

Automata, Computability, and Complexity
Theory and Applications

自动机理论与应用



Elaine Rich



清华大学出版社

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Elaine Rich 著

清华大学出版社 北京

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出版说明

进入 21 世纪,世界各国的经济、科技以及综合国力的竞争将更加激烈。竞争的中心无疑是对人才的竞争。谁拥有大量高素质的人才,谁就能在竞争中取得优势。高等教育,作为培养高素质人才的事业,必然受到高度重视。目前我国高等教育的教材更新较慢,为了加快教材的更新频率,教育部正在大力促进我国高校采用国外原版教材。

清华大学出版社从1996年开始,与国外著名出版公司合作,影印出版了"大学计算机教育丛书(影印版)"等一系列引进图书,受到国内读者的欢迎和支持。跨入21世纪,我们本着为我国高等教育教材建设服务的初衷,在已有的基础上,进一步扩大选题内容,改变图书开本尺寸,一如既往地请有关专家挑选适用于我国高校本科及研究生计算机教育的国外经典教材或著名教材,组成本套"大学计算机教育国外著名教材系列(影印版)",以飨读者。深切期盼读者及时将使用本系列教材的效果和意见反馈给我们。更希望国内专家、教授积极向我们推荐国外计算机教育的优秀教材,以利我们把"大学计算机教育国外著名教材系列(影印版)"做得更好,更适合高校师生的需要。

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PREFACE

This book has three goals:

- 1. To introduce students to the elegant theory that underlies modern computing.
- 2. To motivate students by showing them that the theory is alive. While much of it has been known since the early days of digital computers (and some of it even longer), the theory continues to inform many of the most important applications that are considered today.
- 3. To show students how to start looking for ways to exploit the theory in their own work.

The core of the book, as a standard textbook, is Parts I through V. They address the first of the stated goals. They contain the theory that is being presented. There is more material in them than can be covered in a one-semester course. Sections that are marked with a • are optional, in the sense that later material does not, for the most part, depend on them. The Course Plans section on page xv suggests ways of selecting sections that are appropriate for some typical computer science courses.

Then there are seventeen appendices:

- Appendix A reviews the mathematical concepts on which the main text relies. Students should be encouraged to review it during the first week of class.
- Appendix B describes techniques for working with logical formulas (both Boolean and first-order).
- Appendices C, D, E and F treat selected theoretical concepts in greater depth. In particular, they contain the details of some proofs that are only sketched in the main text.
- Appendices G through Q address the second and third goals. They describe applications of the techniques that are described in the main body of the book. They also contain some interesting historical material. Although they are long (at least in comparison to the space that is devoted to applications in most other books in this area), they only skim the surface of the applications that they present. But my hope is that that is enough. The World Wide Web has completely changed our ability to access knowledge. What matters now is to know that something exists and thus to look for it. The short discussions that are presented in these appendices will, I hope, give students that understanding.

There is a Web site that accompanies this book: http://www.theoryandapplications.org/. It is organized into the same sections as the book, so that it is easy to follow the two in parallel. The symbol \square following a concept in the text means that additional material is available on the Web site.

Throughout the text, you'll find pointers to the material in the appendices, as well as to material on the book's Web site. There are also some standalone application notes. These pointers and notes are enclosed in boxes, and refer you to the appropriate appendix and page number or to the Web. The appendix references look like this:

This technique really is useful. (H. 1. 2.)

Notation

It is common practice to write definitions in the following form:

A something is a *special something* if it possesses property P.

This form is used even though property P is not only a sufficient but also a necessary condition for being a special something. For clarity we will, in those cases, write "if and only if", abbreviated "iff", instead of "if". So we will write:

A something is a **special something** iff it possesses property P.

Throughout the book we will, with a few exceptions, use the following naming conventions:

		Examples
sets	capital letters, early in the alphabet, plus S	A, B, C, D, S
logical formulas	capital letters, middle of the alphabet	P, Q, R
predicates and relations	capital letters, middle of the alphabet	P, Q, R
logical constants	subscripted X's and specific names	X_1, X_2 , John, Smoky
functions	lower case letters or words	f, g, convert
integers	lower case letters, middle of the alphabet	i, j, k, l, m, n
string-valued variables	lower case letters, late in the alphabet	s, t, u, v, w, x, y
literal strings	written in computer font	abc. aabbb
language-valued variables	upper case letters starting with L	L, L_1, L_2
specific languages	nonitalicized strings	A ⁿ B ⁿ . WW
regular expressions	lower case Greek letters	α, β, γ
states	lower case letters, middle of the alphabet	p, q, r, s, t
nonterminals in grammar rules	upper case letters	A, B, C, S, T
working strings in grammatical derivations	lower case Greek letter	α, β, γ
strings representing a PDA's stack	lower case Greek letter	α, β, γ
other variables	lower case letters, late in the alphabet	<i>x</i> , <i>y</i> , <i>z</i>

Programs and algorithms will appear throughout the book, stated at varying levels of detail. We will use the following formats for describing them:

 Exact code in some particular programming language will be written the same way other strings are written. • Algorithms that are described in pseudocode will be written as:

Until an even-length string is found do:

Generate the next string in the sequence.

