研究生前沿教材书系

Complexity and Criticality

复杂性和临界状态

（英文影印版）

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复杂性和临界状态

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内 容 简 介

本书是作者在从2000年开始给伦敦帝国理工学院研究生讲授统计力学的讲稿基础上形成的。

复杂性是21世纪的重点研究课题之一,而临界状态则是统计物理中已有相当深入研究的一个分支,本书旨在采用统计力学的方法,以渗滤和伊辛模型为范例,讨论突破复杂性研究的途径。全书共分3章,第一章讲述渗滤现象的研究方法,并就一维、二维渗滤的定义、点阵结构、块体的大小和数密度、关联函数、标度函数、临界指数和实空间重整化群的变换方法等方面都作了详尽的介绍。第二章的重点是讲述二维伊辛模型的相变理论,涉及相互作用自旋系统的自由能和配分函数、磁化强度和磁导率、能量和比热、响应函数、平均场理论、相变的朗道-京茨堡理论、Widom标度假设、临界指数和Wilson重整化群论。第三章介绍自组织临界状态。本章从容易想象的所谓“沙堆”模型出发,讨论沙堆崩塌的物理处理方法,从中引入开放系统的平均场理论、二分叉理论和几率分布的矩分析、定态出现的条件等。本章还就地震和降雨的预测预报作了定性讨论。

全书每章之后都有专门设计的练习,为了降低解题难度,每道题都细分为很多小题目,使解题思路十分明确。答案可从 http://www.worldscibooks.com/physics/p365.htm/.查找。

为了使不同学科的读者克服数学上的困难,书后的8个附录把相关的数学知识和物理量都作了补充交代。

Kim Christensen

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1990 年于丹麦 Arhus 大学物理和天文研究所获得科学硕士。

1993 年于丹麦 Arhus 大学物理和天文研究所获得科学博士。

主要研究兴趣是外界因素引起非平衡系统复杂性变化的理论和数值研究,涉及统计力学、复杂性、标度不变性实验现象、自组织临界状态。

Nicholas R. Moloney

伦敦帝国理工学院(Imperial College London)Blackett 实验室教授。

序　言

本书旨在引进临界现象的概念并研究复杂性和临界状态之间的共同基础。

“复杂性”一词,随着表述上下文的不同,会有多种不同的含义,而且它的官方定义一直都处在变化之中。这是因为对复杂性的研究还处于起步阶段,而且又是一个包括数学、物理学、地球物理、经济学和生物学等许多前沿学科迅速发展的领域。相关的研究机构业已形成,学术会议和研究班也组织起来了,已有大量在复杂性名下撰写的图书和文章发表。迄今为止,研究复杂性问题还没有取得采用何种明晰而简洁的理论形式的一致意见。因此,复杂性的研究工作很可能遭遇无序的危险,甚至会误入歧途。至于我们的看法,认为复杂性是指对具有很多自由度的系统,重复使用简单的规则,直到得出规则本身尚未解码的必然发生的行为出现为止。

另一方面,“临界状态”一词,统计物理学家都有明确的定义。临界状态涉及广延系统在相变时的行为,此时可观测量都是无标度的,也就是说,这些可观测量都没有特征标度。相变时,很多组份的微观“部分”的作用所引起的宏观现象,是只考虑单一部分所满足的定律难以理解的。因此,临界状态是对由相互作用“部分”构成的系统重复应用微观定律后出现的一种合作效应。相变的唯象理论已经成熟,描述相变已有很好的理论形式。

本书分为3章,开头两章,我们以渗滤和伊辛模型为孤立平衡系统的范例,向读者详细介绍临界现象的概念。这些系统只有当外界的作用把某些外参量精细地调整到特定的数值时才会发生相变。

渗滤是显示相变最简单的模型。渗滤作用中的相变纯粹是几何性质的,读者从中可以直觉地了解诸如分形、标度律和重整化等重要的概念。

美好的伊辛模型又会进一步深化读者对相变中出现合作现象的直观感觉。

最后一章中,我们考虑自然界中出现的复杂性。我们要阐明,自然界中的系统既不可能是孤立的,也不会是平衡的。我们研究这样一类非平衡系统:约束条件取消后,调节外参量能得到临界行为的系统。我们希望读者想象一下,非平衡系统中的自组织会不会是像统计力学、地球物理学、大气物理学等不同领域里的一种共有的概念?

