数学专业英语

ENGLISH FOR MATHEMATICS

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哈尔滨工业大学出版社

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内 容 提 要

本书为各高等院校数学及相关专业英语基础教材，也适用于从事数学方面理论研究的读者参考。本书共设七个单元，主要内容涉及：数学史、数学分析、高等代数、概率论等学科中的基础经典问题以及具有代表性的内容。另外，本书的词汇表中列出了国家已经确定的和被广泛认同的专业术语，便于读者了解和查阅。

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Preface

With China's joining the WTO and the rapid development of science and technology in modern society, English is playing an important role in more and more fields. The international exchange in distinct disciplines is increasing every day. Both mathematical knowledge and foreign language ability are required of mathematician in our times. Teaching of English for mathematics thus becomes essential. This book is compiled to meet the requirements of the students in mathematics department. It is based on College English Curriculum(CEC). According to CEC, the aim of college English teaching is to cultivate students to gain strong ability of reading, a fair ability of listening & translating and the basic mastery of skills of writing & speaking, so that they are able to use English as a tool to obtain new information in their field. This is also expected to lay a foundation for the further improvement of their English ability. The book covers topics of several main branches of mathematics. Grateful acknowledgement is presented here to the staff of School of Mathematical Science of Heilongjiang University for their great support in the compiling process of this book.

Since the writing work was done in a rush, errors and mistakes could hardly be avoided. Suggestions or comments are therefore very welcome.

CONTENTS

1

Math Story and Strategies

1.1 THE NUMBER π

The most famous quantity in mathematics is the ratio of the circumference of a circle to its diameter, which is also known as the number pi and denoted by the Greek letter π

$$\frac{\text{circumference}}{\text{diameter}} = \pi$$

The symbol π was not introduced until just over two hundred years ago. The ancient Babylonians estimated this ratio as 3 and, for their purposes, this approximation was quite sufficient. According to the Bible, the ancient Jews used the same value of π. The earliest known trace of an approximate value of π was found in the Ahmes Papyrus written in about 16th century B.C., in which, indirectly, the number π is referred to as equal to 3.160 5. Greek philosopher and mathematician Archimedes, who lived about 225 B.C., estimated the value of pi to be less than $3\frac{1}{7}$ but more than $3\frac{10}{71}$. Ptolemy of Alexandria (c. 150 B.C.) gives the value of π to be about 3.141 6. In the far East, around 500

A.D., a Hindu mathematician named Aryabhata, who worked out a table of a sines, used for π the value 3.141 6. Tsu Chung-Chih of China, who lived around 470, obtained that π has a value between 3.141 592 6 and 3.141 592 7, and after him no closer calculation of π was made for one thousand years. The Arab Al Kashi about 1 430 obtained the amazingly exact estimated value for π of 3.141 592 653 589 793 2. There were several attempts made by various mathematicians to compute the value of π to 140, then 200, then 500 decimal places. In 1853, William Shanks carried the value of π to 707 decimal places. However, nobody seemed to be able to give the exact value for the number π.

What is the exact value of the number π? A mathematician made an experiment in order to find his own estimation of the number π. In his experiment, he used an old bicycle wheel of diameter 63.7 cm. He marked the point on the tire where the wheel was touching the ground and he rolled the wheel straight ahead by turning it 20 times. Next, he measured the distance traveled by the wheel, which was 39.69 meters. He divided the number 3 969 by 20×63.7 and obtained 3.115 384 615 as an approximation of the number π. Of course, this was just his estimate of the number π and he was aware that it was not very accurate.

The problem of finding the exact value of the number π inspired scientists and mathematicians for many centuries before it was solved in 1 761 by Johann Heinrich Lambert(1728 ~ 1777). Lambert proved that the number π cannot be expressed as a fraction or written in a decimal form using only a finite number of digits. Any such representation would always be only an estimation of the number π. Today, we call such numbers *irrational*. The ancient Greeks already knew about the existence of irrational numbers, which they called *incommensurables*. For example, they knew that the length of the diagonal of a square, with side of length equal to one length unit, is such a value. This value, which is denoted $\sqrt{2}$ and is equal to the number x such that $x^2 = 2$, cannot be expressed as a fraction.

