ENGLISH - CHINESE MODERN PHYSICS EXPERIMENT

英汉近代物理实验

主　编　史庆藩
副主编　鲁长宏　郑宁

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内 容 简 介

本书是在北京理工大学物理实验中心近代物理实验讲义的基础上，结合近年的教育教学改革成果编著而成。全书以开放式和研究型教学模式为前提，采用英汉双语的内容表现形式，力求实现培养学生的创新意识、创新精神和创新能力的教学目标。全书共8个单元26个实验，内容涉及Excel、Origin软件在数据处理和分析中的应用，以及原子及原子核物理、近代光学、激光、真空、X射线、电子衍射、磁共振及成像、微波、低温、半导体、凝聚态物理、等离子体物理等近代物理学著名实验和电子显微镜应用等现代测量技术。

本书可作为高等理工科院校本科生的近代物理实验课程的双语教材或参考书，也可供专业英语爱好者参考。

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前　言

“近代物理实验”是物理类专业高年级学生的重要基础课程之一，其内容主要由近、当代物理学发展过程中的一些著名实验以及导致工业革命的一些关键技术手段而构成。近代物理实验所涉及知识的综合性、操控技术的复杂性以及实验过程的研究性决定了该课程对于提升学生综合素质、加强创新意识、创新精神和创新能力的培养具有重要的作用。

当前国际竞争日趋激烈，对中国学校教育提出了新的挑战。为了培养有创新能力并在国际上有竞争力的一流人才，我们对近代物理实验进行了双语教学的改革。双语教学以外语作为沟通媒介，训练学生用双语思维能力获取物理实验知识。因此双语教学在培养学生创新能力的同时，也大大提高了学生的科技英语水平。

本书第一单元介绍了利用 Excel、Origin 软件处理和分析数据的方法。教学目的是训练学生掌握作图、统计分析、曲线拟合等数据处理的各种技术，使其所写实验报告能够达到科技论文发表所要求的水平。其他单元所精选的实验涵盖了原子及原子核物理、近代光学、激光、真空、X 射线、电子衍射、磁共振及成像、微波、低温、半导体、凝聚态物理及等离子体物理学的各个领域。其中有些实验是由教师的科研项目转化而成。通过这些实验的训练能使学生：①体会物理的思想和实验的技术在科学发现过程中的重要作用；②加深对相关理论的理解；③培养严谨求实的科学作风；④提高对于新的物理现象的敏锐洞察能力；⑤提高实验的操控技能；⑥初步掌握科学研究的方法（包括分析现象、数据采集与处理、科学归纳及发现物理规律等），为以后进入研究生课程的学习或走上工作岗位打下坚实的能力基础。

本书在内容撰写上以培养学生的综合性创新能力为宗旨，以相关理论课的讲述实态为参照，注意了理论与实验两方面的详略程度。此外，为了培养学生的物理兴趣，激发学生的探索精神，每一个实验之前简介一位与内容相关的著名物理学家的事迹，以及这些大家的一段演讲或著述。

本书实验内容的撰写贡献分别为：于广泽（4.1），王荣瑶（1.1、2.2、3.1、8.1），冯璐（2.1、2.4、3.2、3.7），孙镭（3.4、3.8），史庆藩（4.2、7.3、7.4），李林（1.2、2.3、3.6、7.5、8.3），李宾（7.1），郑宁（7.6、8.2），张丛（6.2），欧阳吉庭（5.1），鲁长宏（3.3、3.5、6.1、7.2）。留美博士郑宁统筹了全书的英文翻译。美国犹他大学（The University of Utah）的 Dr. Jon P. Johnson 检查了全书的英文表述。审校定稿由史庆藩、鲁长宏和郑宁完成。另外需要说明的是：本书的某

