

Springer 大学数学图书——影印版

Essential Mathematical Biology

生物数学引论

Nicholas F. Britton 著



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内 容 提 要

生物数学在应用数学中占有日益重要的地位, 数学系培养的学生至少一部分人应当对这个领域有所了解。本书由浅入深讲述生物数学基础理论, 从最经典的问题入手, 最后走向学科前沿和近年的热点问题; 内容先进, 讲述方法科学, 简洁明了, 易读性好。适合用作数学及生命科学专业高年级本科生相关课程教材或参考书。

Nicholas F. Britton
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序 言

在学校教书多年,当学生(特别是本科生)问有什么好的参考书时,我们所能推荐的似乎除了教材还是教材,而且不同教材之间的差别并不明显、特色也不鲜明。所以多年前我们就开始酝酿,希望为本科学生引进一些好的参考书,为此清华大学数学科学系的许多教授与清华大学出版社共同付出了很多心血。

这里首批推出的十余本图书,是从 Springer 出版社的多个系列丛书中精心挑选出来的。在丛书的筹划过程中,我们挑选图书最重要的标准并不是完美,而是有特色并包容各个学派(有些书甚至有争议,比如从数学上看也许不够严格),其出发点是希望我们的学生能够吸纳百家之长;同时,在价格方面,我们也做了很多工作,以使得本系列丛书的价格能让更多学校和学生接受,使得更多学生能够从中受益。

本系列图书按其定位,大体有如下四种类型(一本书可以属于多类,但这里限于篇幅不能一一介绍)。

一、适用面比较广、有特色并可以用作教材或参考书的图书。例如:

● **Lovász et al.: Discrete Mathematics. 2003**

该书是离散数学的入门类型教材。与现有的教材(包括国外的教材)相比,它涵盖了离散数学新颖而又前沿的研究课题,同时还涉及信息科学方面既基本又有趣的应用;在着力打好数学基础的同时,也强调了数学与信息科学的关联。姚期智先生倡导和主持的清华大学计算机科学试验班,已经选择该书作为离散数学课程的教材。

二、在目前国内的数学教育中,课程主要以学科的纵向发展为主线,而对数学不同学科之间的联系讨论很少。学生缺乏把不同学科视为一个数学整体的训练,这方面的读物尤其欠缺。这是本丛书一个重要的着力点。最典型的是:

● **Fine/Rosenberger: The Fundamental Theorem of Algebra. 1997**

该书对数学中最重要的定理——代数基本定理给出了六种证明,方法涉及到分析、代数与拓扑;附录中还给出了 Gauss 的证明和 Cauchy 的证明。全书以一个数学问题为主线展开,纵横数学的核心领域;结构严谨、文笔流畅、浅显易懂、引人入胜,是一本少见的能够让读者入迷的好读物,用它来引导学生欣赏和领会“数学的美”绝对不会落于空谈。该书适于自学、讨论,也是极好的短学期课程教材。

● **Baker: Matrix Groups. 2001**

就内容而言,本书并不超出我国大学线性代数、抽象代数和一般拓扑学课程的内容,但是本书所讲的是李群和李代数的基础理论——这是现代数学和物理学非常重要的工具。各种矩阵群和矩阵代数是李群和李代数最典型和

最重要的例子，同时也能帮助学生建立数学不同学科之间的联系。从矩阵出发，既能把握李群和李代数的实质，又能学会计算和运用，所以这是一本不可多得的好书。

三、科学与技术的发展不断为数学提出新的研究课题，因此在数学学科的发展过程中，来自其他学科的推动力是不能忽视的。本系列中第三种类型的读物就是强调数学与其他学科的联系。例如：

● **Woodhouse: Special Relativity. 2003**

该书将物理与数学有机结合，体现了物理学家伽利略的名言：“大自然是一部用数学语言写成的巨著。”不仅如此，本书作者还通过对线性代数、微积分、场论等数学的运用进一步强调并贯穿这样的观点：数学的真谛和发展存在并产生于物理或自然规律及其发现中。精读此书有助于理解物理学和数学的整体关系。

● **Britton: Essential Mathematical Biology. 2003**

生命科学在本世纪一定会有很大发展，其对数学的需求和推动是可以预见的。因此生物数学在应用数学中占有日益重要的地位，数学系培养的学生至少一部分人应当对这个领域有所了解。随着生命科学的迅速发展，生物数学也发展很快。本书由浅入深，从经典的问题入手，最后走向学科前沿和近年的热点问题。该书至少可以消除学生对生物学的神秘感。