Today in schools we use the estimation 3.14 for the number π, and of course this is completely sufficient for the type of problems we discuss in class. However, it was quickly noticed that in real life we need a better estimate to find more accurate measurements for carrying out construction projects, sea navigations and military applications. For most practical purposes, no more than 10 digits of π are required. For mathematical computation, even with astronomically precise calculations, no more than fifty exact digits of π are really necessary: 3.1415926535 8979323846 2643383279 5028841971 6939937510. However, with the power of today' s supercomputers, we are able to compute more than hundreds of billions of digits of the number π. You can download at the web site http://www.verbose.net

files with the exact digits of the number π up to 200 million decimals. We also have the following approximations of the number π:

3.1415926535 8979323846 2643383279 5028841971 6939937510
5820974944 5923078164 0628620890 8628034825 3421170679
8214808651 3282306647 0938446095 5058223172 5359408128
4811174502 8410270193 8521105559 6446229489 5493038196
4428810975 6659334461 2847564823 3786783165 2712019091
4564856692 3460348610 4543266482 1339360726 0249141273
7245870066 0631558817 4881520920 9628292540 9171536436
7892590360 0113305305 4882046652 1384146951 9415116094
3305727036 5759591953 0921861173 8193261179 3105118548
0744623799 6274956735 1885752724 8912279381 8301194912

Since the number π is the ratio of the circumference of a circle to its diameter, we can write a formula for the circumference of a circle, which is

$$C = \pi d$$

where C denotes the circumference and d denotes the diameter of the circle. If r denotes the radius of the circle then $d = 2r$, and we can rewrite the formula for the circumference as

$$C = 2\pi r$$

Words and Expressions

quantity *n*. 量,数量
ratio *n*. 比,比率
circumference *n*. 周长,圆周
circle *n*. 圆,圈,圆周
the circumference of a circle 圆的周长
diameter *n*. 直径
the ratio of the circumference of a circle to its diameter 圆周长与其直径的比
denote *vt*. 指示,表示
be denoted by 被表示为
approximate *adj*. 逼近的,近似的
approximation *n*. 逼近,近似;近似法;近似值
approximate value 近似值
Archimedes 阿基米德
pi π(圆周率)
divide *vt*. 除,等分
fraction *n*. 分数,小数;分式
decimal *adj*. 小数的;十进制的 *n*. 小数;十进小数
decimal form 小数形式
irrational *adj*. 无理的 *n*. 无理数
irrational number 无理数
length *n*. 长,长度
diagonal *n*. 对角线 *adj*. 对角线的
square *n*. 方,正方形 *vt*. 平方,二次幂
length of the diagonal of a square 正方形对角线的长度
unit *n*. 单位,单元

one length unit 一个长度单位
digit *n*. 数字
compute *vt*. 计算
formula *n*. 公式
radius *n*. 半径

1.2 THE NUMBER *e*

One of the first articles which we included in the "History Topics" section archive was on the history of π. It is a very popular article and has prompted many to ask for a similar article about the number e. There is a great contrast between the historical developments of these two numbers and in many ways writing a history of e is a much harder task than writing one for π. The number e is, compared to π, a relative newcomer on the mathematical scene.

The number e first comes into mathematics in a very minor way. This was in 1618 when, in an appendix to Napier's work on logarithms, a table appeared giving the natural logarithms of various numbers. However, that these were logarithms to base e was not recognized since the base to which logarithms are computed did not arise in the way that logarithms were thought about at this time. Although we now think of logarithms as the exponents to which one must raise the base to get the required number, this is a modern way of thinking. We will come back to this point later in this essay. This table in the appendix, although carrying no author's name, was almost certainly written by Oughtred. A few years later, in 1624, again e almost made it into the mathematical literature, but not quite. In that year Briggs gave a numerical approximation to the base 10 logarithm of e but did not mention e itself in his work.

