Dynamical Systems,” and is a required course in the curriculum of most electrical engineering departments. The book contains a comprehensive and well-integrated treatment of the basic notions of signal theory

and of both the time- and frequency-domain analysis of systems. The discrete- and continuous-time case are treated in parallel, sometimes even typographically in two column format. The book contains many examples, several of which are pursued over many chapters. An extensive collection of homework problems concludes each chapter. In addition, the final section of each chapter provides problems whose solution requires a computer. An unusual feature is that the book comes complete with application software of near-professional quality, provided on a disk, that runs on a personal computer.

It is assumed that the student has a background in basic calculus and algebra, knows how to work with complex numbers, has heard of differential equations and linear algebra, knows the fundamentals of physics and electricity, and possibly has had an introductory course in electrical circuits. To use the software, some familiarity with a personal computer is helpful.

From this book we have taught two-quarter (at the University of Twente) and one-semester (at the Technion) courses to sophomore students. We managed to cover practically all of Chapters 1–8 and a sprinkle of the applications in Chapters 9–11. In departments that devote two semesters to the Signals and Systems course, the entire book may easily be covered. There may even be time to pursue the applications dealt with in Chapters 9–11 in a little more depth, at the discretion of the teacher.

A chapter-by-chapter description of the material follows.

Chapter 1 offers a brief overview of the ideas of signals and systems. By way of motivation, a sketch is given of the application areas that are elaborated in the final three chapters of the book.

Chapter 2 presents an introduction to signals. We describe basic notions such as the time axis of signals, discrete- and continuous-time signals, periodic and harmonic signals, various operations on signals, and signal spaces. The final section is devoted to *generalized* signals. A more elaborate treatment of this material, based on *distribution* theory, is presented in Supplement C.

Chapter 3 deals with a number of fundamental ideas related to systems. Two types of systems are introduced at this point: *input-output* systems and *input-output mapping* systems. The definitions of these systems are *set theoretic*, based on the approach of Jan C. Willems to system theory. These ideas allow an axiomatic approach to system theory even at the level of an undergraduate text. The chapter continues by distinguishing various types of systems. By way of linearity and time-invariance, the discussion arrives at *convolution* systems. The convolution operation is thoroughly treated, and the stability of convolution systems is touched upon. A study of the response of convolution systems to harmonic inputs results in the *frequency response* function. The chapter concludes with a discussion of the response of convolution systems to periodic inputs—leading to *cyclical* convolution—and a brief introduction to the interconnection of systems.

In Chapter 4 we consider systems described by constant coefficient linear difference and differential equations. After a review of elementary material concerning the solution of constant coefficient linear difference and differential equations, we deal with the impulse response, stability, and frequency response of these systems. A central notion is that of the *initially-at-rest system*. In the discussion of stability

we introduce *bounded-input-bounded-output* (BIBO) and *converging-input-converging-output* (CICO) stability.

Chapter 5 deals with the state description of systems. Also, here the initial approach is set theoretic. The definition of the state rests upon the *state matching property* formulated by Willems. After introducing the basic ideas, the realization of linear difference and differential systems as state systems is discussed. A further section is devoted to the existence of solutions of state equations and the numerical integration of state differential equations. The chapter continues with the explicit solution of linear state equations and modal analysis. The treatment of the stability of state systems is integrated with that of difference and differential systems.

Chapters 6–8 are devoted to the *frequency domain* description of signals and systems. Chapter 6 begins with a concise but fundamental presentation of signal *expansion*. The theory is made concrete by the introduction of *harmonic* bases for signal spaces and the associated finite and infinite *Fourier series expansions*. Various aspects of these expansions, such as the identities of Plancherel and Parseval, convergence properties, the trigonometric form, symmetry properties, and generalized infinite Fourier series, are carefully presented. The chapter concludes with a treatment of the response of convolution systems to periodic inputs.

Chapter 7 is devoted to *Fourier transforms*. First, transform theory is explained in an abstract setting. The discussion soon focuses on *expansion transforms* (i.e., the transformation from a signal to its expansion coefficients). In this way, the Fourier series expansions of finite-time and periodic signals of Chapter 6 immediately lead to two of the four Fourier transforms that are considered, namely, the discrete-to-discrete Fourier transform (abbreviated DDFT, more conventionally known as the discrete Fourier transform) and the continuous-to-discrete Fourier transform (CDFT). The properties of these transforms are reviewed. Following this it is shown that the expansion of aperiodic rather than periodic or finite-time signals leads to *Fourier integral expansions*. These result in the discrete-to-continuous Fourier transform (DCFT, also known as the discrete-time Fourier transform) and the continuous-to-continuous Fourier transform (CCFT, commonly known as the Fourier integral transform). Also, the properties of these last two transforms are discussed in detail, emphasizing parallels. The chapter ends with showing how Fourier transform theory is used in the frequency domain analysis of convolution systems.