四、最后一类是适合本科学生的课外读物。这类图书对激发和引导学生学习数学的兴趣会非常有帮助，而且目前国内也急需这样的图书。例如：

● **Daepf/Gorkin: Reading, Writing and Proving. 2003**

该书对初学高等数学的读者来说特别有意义。它的基本出发点是引导读者以研究的心态去学习，让读者养成独立思考的习惯，并进而成为研究型的学习者。该书将一个学习数学的过程在某种意义上程序化，努力让学习者养成一个好的学习习惯，以及学会如何应对问题。该书特色鲜明，类似的图书确实很少。

● **Brzezniak/Zastawniak: Basic Stochastic Processes. 1998**

随机过程理论在数学、科学和工程中有越来越广泛的应用，本书适合国内的需要。其主要特点是：书中配有的习题是巩固和延伸学习内容的基本手段，而且有十分完整的解答，非常适合自学和作为教学参考书。这是一本难得的好书，它 1999 年出版，到 2000 年已经是第 3 次印刷，到 2003 年则第 6 次重印。

● **Anglin/Lambek: The Heritage of Thales. 1995**

该书的基本内容是数学的历史和数学的哲学。数学历史是该书的线索，数学是内容的主体，引申到的是数学哲学。它不是一本史论型的著作，而是采用专题式编写方式，每个专题相对独立，所以比较易读、易懂，是本科生学习数学过程中非常好的课外读物。

本系列丛书中的大部分图书还将翻译为中文出版，以适应更多读者的需要。丛书筹划过程中，冯克勤、郑志勇、卢旭光、郑建华、王殿军、杨利军、叶俊、扈志明等很多清华大学的教授都投入了大量精力。他们之中很多人也将是后面中文版的译者。此外，我们今后还将不断努力丰富引进丛书的种类，同时也会将选书的范围在可能情况下进一步扩大到其他高水平的出版机构。

教育是科学技术发展的基石，数学教育更是基石的基础。因为是基础所以它重要；也因为基础所以它显示度不高，容易不被重视。只有将人才培养放到更高的地位上，中国成为创新型国家的目标才会成为可能。

本系列丛书的正式推出，圆了一个我们多年的梦，但这无疑仅仅是开始。

白峰杉

2006年6月于清华园

Preface

This book aims to cover the topics normally included in a first course on Mathematical Biology for mathematics or joint honours undergraduates in the UK, the USA and other countries. Such a course may be given in the second year of an undergraduate degree programme, but more often appears in the third year.

Mathematical Biology is not as hierarchical as many areas of Mathematics, and therefore there is some flexibility over what is included. As a result the book contains more than enough for two one-semester courses, e.g. one based on Chapters 1 to 4, mainly using difference equations and ordinary differential equations, and one based on Chapters 5 to 8, mainly using partial differential equations. However there are some classic areas that are covered in almost every course, the most obvious being population biology, often including epidemiology, and mathematical ecology of one or two species. Population genetics is also a classic area of application, although it does not appear in every first course. In spatially non-uniform systems there is even more choice, but reaction-diffusion equations are almost always included, with applications at the molecular and population level. Some large areas have been excluded through lack of space, such as bio-fluid dynamics and most of mathematical physiology, each of which could fill an undergraduate textbook on its own.

To cover the whole book the student will need a background in linear algebra, vector calculus, difference equations, and ordinary and partial differential equations, although a one-semester course without vector calculus and partial differential equations could easily be constructed. Methods only are required, and the necessary results are collected together in appendices. Some additional material appears on an associated website, at

<http://www.springeronline.com/1-85233-536-X>

This site includes more exercises, more detailed answers to the exercises in the book, and links to other useful sites. Some parts of it require more mathematical background than the book itself, including stochastic processes and continuum mechanics.

I would like to thank Jim Murray for his support and advice since my undergraduate days. I am grateful to all the (academic and non-academic) staff of the Department of Mathematical Sciences at the University of Bath for making this a genial place to work, the past and present members of the Centre for Mathematical Biology at the University of Bath for stimulating discussions, my students for their feedback on the lecture notes from which this book was developed, and Springer Verlag. Finally I would like to thank Suzy and Rachel for their love and support.

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