The next possible occurrence of e is again dubious. In 1647 Saint Vincent computed the area under a rectangular hyperbola. Whether he recognised the connection with logarithms is open to debate, and even if he did there was little reason for him to come across the number e explicitly. Certainly by 1661 Huygens understood the relation between the rectangular hyperbola and the logarithm. He examined explicitly the relation between the area under the rectangular hyperbola $yx = 1$ and the logarithm. Of course, the number e is such that the area under the rectangular hyperbola from 1 to e is equal to 1. This is the property that makes e the base of natural logarithms, but this was not understood by mathematicians at this time, although they were slowly approaching such an understanding.

Huygens made another advance in 1661. He defined a curve which he calls "logarithmic" but in our terminology we would refer to it as an exponential curve, having the form $y = ka^x$. Again out of this comes the logarithm to base of e, which Huygens calculated to 17 decimal places. However, it appears as the calculation of a constant in his work and is not recognized as the logarithm of a number (so again it is a close call but e remains unrecognized).

Further work on logarithms followed which still does not see the number e appear as such, but the work does contribute to the development of logarithms. In 1668 Nicolaus Mercator published *Logarithmotechnia* which contains the series expansion of $\log(1 + x)$. In this work Mercator uses the term "natural logarithm" for the first time for logarithms to base e. The number e itself again fails to appear as such and again remains elusively just round the corner.

Perhaps surprisingly, since this work on logarithms had come so close to recognizing the number e, when e is first "discovered" it is not through the notion of logarithm at all but rather through a study of compound interest. In 1683 Jacob Bernoulli looked at the problem of compound interest and, in examining continuous compound interest, he tried to find the limit of $\left(1+\frac{1}{n}\right)^n$ as n tends to infinity. He used the binomial theorem to show that the limit had to lie between 2 and 3 so we could consider this to be the first approximation found to e. Also if we accept this as a definition of e, it is the first time that a number was defined by a limiting process. He certainly did not recognise any connection between his work and that on logarithms.

We mentioned above that logarithms were not thought of in the early years of their development as having any connection with exponents. Of course from the equation $x = a^t$, we deduce that $t = \log_a x$ where the log is to base a, but this involves a much later way of thinking. Here we are really thinking of log as a function, while early workers in logarithms thought purely of the log as a number which aided calculation. It may have been Jacob Bernoulli who first understood the way that the log function is the inverse of the exponential function. On the other hand the first person to make the connection between logarithms and exponents may well have been James Gregory. In 1684 he certainly recognized the connection between logarithms and exponents, but he may not have been the first.

As far as we know the first time the number e appears in its own right is in 1690. In that year Leibniz wrote a letter to Huygens and in this he used the notation b for what we now call e. At last the number e had a name (even if not its present one) and it was recognized. Now the reader might ask, not unreasonably, why we have not started our article on the history of e at the point where it makes its first appearance. The reason is that although the work we have described previously never quite managed to identify e, once the number was identified then it was slowly realised that this earlier work is relevant. Retrospectively, the early developments on the logarithm became part of an understanding of the number e.

We mentioned above the problems arising from the fact that log was not thought of as a function. It would be fair to say that Johann Bernoull began the study of the calculus of the exponential function in 1697 when he published *Principia calculi exponentialum seu percurrentiuw*. The work involves the calculation of various exponential series and many results are achieved with term by term integration.

So much of our mathematical notation is due to Euler that it will come as no surprise to find that the notation e for this number is due to him. The claim which has sometimes been made, however, that Euler used the letter e because it was the first letter of his name is ridiculous. It is probably not even the case that the e comes from "exponential", but it may have just be the next vowel after "a" and Euler was already using the notation "a" in his work. Whatever the reason, the notation e made its first appearance in a letter Euler wrote to Goldbach in 1731. He made various discoveries regarding e in the following years, but it was not until 1748 when Euler published *Introduction in Analysin infinitorum* that he gave a full treatment of the ideas surrounding e. He showed that

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$$

and that e is the limit of $\left(1 + \frac{1}{n}\right)^n$ as n tends to infinity. Euler gave an approximation for e to 18 decimal places

$$e = 2.718\ 281\ 828\ 459\ 045\ 235$$

without saying where this came from. It is likely that he calculated the value himself, but if so there is no indication of how this was done. In fact taking about 20 terms of $1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$ will give the accuracy which Euler gave. Among other interesting results in this work is the connection between the sine and cosine functions and the complex exponential function, which Euler deduced using De Moivre's formula.