The third chapter dealing with frequency domain analysis, Chapter 8, is devoted to the *z*-transform and the Laplace transform. First, it is explained that Fourier transform theory cannot handle exponentially increasing signals. To overcome this difficulty, the DCFT and CCFT are modified to the (two-sided) *z*-transform and Laplace transform. To deal with initial value problems, the one-sided *z*-transform and Laplace transform are introduced. The existence, properties and inversion of the *z*-transform and Laplace transform are treated with considerable completeness. Three separate sections of Chapter 8 are devoted to the application of these transforms to the analysis of convolution systems, difference and differential systems, and state systems.

The final chapters of the book present applications of the theory to three important areas for which a course on Signals and Systems forms a prerequisite: signal

processing and digital filtering, communication, and feedback and automatic control. Indeed, teachers of courses on these subjects may find that part of their material is covered in this book.

Chapter 9 is the first of the applications chapters. It concerns signal processing and digital filtering. The chapter begins with a discussion of the effect of sampling and interpolation on the frequency content of signals. This leads to a transparent derivation of the sampling theorem. Next on- and off-line signal processing are introduced. After a discussion of windows and windowing, two sections are devoted to an elementary treatment of various methods to design finite and infinite impulse response digital filters. The chapter ends with a derivation of the fast Fourier transform and some considerations about the numerical computation of transforms and convolutions.

Chapter 10 outlines some basic applications to communication theory. The chapter begins with the application of Fourier transforms to the description of narrow-band signals. This is a beautiful theory, which is indispensable in the analysis of modulation and demodulation. Following this, various well-known modulation schemes, including amplitude and frequency modulation, are presented. The chapter ends with a brief discussion of multiplexing.

Chapter 11, finally, contains a concise outline of feedback theory and automatic control. The potential benefits of feedback are demonstrated at a fairly abstract level, illustrated by simple concrete examples. The final section of this chapter reviews various important results on the stability of feedback systems.

The body of the text is complemented with five supplements.

As was mentioned earlier, the book comes with computer software, provided on a disk. The software consists of a powerful interpreter named SIGSYS, which offers a wide and flexible range of operations to generate and handle signals, including Fourier transformation, convolution, and the integration of differential equations. The software has been strongly inspired by MATLAB, which is a widely used computational tool. The difference is that where the main data type of MATLAB is a matrix, that of SIGSYS is a real- or complex-valued *signal*. In addition, complex and real scalars and polynomials are supported. SIGSYS offers an opportunity to do calculations of a widely varying nature interactively. The graphic support provides instantaneous visualization. An extensive Tutorial, describing and illustrating all the operations and commands of SIGSYS, is included at the end of the book. The READ.ME file on the disk that contains SIGSYS should be consulted before running the program. It contains instructions how to set up and start SIGSYS and also lists corrections and modifications to the Tutorial. The disk furthermore contains a number of demos.

We have used SIGSYS for several years to run a laboratory course in parallel with the Signals and Systems course as follows. During the first weeks of the term, students were instructed to go through the first 16 sections of the Tutorial and to do all the exercises provided by the Tutorial. In addition, the students were assigned two or three Computer Exercises from Chapters 2 and 3, whose main purpose is to become familiar with the computer. Depending on the available time, during the rest of the course students were assigned a number of other problems from the Computer Exercises for the rest of the chapters.

A solutions manual for the Problems and Computer Exercises is available to teachers who adopt the book and may be obtained from the publisher. The manual comes with a disk that provides solutions to all the Computer Exercises in macro form. Also, hard-copy solutions are supplied.

The story of the software is this: Right at the initial stages of planning the book it occurred to us that supporting the text with software would be timely and instructive. The success of MATLAB was a powerful stimulus, and it stood example for SIGSYS in many ways. We were lucky to get Rens Strijbos, a long-time associate, to develop the program for us. The time and effort spent on this work are fully comparable with our own exertions. The software was written in the C language and developed in a UNIX environment by using a variety of programming tools.

Writing this book also got us in other ways involved in high technology. The early versions of the text were prepared by using the text formatting language Troff, and the figures were produced with a drawing program. Being able to keep in daily touch by electronic mail was a forceful incentive. Fax and international courier mail from time to time supplemented communication by ordinary mail. The work on the book took us on numerous trips back and forth between Israel and The Netherlands and also to such places as Amherst, Mass., Berlin, Bern, Englewood Cliffs, N.J., and, last but not least, the Sinai desert.

We are grateful to our teachers, in particular Lotfi Zadeh and Charlie Desoer, who showed us what perspicuity means, and to our students, who keep insisting on getting things clear. Our departments deserve credit for accepting our absorption in the project of writing the book, our periods of absence, footing the bills for computer time, and supporting part of the traveling. In particular, we acknowledge the support of the Fund for the Promotion of Research of the Technion. We thank our secretaries, Marja Langkamp and Annette Berg, for their invaluable help. In conclusion, we wish to express our sincere appreciation to Tim Bozik of Prentice Hall for his continued belief in the project even when we started missing deadlines.

Enschede and Haifa

Huibert Kwakernaak
Raphael Sivan

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