第十七届全国水动力学研讨会暨第六届全国水动力学学术会议文集

周连第　邵维文　鲁传敬
许为厚　李毓湘　詹杰民　主编

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内容简介

本书是《水动力学研究与进展》编委会、中国力学学会，中国造船工程学会、香港科技大学、香港理工大学、中山大学、香港力学学会、上海交通大学、上海大学、上海市力学学会联合举办的第十七届全国水动力学研讨会暨第六届全国水动力学学术会议文集，共选收论文130多篇，主要反映水动力学基础；计算流体力学；船舶与海洋工程水动力学；水电与河流动力学；海岸、环境与地球物理流体力学；工业流体力学；近代测试设备与技术等方面的新进展、新水平、新面貌，可供有关专业的科研和教学人员参考。

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大会报告(续)

Shallow Water Flow Computation Using the Unified Coordinates*

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Abstract Two general coordinate systems have been used extensively in computational fluid dynamics: the Eulerian and the Lagrangian. The Eulerian coordinates cause excessive numerical diffusion across flow discontinuities, slip lines in particular. The Lagrangian coordinates, on the other hand, can resolvp slip lines sharply but cause severe grid deformation, resulting in large errors and even breakdown of the computation. Recently, Hui ea al.[1] have introduced a unifled coordinate sqstem which moves with velocity hq, q being the velocity of the fluid particle. It includes the Eulerian system as a special case when $h = 0$, and the Lagrangian when $h = 1$, and was shown for the multi-dimensional Euler equations of gas dynamics to be superior to both Eulerian and Lagrangian systems. The main Purpose of this paper is to adopt this unifled coordinate system to solve the shallow water equations. It will be shown that computational results using the unified system are superior to existing results based on either the Eulerian system or Lagrangian system in that it (a) resolves slip lines sharply, especially for steady flow, (b) avoids grid deformation and computation breakdown in Lagrangian coordinates.

Keywords unified description, Eulerian description, Lagrangian description, two-dimensional shallow water equations, slip lines

1 INTORDUCTION

Two general coordinate systems have been used extensivly for describing fluid motion: the Eulerian and the Lagrangian. Computationally, each system has its advantages as well as disadvantages.

Eulerian coordinates are simple but cause excessive numerical diffusion across discontinuities, slip lines in aprticular. In contrast, Lagrangian coordiantes can resolve slip lines sharply, but cuase large grid deformation, resulting in large errors and even breakdown of

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Recently, Hui and Li[1] have introduced a unified coordinate system which moves with velocity hq , where q is velocity of the fluid particle. It includes the Eulerian coordinates as a special case when $h=0$ and the Lagrangian when $h=1$, and more importantly, it has a new degree of freedom in choosing the arbitrary function h to improve the quality of computational results. In particular, it was shown in Ref. [1]that for the two-dimensional Euler equations of gas dynamics, choosing the function h to preserve grid angles results in a coordinate system which is superior to both Eulerian and Lagrangian systems.

The purpose of this paper is to adopt this unified coordinates approach to solve the shallow water equations; it will be shown that computational results using the unified system are superior to existing results based on either the Eulerian or Lagrangian system.

2 SHALLOW WATER EQUATIONS IN THE UNIFIED COORDINATES

The shallow water equations in conservation form using Cartesian coordinates are

$$\frac{\partial}{\partial t}\begin{pmatrix}\zeta\\ \zeta u\\ \zeta v\end{pmatrix}+\frac{\partial}{\partial x}\begin{pmatrix}\zeta u\\ \zeta u^2+\frac{1}{2}g\zeta^2\\ \zeta uv\end{pmatrix}+\frac{\partial}{\partial y}\begin{pmatrix}\zeta v\\ \zeta uv\\ \zeta v^2+\frac{1}{2}g\zeta^2\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix} \tag{1}$$

where g is the acceleration due to gravity, $\zeta(x, y, t)$ is the total water height measured from the horizontal bottom $u(x, y, t)$ and $v(x, y, t)$ are the comppoments of the fluid velocity in the horizontal x and y direction, respectively. We have neglected the bottom friction force.