些章节或语句并没有采取完全英汉对照的格式，这是考虑到高年级学生已普遍具备良好的英文阅读能力。

本书是北京市教育委员会精品教材建设计划支持项目之一，并且自始至终得到了北京理工大学教务处、实验室设备处的大力支持，作者表示衷心感谢。本书还得到了国家自然科学基金的资助（项目号：10975014）。在成书过程中，还得到了研究生吴宇航和留学生 D. Ntirikwendera 的大力支持和帮助，在此一并致谢。

我们期待本书的出版能够有助于培养学生科学研究所需的综合素质。由于作者水平所限，书中不足之处在所难免，恳请读者提出宝贵的意见和建议。

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CONTENTS

Unit 1

Processing and Analysis of Data
(数据处理与分析)

1.1 APPLICATION OF EXCEL

Nowadays, well-developed computer technology has been extensively involved in a variety of scientific activities. For experimental data analysis, the commonly used software packages like Origin and Microsoft Excel have been used as powerful computational and graphical tools. As a good spreadsheet package Excel provides a convenient method for entering raw data, performing calculations, and illustrating both the numerical and graphical forms of experimental results. In this section, we will give some guides on how to perform the basic analysis and presentation of experimental data by means of Excel, including some tips on the use of statistical analysis, graphical presentation, and linear regression. A wide range of examples can be found in a Guide to Microsoft Excel for Scientists and Engineers[3].

I. *Statistical analysis of experimental data*

All the usual mathematical and statistical functions have been built into Excel spreadsheets. With the built-in functions, we can quickly perform various calculations involved in data analysis. An example is shown below to illustrate how to perform the statistical analysis of experimental data by Excel to obtain the estimated uncertainty in repeated measurements.

Example 1

Twenty repeated measurements (in mm) of the diameter of a metal bead using a micrometer are given in figure 1. The offset uncertainty and the resolution limit of the micrometer are known to be −0.003 mm and 0.004 mm, respectively.

(1) Calculate the mean, variance, standard deviation and standard error in the mean of the measurements.

(2) Considering the resolution limit of the micrometer (0.004 mm), calculate the combined uncertainty in measurements of the metal bead diameter.

(3) Give the final result of the measurements including the mean and the uncertainty to the appropriate number of significant figures.

In the data from twenty repeated measurements, 6.965 looks like a spurious data point. The reason to reject this bad data point will be given later. In the Excel worksheet, B6 to B24 show the 19 numbers of raw data without the spurious one, and C6 to C24 gives the calculations of $x' = x - (-0.003)$ mm. Now we calculate the mean and the standard deviation of the 19 numbers in column C.

(1) B28 is the mean of the sample obtained from the data of C6 to C24.

(2) The Variance and the standard deviation (STDEV) are shown respectively in D28 and C28 by using the built-in functions of Excel. The formulae in the calculations are

$$Variance = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$STDEV = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

(3) C31 is the standard error on the mean, which is given by the formula

$$Stderr = STDEV/\sqrt{n}$$

From the above analysis, we know all 19 data points are well scattered about the mean (6.9978 mm) with a standard deviation of about 0.0004 mm. However, for the suspect data point 6.968 (= 6.965 + 0.003) mm, the calculation $\left(\frac{6.9978 - 6.968}{0.00039} \approx 76\right)$ means that this data point lies far away from the mean. Assuming that a normal distribution for the sampled data is valid in the case of repeated measurements, there is only a very low possibility ~ 0.006% that the true value of the measurement lies outside the confidence limit $x_m - 4\sigma_s$ to $x_m + 4\sigma_s$. Therefore it is highly possible that the data point 6.965 occurs by chance, for instance, it is due to a transcription error when recording the data.

