Springer 大学数学图书——影印版

A First Course in Discrete Mathematics

离散数学引论

Ian Anderson 著

TSINGHUA UNIVERSITY PRESS Springer

Springer 大学数学图书——影印版

A First Course in Discrete Mathematics

离散数学引论

TSINGHUA UNIVERSITY PRESS
北京

Springer

内 容 提 要

本书以简洁和通俗的形式介绍组合数学的一些本质性内容：图论的重要问题，计数方法和试验设计，其中图论约占一半篇幅。书中有大量习题和例题，习题附有部分解答和提示，适于自学。本书可用作数学、计算机科学、信息科学专业大学本科生的组合数学教材，可在大学一年级讲授。

Ian Anderson
A First Course in Discrete Mathematics
EISBN:1-85233-236-0

图书在版编目（CIP）数据

离散数学引论 =A First Course in Discrete Mathematics: 英文 /（美）安德逊（Anderson, I.）著. —影印本. —北京：清华大学出版社，2009.11
（Springer 大学数学图书）

ISBN 978-7-302-21482-3

Ⅰ. ①离… Ⅱ. ①安… Ⅲ. ①离散数学－高等学校－教材－英文 Ⅳ. ①O158

中国版本图书馆 CIP 数据核字（2009）第 200087 号

责任编辑： 陈朝晖
责任印制： 孟凡玉
出版发行： 清华大学出版社
http://www.tup.com.cn
社 总 机：010-62770175
地 址：北京清华大学学研大厦 A 座
邮 编：100084
邮 购：010-62786544
投稿与读者服务：010-62776969，c-service@tup.tsinghua.edu.cn
质 量 反 馈：010-62772015，zhiliang@tup.tsinghua.edu.cn
印 刷 者： 清华大学印刷厂
装 订 者： 三河市金元印装有限公司
经 销： 全国新华书店
开 本： 175×245 **印 张：** 13.5
版 次： 2009 年 11 月第 1 版 **印 次：** 2009 年 11 月第 1 次印刷
印 数： 1~4000
定 价： 34.00 元

本书如存在文字不清、漏印、缺页、倒页、脱页等印装质量问题，请与清华大学出版社出版部联系调换。联系电话：010-62770177 转 3103 产品编号：023896-01

Springer 大学数学图书——影印版

- Information and Coding Theory（信息和编码理论）
  Jones, Gareth A., Jones, J.Mary
- Symmetries（对称）
  Johnson, D.L.
- Matrix Groups: An Introduction to Lie Group Theory（矩阵群：李群理论基础）
  Baker, Andrew
- Essential Mathematical Biology（生物数学引论）
  Britton, Nicholas F.
- Special Relativity（狭义相对论）
  Woodhouse, N.M.J.
- Second Year Calculus（高等微积分）
  Bressoud, David M.
- Linear Functional Analysis（线性泛函分析）
  Rynne, Bryan P., Youngson, Martin A.
- A First Course in Discrete Mathematics（离散数学引论）
  Anderson, Ian
- Discrete Mathematics（离散数学）
  Lovász, L., Pelikán, J., Vesztergombi, K.
- Reading, Writing, and Proving（数学的读写和证明）
  Daepp Ulrich, Gorkin Pamela
- Basic Stochastic Processes（随机过程基础）
  Brzeźniak Zdzisław, Zastawniak Tomasz
- The Fundamental Theorem of Algebra（代数基本定理）
  Fine Benjamin, Rosenberger Gerhard
- The Heritage of Thales（泰勒斯的遗产）
  Anglin W. S., Lambek J.

Springer 大学数学图书——翻译版

- 信息和编码理论（Information and Coding Theory）
  Jones, Gareth A., Jones, J.Mary
- 对称（Symmetries）
  Johnson, D.L.
- 矩阵群：李群理论基础（Matrix Groups: An Introduction to Lie Group Theory）
  Baker, Andrew
- 生物数学引论（Essential Mathematical Biology）
  Britton, Nicholas F.
- 狭义相对论（Special Relativity）
  Woodhouse, N.M.J.
- 高等微积分（Second Year Calculus）
  Bressoud, David M.
- 线性泛函分析（Linear Functional Analysis）
  Rynne, Bryan P., Youngson, Martin A.
- 离散数学引论（A First Course in Discrete Mathematics）
  Anderson, Ian
- 离散数学（Discrete Mathematics）
  Lovász, L., Pelikán, J., Vesztergombi, K.
- 数学的读写和证明（Reading, Writing, and Proving）
  Daepp Ulrich, Gorkin Pamela
- 随机过程基础（Basic Stochastic Processes）
  Brzeźniak Zdzisław, Zastawniak Tomasz
- 代数基本定理（The Fundamental Theorem of Algebra）
  Fine Benjamin, Rosenberger Gerhard
- 泰勒斯的遗产（The Heritage of Thales）
  Anglin W. S., Lambek J

