

Trellis Decoding of Block Codes
A Practical Approach



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**TRELLIS DECODING OF
BLOCK CODES**
A Practical Approach

by



**Bahram Honary
Garik Markarian**
*Communications Research Centre
Lancaster University
Lancaster, United Kingdom*



E9960150



KLUWER ACADEMIC PUBLISHERS
Boston / Dordrecht / London

Distributors for North America:

Kluwer Academic Publishers
101 Philip Drive
Assinippi Park
Norwell, Massachusetts 02061 USA

Distributors for all other countries:

Kluwer Academic Publishers Group
Distribution Centre
Post Office Box 322
3300 AH Dordrecht, THE NETHERLANDS

Library of Congress Cataloging-in-Publication Data

Honary, Bahram.

Trellis decoding of block codes : a practical approach / by Bahram
Honary, Garik Markarian.

p. cm. -- (Kluwer international series in engineering and
computer science ; SECS 391)

Includes bibliographical references and index.

ISBN 0-7923-9860-2 (alk. paper)

1. Error-correcting codes (Information theory) I. Markarian,
Garik. II. Title. III. Series.

TK5102.96.H66 1997

621.382'2'01154--dc21

96-40193

CIP

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Printed on acid-free paper.

Printed in the United States of America

**TRELLIS DECODING OF
BLOCK CODES**
A Practical Approach

**THE KLUWER INTERNATIONAL SERIES
IN ENGINEERING AND COMPUTER SCIENCE**

Bahram Honary dedicates this book to his wife Farideh and his son Souroush

Garik Markarian dedicates this book to his wife Karina, to his daughters Rimma and Nina and to his Parents

PREFACE

It is a great pleasure to be asked to write the Preface for this book on trellis decoding of error correcting block codes. The subject is extremely significant both theoretically and practically, and is very timely because of recent developments in the microelectronic implementation and range of application of error-control coding systems based on block codes. The authors have been notably active in signal processing and coding research and development for several years, and therefore very well placed to contribute to the state of the art on the subject of trellis decoding. In particular, the book represents a unique approach to many practical aspects of the topic.

As the authors point out, there are two main classes of error control codes: block codes and convolutional codes. Block codes came first historically and have a well-developed mathematical structure. Convolutional codes come later, and have developed heuristically, though a more formal treatment has emerged via recent developments in the theory of symbolic dynamics. Maximum likelihood (ML) decoding of powerful codes in both these classes is computationally complex in the general case; that is, ML decoding falls into the class of NP-hard computational problems. This arises because the decoding complexity is an exponential function of key parameters of the code. The structure of convolutional codes is such that they can be represented by means of compact, regular trellises, however; which in turn leads to an efficient ML decoding method, the Viterbi algorithm (VA). In addition, efficient near-ML decoding algorithms exist for both tree and trellis representations of convolutional codes. Though the complexity of these algorithms remains asymptotically exponential, the way in which the computation is organised means that it is feasible to implement practical decoders for powerful convolutional codes. Until recently, corresponding methods for the decoding of block codes did not exist, because the trellis (and tree) structure of these codes was not well understood. This is the reason for the current dominance of convolutional codes in applications requiring ML or near-ML decoding performance.

TRELLIS DECODING OF BLOCK CODES...

Interestingly, trellis representations of block codes began to emerge in the 1970's, but their implications for the efficient decoding were not properly recognised until quit recently. Effective near-ML decoding algorithms based on both trellis and tree representations of block codes were developed first (just as sequential decoding preceded the VA), and quit recently, as a result of the work of the authors of this book and other researchers, representations of block codes by means of compact, regular trellises have been devised. These trellises are based on the generalised array code (GAC) construction of block codes, as described in this text. Almost all members of the most important classes of error-control codes can be constructed in the form of GACs, and the GAC construction joins the squaring, cubing and generalised concatenated code (GCC) construction as a way of facilitating ML (and near-ML) trellis decoding of block codes using the VA. Previously intractable problems, like the soft decision decoding of non-binary block codes (e.g., the Reed-Solomon (RS) codes) also yield to the power of the GAC approach.

All aspects of the trellis decoding of block codes based on the GAC construction are described in this book, with the emphasis on the practical approach. The authors are to be congratulated for bringing together the results of many research investigations into a clear, coherent, and up-to-date presentation of great value to the community.

Prof. Farrell P.G.

The University of Manchester, UK.