	A	B	C	D
4		*Experimental data*		
5		Diameter x (mm)	Diameter x' (mm)	
6		6.995	6.998	
7		6.993	6.996	
8		6.998	7.001	
9		6.997	7.000	
10		6.994	6.997	
11		6.993	6.996	
12		6.994	6.997	
13		6.993	6.996	
14		6.995	6.998	
15		6.996	6.999	
16		6.995	6.998	
17		6.994	6.997	
18		6.994	6.997	
19		6.993	6.996	
20		6.997	7.000	
21		6.994	6.997	
22		6.996	6.999	
23		6.998	7.001	
24		6.993	6.996	
25				
26		*The statistic quantities obtained from EXCEL functions*		
27	Count n=	Mean (x_m)	STDEV (s)	Variance
28	19	6.9978	0.001708	2.9181E-06
29				
30		Standard Error on the mean (σ_s)		
31		from STDEV	0.000391900	

Figure 1　Excel worksheet

Combining the uncertainty arising from the random error in the measurements with the systematic uncertainty due to the resolution limit of instrument, we have

$$U = \sqrt{U_a^2 + U_b^2} = \sqrt{0.00039^2 + \left(\frac{0.004}{\sqrt{3}}\right)^2} = \sqrt{0.0000001521 + 0.000005333} \approx 0.0023\,(\text{mm})$$

Note that the systematic uncertainty of the instrument resolution has the dominant contribution to the combined uncertainty. Based on the above analysis, the final result of the metal bead diameter is: (6.9978 ± 0.0023) mm.

Ⅱ. *Graphical presentation of experimental data*

Excel spreadsheet offers a variety of options for analyzing and presenting experimental data in graphical form. In the following examples, we give some typical applications of spreadsheets in the graphical presentation of the important features of an experiment.

Example 2

In an experiment to determine the crystal spacing of NaCl, 200 data values were recorded for the intensity of X-ray diffraction as a function of incident angle.

(1) Use an Excel spreadsheet to display the X-ray diffraction spectrum.

(2) Find the incident angles at the spectral peaks of K_α and K_β rays, and determine the crystal spacing of NaCl using Bragg's Law. (Hint: use the least squares method).

By transferring the raw data values into the spreadsheet (Table 1 in figure 2), we can plot the diffraction intensity of X-ray versus the incident angles, which gives the X-ray diffraction spectrum of the NaCl crystal (Graph 1 in figure 2). From the spectrum, the incident angles at which the K_α and K_β rays show the first- to third-order diffraction maxima in intensity are clearly seen. It reveals the crystal spacing 2d of NaCl relating to the incident angles, according to Bragg's Law

$$2d \cdot \sin\theta = n\lambda$$

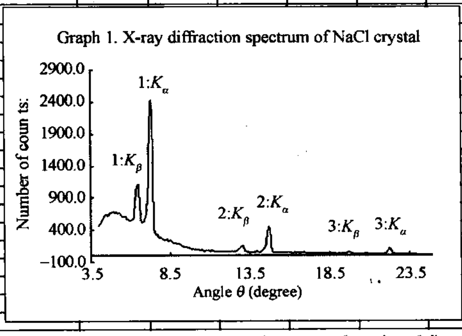


Table 2. The incident angles at the spectral peaks of K_α and K_β rays (from first order to the third order of diffraction pattern)

θ (degree)	Sinθ	X-ray	n	nλ (nm)
6.5	0.113	K_β	1	0.06306
7.3	0.127	K_α	1	0.07108
13.0	0.225	K_β	2	0.12612
14.7	0.254	K_α	2	0.14216
19.7	0.337	K_β	3	0.18918
22.3	0.379	K_α	3	0.21324