序　言

在学校教书多年，当学生（特别是本科生）问有什么好的参考书时，我们所能推荐的似乎除了教材还是教材，而且不同教材之间的差别并不明显、特色也不鲜明。所以多年前我们就开始酝酿，希望为本科学生引进一些好的参考书，为此清华大学数学科学系的许多教授与清华大学出版社共同付出了很多心血。

这里首批推出的十余本图书，是从 Springer 出版社的多个系列丛书中精心挑选出来的。在丛书的筹划过程中，我们挑选图书最重要的标准并不是完美，而是有特色并包容各个学派（有些书甚至有争议，比如从数学上看也许不够严格），其出发点是希望我们的学生能够吸纳百家之长；同时，在价格方面，我们也做了很多工作，以使得本系列丛书的价格能让更多学校和学生接受，使得更多学生能够从中受益。

本系列图书按其定位，大体有如下四种类型（一本书可以属于多类，但这里限于篇幅不能一一介绍）。

一、适用面比较广、有特色并可以用作教材或参考书的图书。例如：

- Lovász et al.: Discrete Mathematics. 2003

  该书是离散数学的入门类型教材。与现有的教材（包括国外的教材）相比，它涵盖了离散数学新颖而又前沿的研究课题，同时还涉及信息科学方面既基本又有趣的应用；在着力打好数学基础的同时，也强调了数学与信息科学的关联。姚期智先生倡导和主持的清华大学计算机科学试验班，已经选择该书作为离散数学课程的教材。

二、在目前国内的数学教育中，课程主要以学科的纵向发展为主线，而对数学不同学科之间的联系讨论很少。学生缺乏把不同学科视为一个数学整体的训练，这方面的读物尤其欠缺。这是本丛书一个重要的着力点。最典型的是：

- Fine/Rosenberger: The Fundamental Theorem of Algebra. 1997

  该书对数学中最重要的定理——代数基本定理给出了六种证明，方法涉及到分析、代数与拓扑；附录中还给出了 Gauss 的证明和 Cauchy 的证明。全书以一个数学问题为主线展开，纵横数学的核心领域；结构严谨、文笔流畅、浅显易懂、引人入胜，是一本少见的能够让读者入迷的好读物，用它来引导学生欣赏和领会“数学的美”绝对不会落于空谈。该书适于自学、讨论，也是极好的短学期课程教材。

- Baker: Matrix Groups. 2001

  就内容而言，本书并不超出我国大学线性代数、抽象代数和一般拓扑学课程的内容，但是本书所讲的是李群和李代数的基础理论——这是现代数学和物理学非常重要的工具。各种矩阵群和矩阵代数是李群和李代数最典型和

最重要的例子，同时也能帮助学生建立数学不同学科之间的联系。从矩阵出发，既能把握李群和李代数的实质，又能学会计算和运用，所以这是一本不可多得的好书。

三、科学与技术的发展不断为数学提出新的研究课题，因此在数学学科的发展过程中，来自其他学科的推动力是不能忽视的。本系列中第三种类型的读物就是强调数学与其他学科的联系。例如：

- Woodhouse: Special Relativity. 2003

该书将物理与数学有机结合，体现了物理学家伽利略的名言：“大自然是一部用数学语言写成的巨著。” 不仅如此，本书作者还通过对线性代数、微积分、场论等数学的运用进一步强调并贯穿这样的观点：数学的真谛和发展存在并产生于物理或自然规律及其发现中。精读此书有助于理解物理学和数学的整体关系。

- Britton: Essential Mathematical Biology. 2003

生命科学在本世纪一定会有很大发展，其对数学的需求和推动是可以预见的。因此生物数学在应用数学中占有日益重要的地位，数学系培养的学生至少一部分人应当对这个领域有所了解。随着生命科学的迅速发展，生物数学也发展很快。本书由浅入深，从经典的问题入手，最后走向学科前沿和近年的热点问题。该书至少可以消除学生对生物学的神秘感。