CONTENTS

LIST OF FIGURES	ix
LIST OF TABLES	xiv
PREFACE	xv
1 INTRODUCTION	1
1.1 About The Book	1
1.2 Acknowledgments	6
2 GENERALISED ARRAY CODES	7
2.1 Array Encoding Technique	7
2.2 Generalised Array Codes	11
2.3 Recursive Encoding Technique For Some Classes of Block Codes Based On Array Decomposition	33
2.4 Generalised Array Codes For Partial Response Channels	39
3 TRELLIS STRUCTURE OF BLOCK CODES	59
3.1 Introduction to the Problem	59
3.2 Fundamental Definitions	60
3.3 Trellis Representation Of Array Codes	63
3.4 Trellis Decoding of Generalised Array Codes	80
3.5 Trellis Decoding of Block Codes in PR Channels	96
4 ADAPTIVE ENCODING AND TRELLIS DECODING TECHNIQUES	111
4.1 Adaptive Encoding/Decoding In Digital Communications	111

TRELLIS DECODING OF BLOCK CODES...

4.2	Adaptive Block Codes With Constant Dimension And Their Trellis Decoding	113
4.3	Adaptive Block Codes With Constant Code Length And Their Trellis Decoding	119
4.4	Nested Encoding And Trellis Decoding Technique	123
4.5	Application of Adaptive Encoding and Trellis Decoding In ARQ Systems	131
4.6	Adaptive Spectral Shaping Codes And Their Trellis Decoding	136
5	REED-SOLOMON CODES AND THEIR TRELLIS DECODING	149
5.1	Finite Fields and Reed-Solomon Codes	149
5.2	Array Decomposition Of Reed-Solomon Codes	154
5.3	Trellis Decoding of RS Codes	159
5.4	Shannon Product of Trellises	161
5.5	Syndrome Trellis Design for RS Codes	163
5.6	Coset trellises for RS Codes	172
5.7	Sub-Optimum Trellis Decoding of Reed-Solomon Codes	183
6	APPLICATION OF TRELLIS DECODERS: MULTI-FUNCTIONAL TRELLIS DECODING OF BLOCK CODES	189
6.1	Elements of Multi-Functional Coding	189
6.2	Block-Coded Trellis Modulation	190
6.3	Synchronisation Protocols For Multi-Functional Trellis Decoding	215
6.4	Real-Time Channel Estimation In Multi-Functional Trellis Decoders	230
	REFERENCES	239
	A GRAPHIC ILLUSTRATION OF THE TRELLIS DESIGN PROCEDURE FOR ARRAY CODES	257

Contents

B GRAPHIC ILLUSTRATION OF THE TRELLIS DESIGN PROCEDURE FOR BLOCK CODES	259
C GRAPHIC SIMULATION OF TRELLIS DECODERS	261
INDEX	263

LIST OF FIGURES

Chapter 1

Chapter 2

2.1	Construction of Generalised Array Codes	12
2.2	Weight Distribution Function of the (8,4,4) GAC	15
2.3	Weight Distribution Function of the (15,5,7) GAC	17
2.4	Weight Distribution Functions of the Golay Codes	26
2.5	P.G.Farrell's Construction Of The (32,6,16) GAC	28
2.6	Decomposition of Reed-Muller Codes	35
2.7	Uncoded (1-D) PR Channel	40
2.8	Two-State Trellis Diagram of the (1-D) PR Channel	41

Chapter 3

3.1	Trellis Diagram Of The (3,2)(3,2) Array Code	68
3.2	Trellis Diagram Of The (3,2)(4,3) Array Code	70
3.3	Trellis Diagram Of The (3, 2)(n_2, k_2) Array Codes	71
3.4	Trellis Diagram Of The (4,3)(4,3) Array Code	72
3.5	Trellis Diagram Of The (4, 3)(n_2, k_2) Array Codes	72
3.6	Division Of The Received Sequence	74
3.7	Calculation Of The Branch Metrics	74
3.8	Trellis Decoding Of The (3,2)(3,2) Array Code	76
3.9	Surviving Path For Trellis Decoding Of The (3,2)(3,2) Array Code	76
3.10	Performance Of Trellis Decoding For Some Array Codes	77
3.11	Performance Of Trellis Decoding For Some Array Codes	78
3.12	Performance Of Trellis Decoding For Some Array Codes	78
3.13	Trellis Structure Of The (8,4,4) GAC	83

TRELLIS DECODING OF BLOCK CODES...