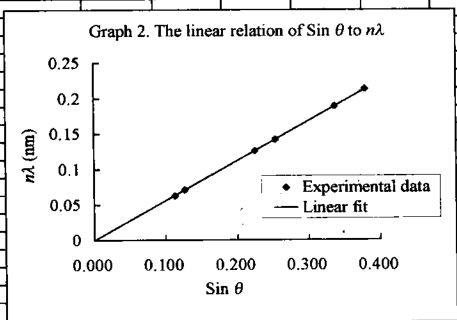


Table 1. Experimental data

Angle (degree)	Counts	Angle (degree)	Counts	Angle (degree)	Counts	Angle (degree)	Counts
4.0	460.3	9.0	176.0	14.0	55.7	19.0	21.7
4.1	501.0	9.1	175.0	14.1	49.7	19.1	26.3
4.2	546.7	9.2	154.7	14.2	67.7	19.2	19.3
4.3	587.0	9.3	135.3	14.3	77.7	19.3	29.3
4.4	639.0	9.4	131.3	14.4	108.7	19.4	28.0
4.5	635.3	9.5	132.0	14.5	186.0	19.5	34.3
4.6	630.0	9.6	120.7	14.6	334.3	19.6	33.3
4.7	662.7	9.7	118.0	14.7	446.3	19.7	50.7
4.8	662.7	9.8	96.0	14.8	221.3	19.8	40.7
4.9	684.0	9.9	103.3	14.9	67.7	19.9	26.0
5.0	664.7	10.0	95.7	15.0	55.0	20.0	23.0
5.1	689.3	10.1	89.0	15.1	43.0	20.1	20.7
5.2	658.7	10.2	93.3	15.2	42.3	20.2	19.0
5.3	669.0	10.3	95.0	15.3	43.3	20.3	24.7
5.4	631.0	10.4	81.0	15.4	42.0	20.4	23.0
5.5	609.3	10.5	93.0	15.5	47.3	20.5	21.0
5.6	604.0	10.6	67.0	15.6	48.0	20.6	22.0
5.7	607.7	10.7	79.3	15.7	40.7	20.7	23.0
5.8	561.0	10.8	73.3	15.8	44.0	20.8	26.3
5.9	549.3	10.9	79.3	15.9	40.7	20.9	19.0
6.0	549.7	11.0	69.0	16.0	43.7	21.0	23.3
6.1	568.3	11.1	73.3	16.1	42.7	21.1	22.0
6.2	575.3	11.2	67.3	16.2	32.0	21.2	20.3
6.3	797.7	11.3	61.0	16.3	41.0	21.3	19.0
6.4	1062.0	11.4	62.0	16.4	32.7	21.4	20.3
6.5	1114.0	11.5	59.3	16.5	41.3	21.5	19.7
6.6	787.0	11.6	69.7	16.6	27.3	21.6	22.7
6.7	515.0	11.7	64.3	16.7	29.7	21.7	19.7
6.8	528.7	11.8	66.0	16.8	41.0	21.8	24.7
6.9	536.7	11.9	65.3	16.9	42.0	21.9	24.0
7.0	707.0	12.0	57.0	17.0	31.3	22.0	26.3
7.1	1307.7	12.1	63.0	17.1	36.3	22.1	48.7
7.2	1982.7	12.2	55.7	17.2	30.0	22.2	85.3
7.3	2405.3	12.3	66.0	17.3	30.7	22.3	107.7
7.4	1489.0	12.4	59.7	17.4	32.7	22.4	65.0
7.5	430.0	12.5	65.7	17.5	33.0	22.5	37.7
7.6	335.7	12.6	64.0	17.6	29.7	22.6	16.0
7.7	331.0	12.7	82.3	17.7	32.3	22.7	19.0
7.8	287.0	12.8	86.7	17.8	29.0	22.8	15.0
7.9	289.3	12.9	120.0	17.9	31.0	22.9	19.3
8.0	258.0	13.0	156.3	18.0	28.7	23.0	15.3
8.1	247.7	13.1	153.7	18.1	26.0	23.1	19.0
8.2	237.0	13.2	67.7	18.2	27.0	23.2	13.3
8.3	249.3	13.3	63.3	18.3	27.7	23.3	19.0
8.4	230.3	13.4	55.0	18.4	26.7	23.4	15.7
8.5	237.3	13.5	51.7	18.5	27.0	23.5	15.3
8.6	209.7	13.6	53.0	18.6	26.7	23.6	15.0
8.7	192.7	13.7	46.3	18.7	33.0	23.7	19.0
8.8	191.0	13.8	50.3	18.8	26.7	23.8	19.7
8.9	187.0	13.9	60.3	18.9	25.0	23.9	17.3

Figure 2 Experiment of the dystal spacing of NaCl

In table 2 of Figure 2, the individual incident angles corresponding to the peaks of the K_α and K_β rays are listed, and the quantities sin θ and $n\lambda$ are calculated respectively in column F and I.

