University Civil Engineering Major
Recommended Teaching Material (English Version)
高等学校土建类专业英文版推荐教学用书

CONSTITUTIVE EQUATIONS FOR CONCRETE AND SOIL

（混凝土和土的本构方程）

〔美〕陈惠发　A.F.萨里普　著
余天庆　王勋文
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本书为土木工程专业研究生系列教材之一《混凝土和土的本构方程》的英文版。全书共包括四大部分：混凝土的弹性和破坏准则；土的弹性和破坏准则；混凝土的塑性及应用；土的塑性及应用。其主要内容为：混凝土的线弹性和破坏准则，混凝土的非线性弹性和亚弹性模型，土的弹性应力-应变关系和破坏准则，混凝土的塑性理论，塑性断裂理论在混凝土中的应用，土的塑性理论及其在土体研究中的应用。本书编写简明扼要，可读性强，其中文版本已由中国建筑工业出版社出版。

本书可作为高等院校研究生或大学高年级教材，也可供工程技术人员参考使用。同时为高校师生及相关技术人员提供了一本价值较高的专业外语读物。

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Foreword

These dual-language four-volume textbooks grew out of my two-volume monograph entitled "**Constitutive Equations for Engineering Materials**" with Volume 1 subtitled "**Elasticity and Modeling**" published by John Wiley Inter-Science, New York, in 1982. Volume 2 subtitled "**Plasticity and Modeling**" together with an updated version of Volume 1 were published more than a decade later by Elsevier, Amsterdam, in 1994. The first parts of Volumes 1 and 2 of the original books are intended as textbooks for elasticity and plasticity respectively, while the later parts of each volume is intended as reference books for advanced materials as required for many civil engineering applications.

The materials in my original two-volume monograph are now reorganized in two separate books published in both Chinese version and English version, respectively; one book for the fundamentals of theories of elasticity and plasticity, and the other book for their implementations to soil and concrete materials and their applications. This book entitled "**Constitutive Equations for Concrete and Soil**" is a practical book. It provides civil engineers with a compact and convenient summary of the mathematical modeling techniques for material behavior in nonlinear finite element analysis in the areas of reinforced concrete and soil mechanics. The companion book entitled "**Elasticity and Plasticity**" is an introductory textbook intended for engineers with only a basic background in mechanics, strength of materials, calculus, material behavior of metals, and some basic concept of finite element methods, and wish to learn more on material modeling under three - dimensional stress and strain conditions. Together, these two books will serve as a fundamental framework for further advances on constitutive equations for engineering materials.

Constitutive Equations for Concrete and Soil is organized into four parts consisting of seven chapters, with Part One (Chapters 1—2) devoting to the specific formulations and applications of elasticity - based models to reinforced concrete materials. There are two major sources of nonlinearity in reinforced concrete: cracking of concrete and plasticity of reinforcement and of the compression concrete. The nonlinearity due to cracking and the failure criteria of concrete are considered in Chapter 1, the nonlinearity due to plasticity of compression concrete is treated in Chapter 2. Examples of using these constitutive models for finite element analysis of typical reinforced concrete structures are also given in this part.

Part Two (Chapter 3) is devoted to soil elasticity and failure criteria. Here, as in

Part One, the specific formulations and applications of the elasticity-based models to soil type of media are developed in details. The procedures for fitting various models to a given set of experiments together with their computer implementation and typical finite element applications in geotechnical engineering are plausibly explained and compared with available experimental results.

Part Three(Chapter 4—5) is devoted to concrete plasticity and its implementation and application in concrete, while Part Four (Chapters 6—7) is devoted to soil plasticity and its implementation and application in soils. Together the two parts provide an overview of modern plasticity-based models for concrete and soil materials. Procedures for implementing these models into finite element computer codes are also presented. Numerous examples illustrate applications to practical engineering problems.

The development of these four textbooks in dual language on constitutive modeling of engineering materials was strongly influenced by the following two factors:

(1) The future direction of research and education in solid mechanics and structural engineering is in the area of modeling, simulation and validation. Modeling is mechanics and material science, simulation is computing and software development, and validation is experimentation and field measurements. Constitutive modeling of engineering materials is a critical element for the future advances in civil engineering applications.

(2) The realization of the importance of English language in the globalization of the world economy. The dual-language publication of these textbooks provides an excellent tool for those who wish to learn about constitutive modeling and to use computer simulation in the standard terminology of mechanics, materials, and computing in both English and Chinese.

I express my sincere thanks to Professor T. Q. Yu (Hubei University of Technology) and Dr. X. W. Wang (China Academy of Railway Sciences) for suggesting and carrying out the reorganization of my original two-volume monograph into the present dual-language version of the four books. Student, researcher, or practitioner, novice and expert alike, will profit much from reading these books, either in English of Chinese, and having them for references in the years to come.

W. F. Chen
Honolulu, Hawaii
August 2003

NOTATION

Given below is a list of the principal symbols and notations used in the book. All notations and symbols are defined in the text when they first appear. Symbols which have more than one meaning are defined clearly when used to avoid confusion, and usually the correct meaning will be obvious from the context.