3.14	Trellis Structure Of The (7,4,3) GAC	83
3.15	Trellis Structure Of The (10,5,4) GAC	85
3.16	Trellis Diagrams Of The (12,7,4) And (12,8,3) GACs	86
3.17	Trellis Structure Of The (16,10,4) And ((16,11,4) GACs	87
3.18	Trellis Diagrams Of The (16,5,8) And (15,5,7) GACs	88
3.19	Trellis Diagrams Of The (15,7,5) GAC	89
3.20	Trellis Diagram Of The (31,6,15) GAC	90
3.21	Trellis Structure Of The Golay Codes	91
3.22	Trellis Structure Of The (16,8,6) Nordstrom-Robinson Code	92
3.23	Trellis Diagram Of The (17,9,5) GAC	93
3.24	Error Performance Of Some GACs	95
3.25	Trellis Structure Of The (8,4,4) And (7,4,3) RLL Codes	98
3.26	Trellis Structure Of The (16,5,8) RLL GAC	99
3.27	Trellis Design For The (8,3,4) Balanced Code	102
3.28	Another Trellis Structure Of The (8,3,4) Balanced Code	102
3.29	Trellis Diagram Of The (16,6,4) Balanced Code (First Sub-Trellis	103
3.30	Trellis Diagram Of The (16,7,4) Balanced Code (First Two Sub-Trellises	104
3.31	Trellis Structure Of The (16,9,4) Balanced Code	105
3.32	Error Performance Of The (16,9,4) RLL Code In PR Channel	107
3.33	Trellis Diagram Of The (16,5,8) Code In (1-D) PR Channel	108
3.34	Trellis Diagram Of The (8,3,4) Balanced Code In (1-D) PR Channel	108

Chapter 4

4.1	Adaptive Trellis Decoding With $k = 4$, Based On Array Codes	117
4.2	Adaptive Trellis Decoding With $k = 4$, Based On GACs	119
4.3	Trellis Diagram Of The (16,3,8) GAC	122
4.4	Adaptive Nested Trellis Decoding	127
4.5	Trellis Diagram Of The (12,3,6) GAC	130
4.6	Trellis Decoder For Type-II ARQ Protocol	135
4.7	Spectral Shaping For The (8,4,4) GAC	143
4.8	Spectral Shaping For The (16,5,8) GAC	145

List of Figures

4.9 Block Diagram Of The Adaptive Communications System based On SSCs	146
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Chapter 5

5.1 Shannon Product of Trellises	164
5.2 Component Syndrome Trellises For The (7,5,3) RS Code	168
5.3 Syndrome Trellis For The (7,5,3) RS Code	169
5.4 Component Syndrome Trellises For The (15,3,13) RS Code	171
5.5 Syndrome Trellis For The (15,3,13) RS Code	172
5.6 Component Coset Trellises For The (7, 3, 5) RS Code	175
5.7 Coset Trellis For The (7, 3, 5) RS Code	176
5.8 Component Trellises For The (15,3,13) RS Code	179
5.9 Coset Trellis Of The (15,3,13) RS Code	180
5.10 Coset Trellis Of The (15,13,3) RS Code	183
5.11 Two-Stage Decoding of the (7,5,3) RS Code Using a Hard-Decision Sub-Trellis Predictor	186
5.12 Two-Stage Decoding of the (7, 5, 3) RS Code Using a Soft-Decision Sub-Trellis Predictor	187

Chapter 6

6.1 Signal Mapping in BCM, Based on $(3, 2)(n_2, k_2)$ Array Codes and 4-PSK Modulation Format	195
6.2 Trellis Diagram of the BCM, Based on $(3, 2)(n_2, k_2)$ Array Codes and 4-PSK Modulation Format	196
6.3 Signal Mapping in BCM, Based on $(4, 3)(n_2, k_2)$ Array Codes and 8-PSK Modulation Format	197
6.4 Trellis Diagram of the BCM, Based on $(4, 3)(n_2, k_2)$ Array Codes and 8-PSK Modulation Format	197
6.5 Signal Mapping in BCM, Based on $(3, 2)(n_2, k_2)$ Array Codes and 2FSK/2PSK Modulation	200
6.6 Signallin Set and Euclidean Distances for 8-PSK Modulation	205
6.7 Trellis Structure of the $R = 1, k = 4$ Trellis BCM and Labelled 4-PSK Signal Set	207
6.8 Error Performance of the Proposed $R = 1, k = 4$ Trellis BCM	207

TRELLIS DECODING OF BLOCK CODES...