Interestingly Euler also gave the continued fraction expansion of e

and noted a pattern in the expansion. In particular he gave

$$\frac{e-1}{2}=\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{10+\cfrac{1}{14+\cfrac{1}{18+\cdots}}}}}$$

and

$$e-1=1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{6+}}}}}}}}$$

Euler did not give a proof that the patterns he spotted continue (which they do) but he knew that if such a proof were given it would prove that e is irrational. For, if the continued fraction for $\frac{e-1}{2}$ were to follow the pattern shown in the first few terms, 6, 10, 14, 18, 22, 26, $\cdots$ (add 4 each time) then it will never terminate so $\frac{e-1}{2}$ (and so e) cannot be rational. One could certainly see this as the first attempt to prove that e is not rational.

The same passion that drove people to calculate to more and more decimal places of π never seemed to take hold in quite the same way for e. There were those who did calculate its decimal expansion, however, and the first to give e to a large number of decimal places was Shanks in 1854. It is worth noting that Shanks was an even more enthusiastic calculator of the decimal expansion of π. Glaisher showed that the first 137 places of Shanks calculations for e were correct but found an error which, after correction by Shanks, gave e to 205 places. In fact one needs about 120 terms of $1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots$ to obtain e correct to 200 places.

Further calculations of decimal expansions followed. In 1884 Doorman calculated e to 346 places and found that his calculation agreed with that of Shanks as far as place 187 but then became different. In 1887 Adams calculated the base 10 log of e to 272 places.

Words and Expressions

prompt *adj*. 迅速的,敏捷的;即时的,立刻的 *vt*. 激励,鼓动 *n*. 提示
appendix *n*. 附录,附属物
occurrence *n*. 发生;事件
explicitly *adv*. 明确地
rectangular *adj*. 矩形的,长方形的,直角的
rectangular hyperbola 等轴双曲线,直角双曲线
terminology *n*. 术语,专门名词
dubious *adj*. 怀疑的,可疑的
elusively *adv*. 难懂地,令人困惑地
process *n*. 过程;方法,步骤
limiting process 极限过程
deduce *vt*. 演绎,推演
log funtion 对数函数
exponential function 指数函数
exponential series 指数级数
retrospectively *adv*. 回顾
accuracy *n*. 精度,精确度;准确,准确性
sine *n*. 正弦
sine function 正弦函数
cosine *n*. 余弦
cosine function 余弦函数

complex exponential function 复指数函数
De Moivre's formula 德·摩根公式
terminate *adj*. 有限的,有结尾的 *vt*. 终止,结束 *vi*. 结束,结局
enthusiastic *adj*. 热心的,满腔热忱的
enthusiastic calculator 满腔热忱的计算者

1.3 INEQUALITIES FOR CONVEX FUNCTION

Convex functions

Convex functions are powerful tools for proving a large class of inequalities. They provide an elegant and unified treatment of the most important classical inequalities.

A real-valued function on an interval I is called *convex* if

$$f(\lambda x+(1-\lambda)y)\leqslant\lambda f(x)+(1-\lambda)f(y) \qquad (1)$$

for every $x, y\in I$ and $\lambda\in[0,1]$; it is called *strictly convex* if

$$f(\lambda x+(1-\lambda)y)<\lambda f(x)+(1-\lambda)f(y) \qquad (2)$$

for every $x, y\in I, x\neq y$ and $\lambda\in(0,1)$.

Notice

f is called *concave* (*strictly concave*) on I if $-f$ is *convex* (*strictly convex*) on I.

The geometrical meaning of convexity is clear: f is strictly convex if and only if for every two points $P=(x,f(x))$ and $Q=(y,f(y))$ on the graph of f, the point $R=(z,f(z))$ lies below the segment PQ for every z between x and y.

How to recognize a convex function without the graph? We can use (1) directly, but the following criterion is often very useful:

Test for Convexity

Let f be a twice differentiable function on I. Then f is convex on I