Introduce a transformation of coordinates from (t, x, y) to (λ, ξ, η) ,

$$\begin{cases} dt=d\lambda, \\ dx=hud\lambda+Ad\xi+Ld\eta \\ dy=hvd\lambda+Bd\xi+Md\eta \end{cases} \tag{2}$$

where h is arbitrary. Let

$$\frac{D_h}{Dt}\equiv\frac{\partial}{\partial t}+hu\frac{\partial}{\partial x}+hv\frac{\partial}{\partial y} \tag{3}$$

denote the time derivative following the pseudopartical, whose velocity is $hq, q=(u,v)$. Then, under the assumption $AM-BL\neq 0$ (nonsingularity of the transformation (2)), it is easy to show

$$\frac{D_h\xi}{Dt}=0, \qquad \frac{D_h\eta}{Dt}=0 \tag{4}$$

that is to say, the coordiantes (ξ, η) are material functions of the pseudoparticles. Accordingly, computational cells move and deform with pseudoparticles, rather than with fluid particles as in Lagrangian coordinates.

Under transformation (2) the shallow water Eq. (1) become

$$\frac{\partial \boldsymbol{E}}{\partial \lambda}+\frac{\partial \boldsymbol{F}}{\partial \xi}+\frac{\partial \boldsymbol{G}}{\partial \eta}=0 \tag{5}$$

where

$$\boldsymbol{E}=\begin{pmatrix}\zeta\Delta\\ \zeta\Delta u\\ \zeta\Delta v\\ A\\ B\\ L\\ M\end{pmatrix}, \quad \boldsymbol{F}=\begin{pmatrix}\zeta(1-h)I\\ \zeta(1-h)Iu+\frac{1}{2}g\zeta^2M\\ \zeta(1-h)Iv-\frac{1}{2}g\zeta^2L\\ -hu\\ -hv\\ 0\\ 0\end{pmatrix}, \quad \boldsymbol{G}=\begin{pmatrix}\zeta(1-h)J\\ \zeta(1-h)Iu-\frac{1}{2}g\zeta^2B\\ \zeta(1-h)Iv-\frac{1}{2}g\zeta^2A\\ 0\\ 0\\ -hu\\ -hv\end{pmatrix}$$

with

$$\Delta=AM-BL, \quad I=uM-vL, \quad J=vA-uB \tag{6}$$

We note that the shallow water Eq. (5) written in the unified cocrdinates are in conservation from. We also point out that although Eq. (5) is larger than Eq. (1), compuationally the extra computing time required for solving the last four compatibility equaitons of Eq. (5) is very small, typically 3~5%, becuse the bulk of computing time is spent on solving the Riemann problems for the first three equations of Eq. (5), which require the same amount of computing time as system Eq. (1).

3 ONE-DIMENSIONAL SHALLOW WATER FLOW

For the special case of one-dimensional flow, transofrmation (2) simplifes to

$$\begin{cases} dt = d\lambda \\ dx = hud\lambda + Ad\xi \end{cases} \tag{7}$$

and the shallow water Eq. (5) become

$$\frac{\partial \boldsymbol{E}}{\partial \lambda} + \frac{\partial \boldsymbol{F}}{\partial \xi} = 0 \tag{8}$$

where

$$\boldsymbol{E} = \begin{bmatrix} \zeta A \\ \zeta A u \\ A \end{bmatrix}, \quad \boldsymbol{F} = \begin{bmatrix} \zeta(1-h)u \\ \zeta(1-h)u^2 + \frac{1}{2}g\zeta^2 \\ -hu \end{bmatrix} \tag{9}$$

It can be shown easily that Eq. (8) is hyperbolic. indeed the eigenvalues of Eq. (8) are:

$$\sigma_1 = 0 \tag{10}$$

$$\sigma_{\pm} = \frac{(1-h)u \pm \sqrt{g\zeta}}{A} \tag{11}$$

The corresponding right eigenvectors, when the primitive variables $\boldsymbol{U} = (\zeta, u, A)^T$ are used, are

$$r_1 = (0,0,1)^T \tag{12}$$

$$r_{\pm} = (1, \pm\sqrt{g/\zeta}, \mp\frac{h}{\sigma_{\pm}}\sqrt{g/\zeta})^T \tag{13}$$

Direct computation shows that the one-dimensional shallow water equations have juct one type of flow discontinuity (shocks), but there is no flow contact (slip) lines which exist in the one-dimensional Euler equations of gas synamics. With no contact line to resolve, Eulerian and Lagrangian coordinates are on equal footing for accuracy, as is verified in our computation. The adaptive Godunov scheme[2,3], which resolves shock crisply, can now be applied to either the Lagrangian coordiantes or the Eulerian one, or indeed for any h. Our compuations for 1-D problem use $h = 0$.