虽然描述复杂性和临界状态的数学方法已经发展起来了,根据我们的经验,这些方法行业之外的科学家是不熟悉的。我们希望本书将有助于学生和研究人员更加定量地处理复杂性和临界状态。因此,本书自始至终都强调了采用数学定量技巧的重要性。

本书是完整而封闭式的,因此即使不熟悉复杂性和临界状态概念的读者也易于接受。这本教科书既能作为高年级大学生和研究生的基础教程,又能作为不同领域研究人员的导介性参考书。每章结束都有练习,练习的完整解答可通过图书联合网与作者联系得到:http://www.worldscibooks.com/physics/p365.htm/. 在这一网页上,读者还可以找到用动画来使所考虑模型可视化的代码。引用的文献是最早的出处和关系最大的出版物,但必须声明,这既不可能是完备的,甚至很可能是偏颇的。如果因为我们的疏忽和偏见而引起读者的麻烦,实在抱歉。

本书的基础是 2000 年以来在伦敦帝国理工学院讲授统计力学的讲稿,我们希望向参与这门课程的全体学生献礼,他们的建设性反馈意见都一一地被吸收到本书的表述之中了。我们特别要向 Arno Proeme 和 George Schusterisch 表示感谢,他们看出了许多差错和排版上的问题。我们同样期盼读者热心地关注各处细节并将发现的差错转告作者。

我们很感激 Dimitri Vvedensky 首先提出写书的建议。此外,我们对帝国学院出版社在整个写作过程中的有效和高度敬业的精神表示感谢,编辑 Katie Lydon 以极大的耐心等待最后的手稿很使我

们感动。

我们要对整个写作过程中提供这样那样帮助的大量人员表示感激。尤其是伦敦帝国理工学院在读和读完了 Ph. D 的学生,他们是:Nadia Farid, Vera Pancaldi, Ole Peters, Gunnar Pruessner, 以及 Matthew Stapleton。没有他们持之以恒的支持和热情,我们决不可能完成这本书。事实上,伦敦帝国理工学院凝聚态理论教研组的奇思妙想的氛围,也是我们写作灵感的源泉。我们十分感激挪威奥斯陆大学 PGP 的好客精神,在那里我们完成了本书的部分写作工作。Álvaro Corral 和 Jens G. Feder 都提出过许多深刻的建议。

作者之一 Kim Christensen(K. C.)要对 Hugo S. Jensen, Hans C. Fogedby, Henrik J. Jensen, Per Bak, Zeev Olami 及 Amnon Aharony 表示特别的敬意。Hugo S. Jensen 是一位声誉卓著的高中物理教师,他是使我成为物理学家的鞭策者,Hans Fogedby 和 Henrik Jensen 的热情和对物理的深刻洞察力总是使我敬仰。他们都像 Per Bak 一样,既是杰出的物理学家,又是真诚的朋友。Per Bak 的热情和对科学的洞察力是无人可比的,而 Zeev Olami 同我有过难以忘怀的交往,我们俩与 Per 一道在布鲁克海文国家实验室共事多年。十分感谢 Per 和 Zeev 的鼓励,感谢你们的无私和友情。我为与你们的分离而惋惜,我将永远铭记在心。K. C. 有幸参加 Amnon Aharony 讲授的关于合作现象的博士后课程。渗滤性这一章我就是想千方百计达到他那样高的科学标准,达到他那样的教学效果并完成错综复杂的课堂教学。这本书并没有将我在科学生涯中遇到的一流物理学家的重大影响都完全体现出来。