Graph 2 of Figure 2 is the plot of $n\lambda$ against sin θ, which clearly shows a linear relationship between sin θ and $n\lambda$. Thus we can determine the crystal spacing of NaCl by using the least squares method. A linear regression of data pairs (sin θ, $n\lambda$) gives the gradient value as 0.561043 nm, which is the value of the crystal spacing ($2d$) of NaCl. Compared to the accepted value $2d_{\text{theory}}$ = 0.56402 nm, the fractional error in this measurement is 0.5%.

Ⅲ. *Linear regression analysis of experimental data*

A linear relationship of two quantities exists in many experimental measurements. As was demonstrated in example 2, the best-fitted straight line to the two quantities with a linear relationship was determined by the method of the least squares. From this, the required physical quantities can be derived based on the slope or intercept of a straight-line fitting of experimental data. Excel provides a powerful means for the linear regression of the experimental data.

Example 3

The following data were obtained from a simple pendulum experiment to determine the acceleration due to gravity. The period of the pendulum (P) as a function of the length of the pendulum (L) was shown in the Excel worksheet below (A5 to B12).

(1) Plot these raw data in Excel with error bars.

(2) The formula relating P and L is $P = 2\pi\sqrt{\frac{L}{g}}$. Find a linearised form of the above equation, and use the equation editor to display the linearised equation.

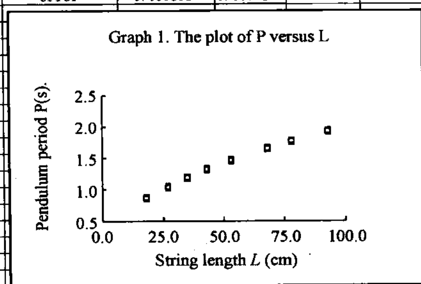
(3) Calculate the quantities in the linearised equation. Plot x-y scattering graph and use the trendline function to fit a straight line passing through data points. Make the fitted equation display on the graph with enough significant figures.

(4) Use the built-in function LINEST to obtain the value of the gradient and intercept and their errors.

(5) Use the values of the gradient and its error to calculate g (including the uncertainty).

(6) Calculate the error bars for each of the points plotted on the linearised graph. Add these error bars to the plot.

	A	B	C	D	E	F
3	*Table 1. The data to be plotted*					
4	L (cm)±0.1 cm	P(s)±0.05 s	ΔP (s)	P^2 (s^2)	Δ (P^2)	
5	18.0	0.8642	0.05	0.7468	0.0864	
6	27.0	1.0398	0.05	1.0812	0.1040	
7	35.0	1.1897	0.05	1.4154	0.1190	
8	43.0	1.3228	0.05	1.7498	0.1323	
9	53.0	1.4643	0.05	2.1442	0.1464	
10	68.0	1.6569	0.06	2.7453	0.1657	
11	78.0	1.7751	0.05	3.1510	0.1775	
12	93.0	1.9375	0.05	3.7539	0.1938	
13						
14						
15						
16	**From LINEST**					
17		gradient	intercept			
18	*value*	0.040247	0.010642			
19	*error*	0.000131	0.007523			