Stresses and Strains

$\sigma_1, \sigma_2, \sigma_3$	Principal stresses
σ_{ij}	Stress tensor
s_{ij}	Stress deviator tensor
σ	Normal stress
τ	Shear stress
$\sigma_{\text{oct}} = \frac{1}{3} I_1$	Octahedral normal stress
$\tau_{\text{oct}} = \sqrt{\frac{2}{3} J_2}$	Octahedral shear stress
$\sigma_{\text{m}} = \sigma_{\text{oct}}$	Mean normal (hydrostatic) stress
$\tau_{\text{m}} = \sqrt{\frac{2}{5} J_2}$	Mean shear stress
s_1, s_2, s_3	Principal stress deviators
$\varepsilon_1, \varepsilon_2, \varepsilon_3$	Principal strains
ε_{ij}	Strain tensor
e_{ij}	Strain deviator tensor
ε	Normal strain
γ	Engineering shear strain
$\varepsilon_{\text{v}} = I'_1$	Volumetric strain
$\varepsilon_{\text{oct}} = \frac{1}{3} I'_1$	Octahedral normal strain
$\gamma_{\text{oct}} = 2\sqrt{\frac{2}{3} J'_2}$	Octahedral engineering shear strain
e_1, e_2, e_3	Principal strain deviators

Invariants

$I_1 = \sigma_1 + \sigma_2 + \sigma_3 = \sigma_{ii}$ = first invariant of stress tensor

$J_2 = \frac{1}{2} s_{ij} s_{ij}$

$= \frac{1}{6}[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2] + \tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2$

= second invariant of stress deviator tensor

$J_3 = \frac{1}{3} s_{ij} s_{jk} s_{ki}$ = third invariant of stress deviator tensor

$\cos 3\theta = \frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}}$ where θ is the angle of similarity defined in Figure 1.13

$I_1' = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \varepsilon_v$ = first invariant of strain tensor

$\rho = \sqrt{2J_2}$ = deviatoric length defined in Figure 1.12

$\xi = \frac{1}{\sqrt{3}} I_1$ = hydrostatic length defined in Figure 1.12

$$J_2' = \frac{1}{2} e_{ij} e_{ij}$$
$$= \frac{1}{6}[(\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2] + \varepsilon_{xy}^2 + \varepsilon_{yz}^2 + \varepsilon_{zx}^2$$

= second invariant of strain deviator tensor

Material Parameters

f_c'	Uniaxial compressive cylinder strength ($f_c' > 0$)
f_t'	Uniaxial tensile strength
f_{bc}'	Equal biaxial compressive strength ($f_{bc}' > 0$)
E	Young's modulus
ν	Poisson's ratio
$K =$	$\frac{E}{3(1-2\nu)}$ = Bulk modulus
$G =$	$\frac{E}{2(1+\nu)}$ = Shear modulus
c, ϕ	Cohesion and friction angle in Mohr–Coulomb criterion
α, k	Constants in Drucker–Prager criterion
k	Yield (failure) stress in pure shear

Miscellaneous

{ }	Vector
[ ]	Matrix
C_{ijkl}	Material stiffness tensor
D_{ijkl}	Material compliance tensor
$f(\ )$	Failure criterion or yield function
x, y, z or x_1, x_2, x_3	Cartesian coordinates
δ_{ij}	Kronecker delta
$W(\varepsilon_{ij})$	Strain energy density
$\Omega(\sigma_{ij})$	Complementary energy density
$l_{ij} =$	$\cos(x_i', x_j)$ = The cosines of the angles between x_i' and x_j axes●
ε_{ijk}	Alternating tensor●

● 陈惠发.余天庆等. "Elasticity and Plasticity" 北京:中国建筑工业出版社, 2005.

CONTENTS

NOTATION

PART ONE

ELASTICITY AND FAILURE CRITERIA FOR CONCRETE

CHAPTER ONE

LINEAR ELASTICITY AND FAILURE CRITERIA FOR CONCRETE

1.1 INTRODUCTION

Nonlinear analysis of reinforced concrete structures has become increasingly important in recent years. It is only by carrying out a complete *progressive failure analysis* of the structure up to collapse that it is possible to assess all safety aspects of a structure and find its deformational characteristics. This type of analysis is particularly desirable for certain structures such as concrete reactor vessels, nuclear containment structures, and parts of offshore platforms. Experimental studies on these structural systems are very expensive, and empirical approaches alone are not sufficient for an adequate evaluation of safety with respect to the limit state.

With the present state of development of computer programs based on the finite element method, inadequate modeling of reinforced concrete material is often one of the major factors in limiting the capability of structural analysis. This is because reinforced concrete has a very complex behavior involving phenomena such as inelasticity, cracking, time dependency, and interactive effects between concrete and reinforcement. The development of material models for uncracked and cracked concrete for all stages of loading is a particularly challenging field in nonlinear analysis of reinforced concrete structures.