4 TWO-DIMENSIONAL SHALLOW WATER FLOW

Direct though tedious computations show that system (5) is hyperbolic for $h \neq 1$, which includes the Eulerian coordinates when $h = 0$, but is only weakly hyperbolic for $h = 1$, i. e. in Lagrangian coordinates, lacking one eigenvector although the eigenvalues are all real. This new finding has important implications.

We now come to the question of the determination of h. As mentioned earlier, the chief advantage of the unifled coordiantes is the new degree of freedom in choosing h. As shown in Ref. [1], a good choice for h is preserve the grid angles in the solution process which marches in λ, i. e.,

$$\frac{\partial}{\partial\lambda}\left[\frac{\nabla\xi}{|\nabla\xi|}\cdot\frac{\nabla\eta}{|\nabla\eta|}\right]=0 \tag{14}$$

Since

$$\nabla\xi = (M, -L)/\Delta$$

$$\nabla\eta = (-B, A)/\Delta$$

condition (14) becomes

$$\frac{\partial}{\partial\lambda}\left[\frac{AL+BM}{\sqrt{A^2+B^2}\sqrt{L^2+M^2}}\right]=0 \tag{15}$$

By making use of the last four equations of Eq. (5), it is easy to show that (15) is equivalent to

$$S^2 J\frac{\partial h}{\partial\xi}+T^2 I\frac{\partial h}{\partial\eta}=\left[S^2\left(B\frac{\partial u}{\partial\xi}-A\frac{\partial v}{\partial\xi}\right)-T^2\left(M\frac{\partial u}{\partial\eta}-L\frac{\partial v}{\partial\eta}\right)\right]h \tag{16}$$

where

$$S^2 = L^2+M^2, \qquad T^2 + A^2 + B^2$$

A consequent of determining h from (16) is that if the grid is orthogonal at $\lambda = 0$ it will remain so for subsequent λ. Orhtogonal grids are know to possess many desirable properties over nonorthogonal grids, e. g., attaining higher accuracy than nonorthogonal grids. Computationally, (16) is to be solved at every time step after the flow variable $Q=(\zeta, u, v)^T$, and the geometric variables $K=(A, B, L, M)^T$ and found. It is thus a first order linear partial differential equation for $h(\xi, \eta; \lambda)$ with λ appearing as a prameter. To find solution h in the range

$$0 \leqslant h \leqslant 1 \tag{17}$$

we note that (16) is linear and homogeneous, and therefore it possesses two properties: (a) positive solution $h > 0$ always exists, and (b) if h is a solution of (16) so is h/C, C being any constant. Making use of property (a), we let $g = \ln(hq)$ to get

$$S^2(A\sin\theta - B\cos\theta)\frac{\partial g}{\partial \xi} + T^2(M\cos\theta - L\sin\theta)\frac{\partial g}{\partial \eta} =$$

$$S^2\left(B\frac{\partial \cos\theta}{\partial \xi} - A\frac{\partial \sin\theta}{\partial \xi}\right) - T^2\left(M\frac{\partial \cos\theta}{\partial \eta} - L\frac{\partial \sin\theta}{\partial \eta}\right) \tag{18}$$

where $q = \sqrt{u^2 + v^2}$ and θ is the flow angle: $u = q\cos\theta$, $v = q\sin\theta$. Now, if g_1 is any solution to (18), then $h = e^{g_1}/qC$ is solution to (16) satisfying condition (17), provided we choose C equal to the maximum of e^{g_1}/q over the whole flow field being computed. The reason to work ln (hq) instead of h is that from our experience with steady flow[4], hq is continuous across slip lines, hence working with hq can minimize the numerical errors.

Numerically, Eq. (18) is solved by the method of characteristics if their slopes do not change sign; otherwise, it is solved by iteration.