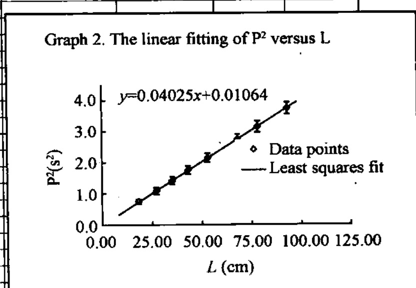


Figure 3 Pendulum experiment

In table 1 of figure 3, A5 to C12 are the raw data of measurements. Graph 1 of figure 3 shows the raw data with error bars at P. The formula relating P and L is

$$P = 2\pi\sqrt{\frac{L}{g}} \tag{1}$$

By linearising the above equation, we obtain a linear relationship between P^2 and L:

$$P^2 = \frac{4\pi^2}{g}L \tag{2}$$

The values of P^2 are calculated in D6 to D12. In the plot of P^2 against L (Graph 2 in figure 3), the linear regression of data pairs (P^2, L) is performed by adding the trendline onto the plot. The gradient of the straight line is $4\pi^2/g$.

From the trendline equation, the gradient is obtained as 0.04025. Hence we have

$$g = 4\pi^2/gradient = 4 \times 9.8696/0.04025 = 980.8348\text{cm} \cdot \text{s}^{-2}$$

By using LINEST function, the values and errors of the gradient and intercept are calculated in cells array B18 to C19. The error of the gradient is obtained as 0.01312.

The error in g is calculated by

$$\Delta g = \frac{\Delta(gradient)}{gradient} \times g = \frac{0.000131}{0.04025} \times 980.8348 = 3.192\text{cm} \cdot \text{s}^{-2}$$

Thus we have the result for the acceleration due to gravity

$$g = (9.808 \pm 0.032)\text{m} \cdot \text{s}^{-2}$$

From the error on P, we can estimate the error on P^2 by using the error propagation formula

$$\Delta(P^2) = 2P\Delta P$$

The errors on P^2 are shown in the data table from E5 to E12 and are plotted in Graph 2 of figure 3.

1.2 APPLICATION OF ORIGIN

Nowadays, since computer technology has developed very well, it has been used widely in a variety of scientific activities. In the field of experimental data analysis, software packages like Origin, MATLAB, Mathematica and Microsoft Excel have been used as effective computational and graphical tools. Origin includes a suite of features that cater to the needs of scientists and engineers alike. Multi-sheet workbooks, publication-quality graphics, and standardized analysis tools provide a tightly integrated workspace for you to import data, create and annotate graphs, explore and analyze data, and publish your work. In this chapter, we will give some guides on how to perform the basic analysis and presentation of experimental data by means of Origin, including some tips on the use of statistical analysis, graphical presentation, and linear regression. The data of all examples of this chapter are from the Origin, which is located in the Samples subfolder of the Origin program folder.

I. *Introduction*

The Origin Workspace contains the following sections: 1, Menu Bar; 2, Toolbars; 3, Project Explorer; 4, Work space; 5, Status Bar, which is also shown in figure 1.