When we want to be able to talk about the steps, they will be numbered, so we will write:

- 1. Until an even-length string is found do:
 - 1.1. Generate the next string in the sequence.
- 2. Reverse the string that was found.

When comments are necessary, as for example in code or in grammars, they will be preceded by the string /*.

Course Plans

Appendix A summarizes the mathematical concepts on which the rest of the book relies. Depending on the background of the students, it may be appropriate to spend one or more lectures on this material. At the University of Texas, our students have had two prior courses in logic and discrete structures before they arrive in my class, so I have found that it is sufficient just to ask the students to read Appendix A and to work a selection of the exercises that are provided at the end of it.

Part I lays the groundwork for the rest of the book. Chapter 2 is essential, since it defines the fundamental structures: strings and languages. I have found that it is very useful to cover Chapter 3, which presents a roadmap for the rest of the material. It helps students see where we are going and how each piece of the theory fits into the overall picture of a theory of computation. Chapter 4 introduces three ideas that become important later in the book. I have found that it may be better to skip Chapter 4 at the beginning of my class and to return to each of its sections once or twice later, as the concepts are required.

If the optional sections are omitted, Chapters 5, 6, 8, 9, 11–14, 17–21, and, optionally, 23 and/or 24 cover the material in a standard course in Automata Theory. Chapter 15 (Context-Free Parsing) contains material that many computer science students need to see and it fits well into an Automata Theory course. I used to include much of it in my class. But that material is often taught in a course on Programming Languages or Compilers. In that case, it makes sense to omit it from the Automata Theory course. In its place, I now cover the optional material in Chapter 5, particularly the section on stochastic finite automata. I also cover Chapter 22. I've found that students are more motivated to tackle the difficult material (particularly the design of reduction proofs) in Chapter 21 if they can see ways in which the theory of undecidability applies to problems that are, to them, more intriguing than questions about the behavior of Turing machines.

This text is also appropriate for a broader course that includes the core of the classic theory of automata plus the modern theory of complexity. Such a course might

cover Chapters 2-3, 5, 8, 11, 13, 17-21, and 27-30, omitting sections as time pressures require.

This text is unique in the amount of space it devotes to applications of the core theoretical material. In order to make the application discussions coherent, they are separated from the main text and occur in the appendices at the end of the book. But I have found that I can substantially increase student interest in my course by sprinkling application discussions throughout the term. The application references that occur in the main text suggest places where it makes sense to do that.

Resources for Instructors

There is a website, www.prenhall.com/rich, that contains materials that have been designed to make it easy to teach from this book. In particular, it contains:

- a complete set of Powerpoint slides,
- solutions to many of the Exercises, and
- additional problems, many of them with solutions.

I would like to invite instructors who use this book to send me additional problems that can be shared with other users.

ACKNOWLEDGMENTS

This book would not have been possible without the help of many people. When I first began teaching CS 341, Automata Theory, at the University of Texas, I was given a collection of notes that had been written by Bob Wall and Russell Williams. Much of the material in this book has evolved from those notes. I first learned automata theory from [Hopcroft and Ullman 1969]. Over the years that I have taught CS 341, I have used several textbooks, most frequently [Lewis and Papadimitriou 1988] and [Sipser 2006]. Much of what I have written here has been heavily influenced by the treatment of this material in those books.

Several of my friends, colleagues, and students have provided examples, answered numerous questions, and critiqued what I have written. I am particularly indebted to Don Baker, Volker Bandke, Jim Barnett, Jon Bentley, Gary Bland, Jaime Carbonell, Alan Cline, Martin Cohn, Dan Connolly, Ann Daniel, Chris Edmonson-Yurkanan, Scott Fahlman, Warren Gish, Mohamed Gouda, Jim Hendler, Oscar Hernandez, David Jefferson, Ben Kuipers, Greg Lavender, Tim Maxwell, Andy Mills, Jay Misra, Luay Nakhleh, Gordon Novak, Gabriela Ochoa, Dewayne Perry, Brian Reid, Bob Rich, Mike Scott, Cathy Stacy, Peter Stone, Lynda Trader, and David Zuckerman. Luay Nakhleh, Dan Tamir, and Bob Wall have used drafts of this book in their classes. I thank them for their feedback and that of their students.

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CREDITS

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A quilt, Blue Tweed (1996, 53" x 80", cotton, machine pieced and quilted), made by the author. Notice that your eye fills in the vertical lines, so they appear to run the length of the quilt, even though the colors in the middle of the quilt are all about the same. Quilt photography by David Reaves.

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xx Credits

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