The nonlinear response of reinforced concrete is caused by four major material effects: (1) cracking of the concrete; (2) plasticity of the reinforcement and of the compression concrete; (3) bond slip between steel and concrete, aggregate interlock, dowel action of reinforcement steel, etc.; and (4) time-dependent effects such as creep, shrinkage, temperature, and load history. Only the cracking and interactive effects between concrete and reinforcement are considered in this chapter. Nonlinear response of compression concrete is discussed in the following chapter. Time-dependent effects are not considered in this book.

In spite of its obvious shortcomings, the *linear theory of elasticity* combined with criteria defining "*failure*" of concrete is the most commonly used material law for concrete in reinforced concrete analysis. This is the subject of this chapter. The linear elastic model can be significantly improved by using the *nonlinear theory of elasticity*. The nonlinear elastic formulations can be of Cauchy type or *hyperelastic* type that can be quite accurate for concrete sustaining proportional loading. However, these formulations fail to identify inelastic deformation, a shortcoming that becomes apparent when the material experiences unloading. This can to some extent be improved by introducing differential or incremental formulation of *hypoelasticity*. All these aspects of stress–strain formulations are discussed in the following chapter. More advanced mathematical modeling of reinforced concrete material based on the *flow theory of plasticity* is presented in this book.

This chapter is divided into five main parts: (1) typical behavior of concrete (Sec. 1.2); (2) failure criteria (Secs. 1.3 to 1.5); (3) linear elastic-fracture models (Secs. 1.6 and 1.7); (4) interaction between concrete and reinforcement (Sec. 1.8); and (5) examples of finite element applications (Sec. 1.9)

In an effort to make this and the following chapter reasonably self-contained, for those who are interested only in the nonlinear analysis of reinforced concrete structures, a number of important concepts involving stress and strain invariants (Sec. 1.3), linear elasticity (Sec. 1.6.3), and nonlinear elasticity (Sec. 2.2) have been briefly reviewed in various subsections. These materials can be found in the foregoing chapters, but they are collected here in a form that is keyed directly to the main exposition of the relevant sections.

1.2 MECHANICAL BEHAVIOR OF CONCRETE

1.2.1 General

Despite the widespread use of concrete as a structural material, our knowledge about its exact physical properties and behavior under various stress combinations is rather deficient. This is not surprising to anyone who is aware of the heterogeneous structure of concrete. During loading concrete suffers not only elastic deformations, but also inelastic and time-dependent deformations caused by microstructural changes. This inelastic deformation is primarily due to microcracking and internal friction sliding. Thus, in order to give physical explanations of the experimentally observed phenomena in test specimens, the knowledge of the microstructure of concrete is fundamental. This knowledge is also important in every constitutive modeling of concrete on a macroscopic dimensional level. Reviews on the microstructure and properties of concrete have been given by Newman (1966) and Brooks and Newman (1968). In addition, several textbooks on the properties of concrete have been published (see, for example, Neville, 1970, 1977). A recent review of current knowledge is given by Aoyama and Noguchi (1979). Extensive reviews have also been given by Shah (1979) on high-strength concrete. Herein only the essential points are summarized.

Concrete is a composite material mainly consisting of different sized aggregate particles which are embedded in a cement paste matrix. From our knowledge about the concrete microstructure, three fundamental properties are emphasized: (1) a large number of bond microcracks exist at the interfaces between coarser aggregates and mortar, (2) the cement paste has a high porosity (about 30%), and these pores are filled with water and/or air; and (3) at all dimensional levels, above the molecule level, air and/or water voids exist. Each of these properties strongly affects the mechanical behavior of concrete. For instance, the propagation of the microcracks during loading contributes to the nonlinear behavior of concrete at low stress levels and causes volume increase (dilatancy) near failure under uniaxial compressive state of stress. For high hydrostatic pressures, the intrusions of voids and paste pores become increasingly important in affecting the behavior and strength of concrete.

Many of the microcracks in concrete are caused by segregation, shrinkage, or thermal expansion in the mortar and therefore exist even before any load has been applied. Some of the microcracks can be developed during loading because of the differences in stiffness between aggregates and mortar. Therefore, the aggregate–mortar interface constitutes the weakest link in the composite system. This is the primary reason for the low tensile strength of concrete materials.

The purpose of this section is to summarize some of the key facets of the experimental behavior of plain concrete under uniaxial, biaxial, and triaxial states of stress. This is essential in the generalized development of various constitutive models for concrete, as described in the subsequent sections and the following chapter. In particular, the test data presented serve the following two major purposes: (1) to give guidance on the proper type of material behavior to be developed in the mathematical modeling; and (2) to provide data for the determination of the various material constants which appear in the mathematical models to be presented later in this and the following chapter.

The discussion that follows is confined mainly to the mechanical behavior of average ordinary (normal weight) concrete under short-term quasi-static loading conditions. Most of the experimental research on concrete has been concerned with these conditions. At present, very few data on the dynamic behavior of concrete are available in the literature, although dynamic loading conditions have a significant effect on the response of concrete to stress and strain. Recently, a study of impact loading on concrete structures has been made by Nilsson (1979).