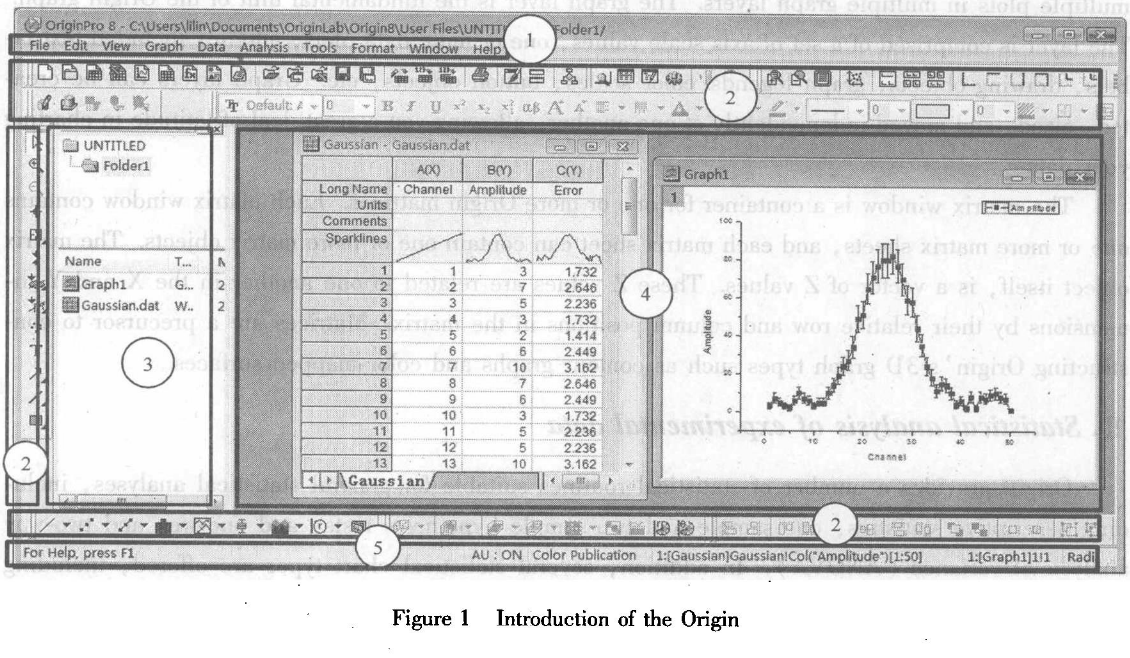


Figure 1 Introduction of the Origin

Project Explorer window is provided to help you organize the contents of your Origin project file. Using Project Explorer, you can create a folder structure inside the Origin Project file that is similar in form and function to the familiar folder structure of Windows Explorer. In addition, you can use Project Explorer to create new project files from a portion of an existing project file or to append the contents of another project file to your current file. The Project Explorer workspace can be hidden or restored as needed.

Tips:

(1) *To create a folder, right-click on the Project Explorer and choose* ***New Folder*** *from the context menu.*

(2) *You can right-click on the folder and select* ***Save As Project*** *to create new project files from a portion of an existing project file.*

(3) *Select* ***File:Append*** *or right-click on the Project Explorer and choose* ***Append Project*** *to append the contents of another project file to your current file.*

Origin has numerous windows and workspaces available for completing various tasks. You can see all types of windows from the ***New*** dialog (**File:New**). The most frequently used windows are **Workbook**, **Graph**, and **Matrix**.

The workbook typically serves as a container for your experimental data and your analysis results. Each Origin workbook is composed of one or more Origin worksheets. Each worksheet, in turn, is composed of one or more worksheet columns or datasets. These datasets may be operated on in full, or in part, from the user interface or from the command line in the Origin Command Window.

The Graph window is a container for graphical depictions of your experimental data and your analysis results. Graph windows may contain a single plot in a single graph layer or they may contain multiple plots in multiple graph layers. The graph layer is the fundamental unit of the Origin graph. The layer is comprised of a set of axis scale values, one or more data plots, and any included text labels, drawing objects, graph legends/color scales, button objects, etc. Graph layers can be created, sized, and moved independently of one another, allowing you a great deal of latitude in charting your data.

The matrix window is a container for one or more Origin matrices. Each matrix window contains one or more matrix sheets, and each matrix sheet can contain one or more matrix objects. The matrix object itself, is a vector of Z values. These Z values are related to one another in the X and Y dimensions by their relative row and column positions in the matrix. Matrices are a precursor to constructing Origin's 3D graph types such as contour graphs and color-mapped surfaces.

Ⅱ. *Statistical analysis of experimental data*

Origin provides a number of statistical routines suitable for general statistical analyses, including descriptive statistics, one-sample and two-sample hypothesis tests, and one-way and two-way analysis of variance (ANOVA). In addition, several statistical chart types are offered, including histograms and box charts. In this section, we will focus on statistics on columns and statistics on rows of descriptive statistics.

The **Statistics on Columns** tool produces a variety of summary statistics for the selected rows of data, including means, sum, variance, standard deviation and standard error etc. An example is shown below to illustrate how to perform the statistical analysis of experimental data.