四、最后一类是适合本科学生的课外读物。这类图书对激发和引导学生学习数学的兴趣会非常有帮助，而且目前国内也急需这样的图书。例如：

- Daepp/Gorkin: Reading, Writing and Proving. 2003

该书对初学高等数学的读者来说特别有意义。它的基本出发点是引导读者以研究的心态去学习，让读者养成独立思考的习惯，并进而成为研究型的学习者。该书将一个学习数学的过程在某种意义下程序化，努力让学习者养成一个好的学习习惯，以及学会如何应对问题。该书特色鲜明，类似的图书确实很少。

- Brzezniak/Zastawniak: Basic Stochastic Processes. 1998

随机过程理论在数学、科学和工程中有越来越广泛的应用，本书适合国内的需要。其主要特点是：书中配有的习题是巩固和延伸学习内容的基本手段，而且有十分完整的解答，非常适合自学和作为教学参考书。这是一本难得的好书，它 1999 年出版，到 2000 年已经是第 3 次印刷，到 2003 年则第 6 次重印。

- Anglin/Lambek: The Heritage of Thales. 1995

该书的基本内容是数学的历史和数学的哲学。数学历史是该书的线索，数学是内容的主体，引申到的是数学哲学。它不是一本史论型的著作，而是采用专题式编写方式，每个专题相对独立，所以比较易读、易懂，是本科生学习数学过程中非常好的课外读物。

本系列丛书中的大部分图书还将翻译为中文出版，以适应更多读者的需要。丛书筹划过程中，冯克勤、郑志勇、卢旭光、郑建华、王殿军、杨利军、叶俊、扈志明等很多清华大学的教授都投入了大量精力。他们之中很多人也将是后面中文版的译者。此外，我们今后还将不断努力丰富引进丛书的种类，同时也会将选书的范围在可能情况下进一步扩大到其他高水平的出版机构。

教育是科学和技术发展的基石，数学教育更是基石的基础。因为是基础所以它重要；也因为是基础所以它显示度不高，容易不被重视。只有将人才培养放到更高的地位上，中国成为创新型国家的目标才会成为可能。

本系列丛书的正式推出，圆了一个我们多年的梦，但这无疑仅仅是开始。

白峰杉

2006 年 6 月于清华园

Preface

This addition to the SUMS series of textbooks is an introduction to various aspects of discrete mathematics. It is intended as a textbook which could be used at undergraduate level, probably in the second year of an English undergraduate mathematics course. Some textbooks on discrete mathematics are written primarily for computing science students, but the present book is intended for students following a mathematics course. The place of discrete mathematics in the undergraduate curriculum is now fairly well established, and it is certain that its place in the curriculum will be maintained in the third millennium.

Discrete mathematics has several aspects. One fundamental part is **enumeration**, the study of counting arrangements of various types. We might count the number of ways of choosing six lottery numbers from $1, 2, \ldots, 49$, or the number of spanning trees in a complete graph, or the number of ways of arranging 16 teams into four groups of four. We develop methods of counting which can deal with such problems.

Next, **graph theory** can be used to model a variety of situations – road systems, chemical molecules, timetables for examinations. We introduce the basic types of graph and give some indication of what the important properties are that a graph might possess.

The third area of discrete mathematics to be discussed in this book is that of **configurations** or **arrangements**. Latin squares are arrangements of symbols in a particular way; such arrangements can be used to construct experimental designs, magic squares and tournament designs. This leads us on to have a look at block designs, which were discussed extensively by statisticians as well as mathematicians due to their usefulness in the design of experiments. The book closes with a brief introduction to the ideas behind error-correcting codes.

The reader does not require a great deal of technical knowledge to be able to cope with the contents of the book. A knowledge of the method of proof by induction, an acquaintance with the elements of matrix theory and of arithmetic modulo n, a familiarity with geometric series and a certain clarity of thought should see the reader through. Often the main problem encountered by the reader is not in the depth of the argument, but in looking at the problem in the "right way". Facility in this comes of course with practice.

Each chapter ends with a good number of examples. Hints and solutions to most of these are given at the end of the book. The examples are a mixture of fairly straightforward applications of the ideas of the chapter and more challenging problems which are of interest in themselves or are of use later on in the book.