6.9	Trellis Structure Of The Proposed $R = 1.25$, $N_{ch} = 4$, $k = 5$ Trellis BCM	208
6.10	Error Performance of the Proposed $R = 1.25$, $N_{ch} = 4$, $k = 5$ Trellis BCM	209
6.11	Trellis Structure of the $\bar{k} = [2, 1, \dots, 1, 0]$ Trellis BCM	211
6.12	Error Performance Of Different Trellis BCM With $R = 1$	211
6.13	Error Performance Of Trellis BCM With Enhanced Perfor- mance For One Bit	213
6.14	Modified Signal Set	214
6.15	CABS Decoding Procedure	220
6.16	Example Of TEST For The (8,4,4) Code	223
6.17	Metric Values For The (8,4,4) Code	224
6.18	Decoder Operations in TEST	225
6.19	Operation of Algorithm (a)	226
6.20	Operation of Algorithm (b)	227
6.21	Block Diagram Of The TEST	228
6.22	Algorithm Performance Using (16,5,8) RM Code	229
6.23	Algorithm Performance Using (15,5,7) GAC	229
6.24	Accuracy Of SNR Measurements	235
6.25	Average Value of Metric (a) and Metric (b)	236
6.26	Average Value Of Metric (b) After 250 Decoder Operations	237

Appendix A

Appendix B

Appendix C

LIST OF TABLES

Chapter 1

Chapter 2

2.1	Code Table of the (8,3,4) Row Code.	25
2.2	Construction of Some GACs With $d_{min} = 3$ and $d_{min} = 4$	29
2.3	Construction of Some GACs With $d_{min} = 5$	30
2.4	Construction of Some GACs With $d_{min} = 7$ and $d_{min} = 8$	30
2.5	Parameters of Some RLL Array Codes	45
2.6	Code Table of the (8,4,4) RLL/GAC	46
2.7	Code Table of the (16,5,8) RLL/GAC	48
2.8	Parameters of Some RLL/GACs	49
2.9	Definition of Functions	51
2.10	Encoding Procedure	51
2.11	Encoding Procedure for the (8,3,4) Balanced Code	52
2.12	Code Table of the (8,3,4) Balanced Code	53
2.13	Encoding Procedure for (8,4,2) Balanced Code	53
2.14	Code Table of the (8,4,2) GAC	54
2.15	Encoding Procedure for (16,9,4) Balanced Code	55

Chapter 3

3.1	The Number Of Trellis States And Available Code Rates For Some Array Codes	73
3.2	Alteration of Branch Labels At The First Depth	106
3.3	Alteration Of Branch Labels For The (8,3,4) Balanced Code	109

Chapter 4

4.1	List Of Codes With Codelength $n \leq 16$	125
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TRELLIS DECODING OF BLOCK CODES...

4.2	List Of Codes With Codelength $n \leq 64$	126
4.2	List Of Codes With Codelength $n \leq 64$ (Cont.)	127
4.3	Trellis Structure Of Codes With Codelength $n \leq 16$	128
4.4	Trellis Structure Of Codes With Codelength $n \leq 64$	129
4.4	Trellis Structure Of Codes With Codelength $n \leq 64$ (Cont.)	130
4.5	Encoding Table For The Manchester Code	140
4.6	Possible Modification Vectors For (8,4,4) GAC	144
4.7	Code Table Of The (6,3,3) GAC	148

Chapter 5

5.1	Relationship Between The Exponential and Polynomial Representations	152
5.2	Addition Table for $GF(2)$, $GF(2^2)$ and $GF(2^4)$	153
5.3	Multiplication Table for $GF(2)$, $GF(2^2)$ and $GF(2^4)$	154

Chapter 6

6.1	Information Rate Per Channel Symbol for the BCM Employing Array Codes and M -ary Signalling Sets	193
6.2	Signal Mapping in BCM, Based on $(3, 2)(n_2, k_2)$ Array Codes And 4-PSK Modulation Format	195
6.3	Signal Mapping in BCM, Based on $(4, 3)(n_2, k_2)$ Array Codes and 8-PSK Modulation Format	198
6.4	Signal Mapping in BCM Based on $(3, 2)(n_2, k_2)$ Array Codes and 2-FSK/2-PSK Modulation Format	200
6.5	Coding Gains for $R = 1$, $\bar{k} = [2, 1, \dots, 1, 0]$ BCM Trellis	213

Appendix A

Appendix B

B.1	Input Parameters For Trellis Design Of Different Codes	260
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Appendix C