Example 1

(1) Start with an empty worksheet, select **File: Import: Single ASCII** to open the *Import Single ASCII* dialog, browse to the *\Samples \Statistics* subfolder of the Origin program folder, and im-

port the file *diameter. dat*.

(2) Highlight columns 1 and select **Statistics**: **Descriptive Statistics**: **Statistics on Columns**. Make sure to check the **N Total**, **Mean**, **Standard Deviation**, **SE of mean**, **Variance** and **Sum** check boxes on *Quantities to Compute* > *Moments* branch to output these results. Select *Book* is <*new*> on *Output Reports To* > *Descriptive Statistics Tables* branch to open the results in the new window. Lists under the *Computation Control* branch provide options for calculating quantities, here we select *Variance Divisor of Moment* is *DF*.

(3) After you click the OK button, a report table sheet is created to show all the specified statistics, shown in the figure 2.

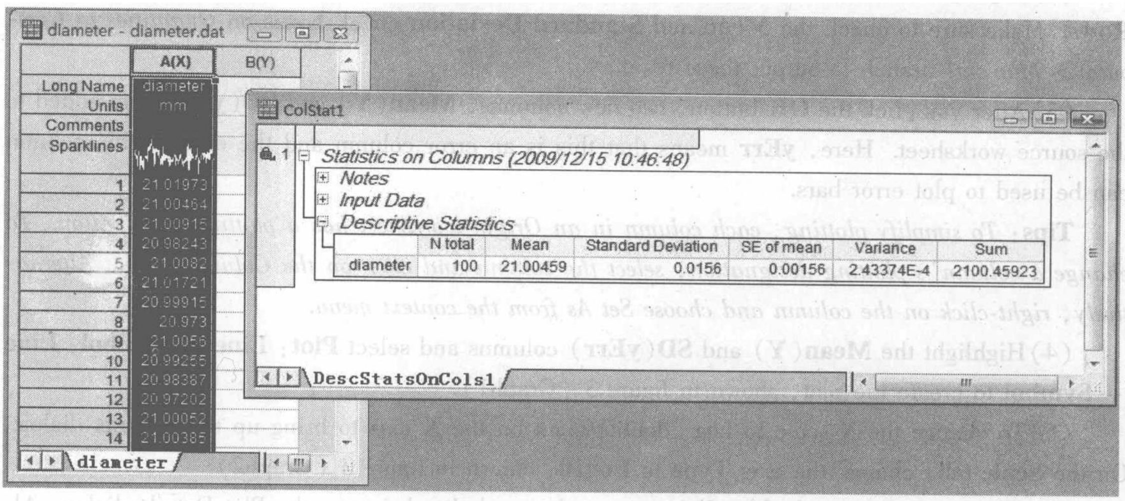


Figure 2 Report table of statistics on columns

The formulae in the calculations are

$$\textit{Standard Deviation},\ s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{d}},\ \textit{Variance} = s^2,\ \textit{SE of mean} = s/\sqrt{n}$$

Where, $d = n - 1$ for *DF*, $d = n$ for *N* and *DF*, *N* is the options in the *Computation Control* > *Variance Divisor of Moment*.

From the above analysis, we know all 100 data points are well scattered about the mean (21.00459 mm) with a standard deviation of about 0.0016 mm. Combining the uncertainty arising from the random error in the measurements with the systematic uncertainty due to the resolution limit of instrument and considering the resolution limit of the micrometer (0.004 mm), we have

$$U = \sqrt{U_a^2 + U_b^2} = \sqrt{0.0016^2 + \left(\frac{0.004}{\sqrt{3}}\right)^2} \approx 0.0028\text{mm}$$

Note that the systematic uncertainty of the instrument resolution has the dominant contribution to the combined uncertainty. Based on the above analysis, the final result of the diameter is: 21.0046 (0.0028) mm.