My hope is that this text will provide the basis for a first course in discrete mathematics. Obviously the choice of material for such a course is dependent on the interests of the teacher, but there should be enough topics here to enable an appropriate choice to be made. The text has been influenced in countless ways by the many texts that have appeared over the years, but ultimately it is determined by my own preferences, likes and dislikes, and by my own experience of teaching discrete mathematics at different levels over many years, from masterclasses for 14-year-olds to final year honours courses.

I would like to thank the Springer staff for their encouragement to write this book and for their help in its production. Thanks is also due to Gail Henry for converting my manuscript into a LaTeX file, and to Mark Thomson for reading and commenting on many of the chapters.

University of Glasgow, June 2000

Contents

1
Counting and Binomial Coefficients

In this chapter we introduce the basic counting methods, the factorial function and the binomial coefficients. These are of fundamental importance to the subject matter of subsequent chapters. We start with two basic principles.

1.1 Basic Principles

(a) **The multiplication principle.** Suppose that an activity consists of k stages, and that the ith stage can be carried out in α_i different ways, irrespective of how the other stages are carried out. Then the whole activity can be carried out in $\alpha_1\alpha_2\ldots\alpha_k$ ways.

Example 1.1

A restaurant serves three types of starter, six main courses and five desserts. So a three-course meal can be chosen in $3 \times 6 \times 5 = 90$ ways.

(b) **The addition principle.** If $A_1, \ldots, A_k$ are pairwise disjoint sets (i.e. $A_i \cap A_j = \emptyset$ wherever $i \neq j$), then the number of elements in their union is

$$|A_1 \cup \cdots \cup A_k| = |A_1| + \cdots + |A_k| = \sum_{i=1}^{k} |A_i|.$$

Example 1.2

In the above example, how many different two-course meals (including a main course) are there?

Solution

There are two types of two-course meal to consider. Let A_1 denote the set of meals consisting of a starter and a main course, and let A_2 denote the set of meals consisting of a main course and a dessert. Then the required number is

$$\begin{aligned} |A_1 \cup A_2| &= |A_1| + |A_2| && \text{(by the addition principle)} \\ &= (3 \times 6) + (6 \times 5) && \text{(by the multiplication principle)} \\ &= 48. \end{aligned}$$

1.2 Factorials

How many ways are there of placing a, b and c in a row? There are six ways, namely

$$abc,\ acb,\ bac,\ bca,\ cab,\ cba.$$

Note that there are three choices for the first place, then two for the second, and then just one for the third; so by the multiplication principle there are $3 \times 2 \times 1 = 6$ possible orderings. In general, if we define $n!$ ("n factorial") by

$$n! = n(n-1)(n-2)\dots 2.1$$

then we have

Theorem 1.1

The number of ways of placing n objects in order is $n!$.

Example 1.3

Four people, A, B, C, D, form a committee. One is to be president, one secretary, one treasurer, and one social convener. In how many ways can the posts be assigned?

Solution

Think of first choosing a president, then a secretary, and so on. There $4! = 24$ possible choices.

The value of $n!$ gets big very quickly:

$$5! = 120, \quad 10! = 3\,628\,800, \quad 50! \cong 3.04 \times 10^{64}.$$

This is an example of what is called *combinatorial explosion*: the number of different arrangements of n objects gets huge as n increases. The enormous size of $n!$ lies behind what is known as the *travelling salesman problem* which will be studied further in Chapter 4. A traveller sets out from home, has to visit n towns and then return home. Given the mileages between the towns, how does the traveller find the shortest possible route? The naive approach of considering each of the $n!$ possible routes is impracticable if n is large, so another approach is needed.

In certain problems, only some of a given set of objects are to be listed.

Example 1.4

A competition on the back of a cereal packet lists ten properties of a car, and asks the consumer to choose the six most important ones, listing them in order of importance. How many different entries are possible?