We now discuss solution strategies. As the system of shallow water equations (5) written in unified coordiantes is in conservation form, any well-established shock-capturing method can be used to solve it. We shall use the Godunov method with MUSCL update to higher resolution to solve system (5). The computation will be done entirely in the λ-ξ-η space. A physical cell in the x-y plane marching along the pseudoparticle's pathline corresponds to a rectangular cell in the ξ-η plane marching in the λ direction in the computational space λ-ξ-η. The superscript k refers to the marching time step number and the subscripts i and j refer to the cell index number on a time plane λ =const. The time step $\Delta\lambda^k = \lambda^{k+1} - \lambda^k$ is uniform for all i and j, but it is always chosen to satisfy the CFL stability conditon. The grid divides the computational domain into cubic control volumes, or cells, which in ξ and η direction are centered at $(\lambda^k, \xi_i, \eta_j)$ and have widths $\Delta\xi_i = \xi_{i+1/2} - \xi_{i-1/2}$ and $\Delta\eta_j = \eta_{j+1/2} - \eta_{j-1/2}$ (for all k). Unless otherwise stated we shall use uniform cell width $\Delta\xi_i$ for all i and $\Delta\eta_j$ for all j.

In the physical space (t, x, y) a cuboid cell marching in (λ, ξ, η) space corresponds to a pseudopatticle marching along its path tube with step Δt $(\Delta t = \Delta\lambda)$. The pseudoparticle is bounded by four path surfaces $\xi = \xi_{i\pm1/2}$ and $\eta = \eta_{j\pm1/2}$ around it. Initially, any curvilinear coordiante grid on the x-y plane may be used as the ξ-η coordinate grid, and the initial geometric variable $\boldsymbol{K}=(A, B, L, M)^T$ can be determined from (2) as part of the initial conidtions. A stationary solid wall is always a path surface of the fluids and hence also of the pseudofluids; it is therefore a coordiante surface of the unifled coordinates.

Applying the divergence theorem to (5) over the cuboid cell (i, j, k) results in

$$\boldsymbol{E}_{i,j}^{k+1} = \boldsymbol{E}_{i,j}^{k} - \frac{\Delta\lambda^k}{\Delta\xi_i}(\boldsymbol{F}_{i+1/2,j}^{k+1/2} - \boldsymbol{F}_{i-1/2,j}^{k+1/2}) - \frac{\Delta\lambda^k}{\Delta\eta_j}(\boldsymbol{G}_{i,j+1/2}^{k+1/2} - \boldsymbol{G}_{i,j-1/2}^{k+1/2}) \tag{19}$$

$$i = 1,2,\cdots,m \qquad j = 1,2,\cdots,n \tag{20}$$

where the notation for the cell average of any quantity f is

$$f_{i,j}^k = \frac{1}{\Delta\xi_i\Delta\eta_j}\int_{\xi_{i-1/2}}^{\xi_{i+1/2}}\int_{\eta_{j-1/2}}^{\eta_{j+1/2}} f(\lambda^k,\xi,\eta)\,\mathrm{d}\xi\mathrm{d}\eta \tag{21}$$

and the notation for the λ average of f is

$$f_{i+1/2,j}^{k+1/2} = \frac{1}{\Delta\lambda^k}\int_{\lambda^k}^{\lambda^{k+1}} f(\lambda,\xi_{i+1/2},\eta_j)\,\mathrm{d}\lambda \tag{22}$$

$$f_{i,j+1/2}^{k+1/2} = \frac{1}{\Delta\lambda^k}\int_{\lambda^k}^{\lambda^{k+1}} f(\lambda,\xi_i,\eta_{j+1/2})\,\mathrm{d}\lambda \tag{23}$$

We shall numerically solve (5) using a Godunov-type scheme based on the dimensional splitting approximation to reduce the two-dimensional flow problem to two one-dimensional flow problems. We shall use the Strang splitting[5] in this paper. Let $L_{\Delta\lambda}^{\xi}$ represent the exact solution operator for the one-dimensional equation in the λ-ξ plane and $L_{\Delta\lambda}^{\eta}$ similarly defined, then according to Strang splitting

$$\boldsymbol{E}^{k+1} = L_{\frac{\Delta\lambda}{2}}^{\xi} L_{\Delta\lambda}^{\eta} L_{\frac{\Delta\lambda}{2}}^{\xi} \boldsymbol{E}^k$$

where $\Delta\lambda = \lambda^{k+1} - \lambda^k$.