Solution

There are ten possibilities for the first choice, then nine for the second, and so on down to five for the sixth. So, by the multiplication principle, the number of possible lists is

$$10 \times 9 \times 8 \times 7 \times 6 \times 5 = \frac{10!}{4!} = 151\,200.$$

In general, we have:

Theorem 1.2

The number of ways of selecting r objects from n, in order but with no repetitions, is

$$n(n-1)\dots(n-r+1) = \frac{n!}{(n-r)!}.$$

Example 1.5

A committee of 4 is to be chosen, as in Example 1.3, but this time there are 20 people to choose from. The number of choices of president, secretary, treasurer, social convener is

$$20 \times 19 \times 18 \times 17 = \frac{20!}{16!} = 116\,280.$$

1.3 Selections

Suppose that in Example 1.5 we wanted just the number of ways of choosing four people for a committee, not bothering about the positions they might fill. Denote by $\binom{20}{4}$ (and read as 20-choose-4) the number of ways in which we can choose 4 from 20 where order does not matter.

Each such choice of four from 20 can be ordered in 4! ways to give an assignment to particular positions within the committee, so, by Example 1.5,

$$4! \times \binom{20}{4} = \frac{20!}{16!}.$$

Thus

$$\binom{20}{4} = \frac{20!}{4!16!} = 4845.$$

This argument is general: $\frac{n!}{(n-r)!} = r! \times \binom{n}{r}$, so we have the following general formula.

Theorem 1.3

Let $\binom{n}{r}$ denote the number of unordered selections of r from n where repetitions are not allowed. Then

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}. \tag{1.1}$$

Since some students find the idea of turning ordered selections into unordered selections confusing, here is another way of deriving the formula (1.1).

Suppose we have to choose a team of r players from a pool of n, one of them to be appointed captain. This can be done by first choosing the team - and there are $\binom{n}{r}$ ways of doing this - and then choosing the captain - there are r ways of doing this. So there are $r\binom{n}{r}$ choices altogether. But we could, instead, first choose the captain - there are n ways of doing this - and then choose the $r-1$ other members of the team - and there are $\binom{n-1}{r-1}$ ways of doing this. So the number of choices is also $n\binom{n-1}{r-1}$. Thus

$$r\binom{n}{r} = n\binom{n-1}{r-1},$$

so that

$$\binom{n}{r} = \frac{n}{r}\binom{n-1}{r-1}. \tag{1.2}$$

But similarly, $\binom{n-1}{r-1} = \frac{n-1}{r-1}\binom{n-2}{r-2}$, on replacing n by $n-1$ and r by $r-1$, so we get

$$\binom{n}{r} = \frac{n}{r} \cdot \frac{n-1}{r-1}\binom{n-2}{r-2}.$$

Continuing in this way we obtain

$$\binom{n}{r} = \frac{n}{r} \cdot \frac{n-1}{r-1} \cdots \frac{n-(r-2)}{2} \cdot \binom{n-(r-1)}{1}.$$

Since $\binom{m}{1}$ is clearly always m, we obtain finally

$$\binom{n}{r} = \frac{n(n-1)\ldots(n-r+1)}{r!} = \frac{n!}{r!(n-r)!}$$

as before.

[Note in passing that this is a good example of the technique of **counting the same thing in two different ways.**]

Example 1.6

In the UK National Lottery, a participant chooses six of the numbers 1 to 49; order does not matter. So the number of possible choices is

$$\binom{49}{6} = \frac{49 \times 48 \times 47 \times 46 \times 45 \times 44}{6!} = 13\,983\,816.$$

So there is roughly one chance in 14 million of winning the jackpot!

Example 1.7

How likely is it that next week's lottery winning numbers will be disjoint from this week's?

Solution

There are $\binom{49}{6}$ possible selections next week. The number of these which are disjoint from this week's must be $\binom{43}{6}$, since six of the 49 numbers are ruled out. Since all $\binom{49}{6}$ selections are equally likely, the required probability is

$$\binom{43}{6} \Big/ \binom{49}{6} = 0.436\ldots.$$

Example 1.8

Binary sequences. There are 2^n n-digit binary sequences since each of the n digits is 0 or 1. For example, the eight binary sequences of length three are

$$000 \quad 001 \quad 010 \quad 011 \quad 100 \quad 101 \quad 110 \quad 111.$$

(a) How many binary sequences of length 12 contain exactly six 0s?

(b) How many have more 0s than 1s?

Solution

(a) The six 0s occupy six of the 12 positions. There are $\binom{12}{6} = 924$ choices of these six positions, and this gives the number required.

(b) There are $2^{12} - 924 = 3172$ sequences with unequal numbers of 0s and 1s. By symmetry, exactly half of these, i.e. 1586, will have more 0s than 1s.