5 TEST EXAMPLES

In this section the unified coordiantes approach is tested numerically on several examples.

Example 1. The first example is purely a one-dimensional two-dam break problem. In a long channel three different heigths of still water are separated by two dams, of which one is located at $x = 0.8$ and the other at $x = 1.2$ (Fig. 1(a)). At $t = 0$ the dam located at $x = 0.8$ is broken instantly and completely, resulting in an expansion wave moving upstream and a bore (shock) rushing downstream (Figs. 1(b), (c), (d)). The bore (shock) then reaches

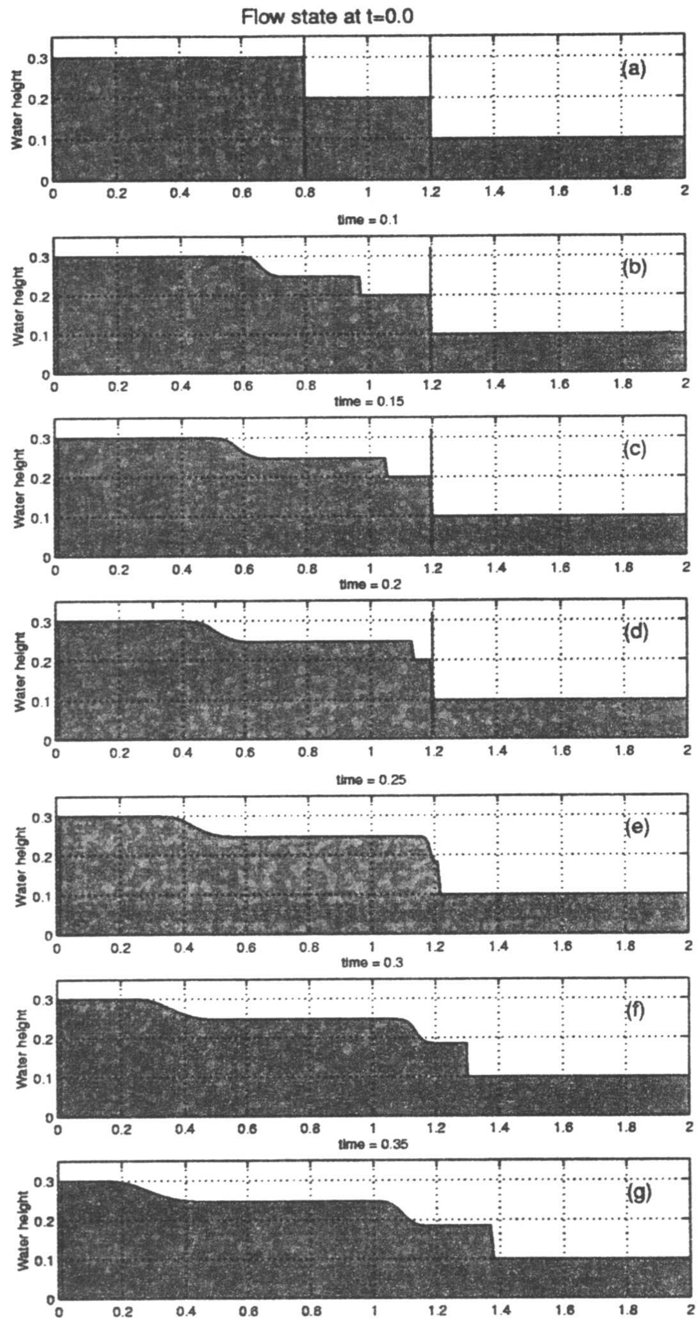

Fig. 1 Evolution of water height in the two-dam problem. Shock-adaptive Godunov scheme. (a) $t = 0.0$; (b) $t = 0.1$; (c) $t = 0.15$; (d) $t = 0.2$; (e) $t = 0.25$; (f) $t = 0.3$; (g) $t = 0.35$