最后,K. C. 要在这里提一下他的美满家庭,正是这个家庭在编书的马拉松过程中提供了巨大的支持,家庭成员总是不失时机地提醒我,书稿不能拖得太久。

Kim Christensen 和 Nicholas R. Moloney

Contents

Chapter 1

Percolation

1.1 Introduction

Take some squared paper and black out a portion of the squares randomly. Consider the clusters of adjacent black squares. If only a small portion of squares are blacked out, it is unlikely that a cluster extends across opposite sides of the paper. However, if a large portion of squares are blacked out, it is likely that a cluster 'percolates' across the paper. Indeed, for a paper of infinite size, there exists a unique portion of randomly blacked out squares that mark a phase transition from paper with no percolating infinite cluster to paper with a percolating infinite cluster. Percolation is the study of the clusters as a function of the portion of black squares. It is a purely geometrical problem and is arguably the simplest model that undergoes a phase transition. The challenge in percolation lies in describing its emergent structures rather than understanding its defining rules. Apart from investigating the geometrical properties of the percolating infinite cluster, we will be specifically interested in the statistical properties of finite clusters. One approach to studying these clusters is simply to calculate their size and number explicitly from the rules of percolation, but we will find that in general this is a hopeless task even for clusters of moderate size. In the vicinity of the phase transition, another approach to describing cluster statistics suggests itself, and this is related to the notion of scale invariance. We will focus on the phase transition and dwell upon scale invariance, since these are unifying themes throughout the book. We will develop a mathematical framework for expressing scale invariance, applicable not only to percolation, but to other models undergoing a phase transition.

Percolation is, in fact, highly relevant in a variety of physical settings: oil recovery from porous media [King *et al.*, 1999], epidemic modelling [Cardy and Grassberger, 1985], networks [Cohen *et al.*, 2002], fragmentation [Herrmann and Roux, 1990], metal-insulator transition [Ball *et al.*, 1994], ionic transport in glasses and composites [Roman *et al.*, 1986], fracture patterns and earthquakes in rocks and ground water flow [Sahimi, 1994] among others. Therefore, the concepts in percolation are not only of academic interest but also of considerable practical value. For an introduction to percolation and some of its applications at the level of our presentation, we can recommend the popular text [Stauffer and Aharony, 1994].

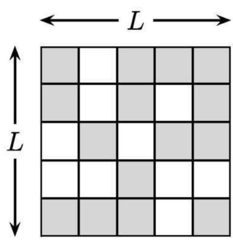
After defining percolation and its main observables, our programme is as follows. First, we will investigate one-dimensional percolation, which is simple and exactly solvable. Although trivial, this exercise is not entirely fruitless because it will expose some of the features of a phase transition. Then we will consider percolation on a tree-like lattice where we will also be able to make considerable analytic progress. This so-called mean-field percolation gives a qualitatively more accurate description of a phase transition, but leaves out important features, both qualitative and quantitative, that are present in percolation in general. In two dimensions, we will initially appear to reach an impasse, since the possibility that clusters 'interact' with themselves will force us to abandon explicit calculations even for clusters of moderate size. Fortunately, however, one-dimensional and mean-field percolation provide clues suggesting that something generic happens to the clusters near a phase transition. We will identify and exploit this generic phenomenon by invoking the powerful concept of scale invariance. This will allow us to propose a particular functional form for the statistics of clusters. While this so-called scaling ansatz will not be able to give us the exact statistics of clusters, we will argue that the concept of scale invariance nonetheless gives us something far more preferable: a statistical framework for unifying the description of clusters in percolation in general.

When studying the geometrical properties of percolation in greater detail, we will find that the root of the simplification of scale invariance lies in the fractal nature of percolation at a phase transition. Loosely speaking, this means that at a phase transition percolating systems look alike on different scales. This in turn will refine our intuition of scale invariance, and we will be in a position to introduce the so-called renormalisation group approach. Such an approach is designed to exploit scale invariance in percolation at the phase transition, by preserving only large-scale features of the clusters. We will illustrate this approach with specific examples of renormalisation group transformations.

1.1.1 *Definition of site percolation*

We now define site percolation, one of the simplest examples of a disordered system. Consider for now a two-dimensional square lattice composed of $L \times L$ sites. We refer to L, measured in units of the lattice spacing, as the lattice size. Note that L is not a length scale as such, but a dimensionless number, L'/a, where L' is the lattice length and a the lattice spacing. Throughout, all lengths are measured in units of the lattice spacing.

To introduce disorder, we occupy sites randomly and independently with occupation probability p, treating all sites equally, see Figure 1.1. The foremost entity of interest is a cluster, which we define as a group of nearest-neighbouring occupied sites. The cluster size, s, is the number of occupied sites in the group.



s	$N(s, p; L)$
1	2
2	1
4	1
7	1

Fig. 1.1 A realisation of two-dimensional site percolation on a square lattice of size $L = 5$. Sites are occupied with probability p and are thus empty with probability $(1 - p)$. Occupied sites are dark grey while unoccupied sites are white. A cluster is defined as a group of nearest-neighbouring occupied sites. In a square lattice, a bulk site has four nearest neighbours: north, south, east, and west and four next-nearest-neighbours: north-east, north-west, south-east, and south-west. The table to the right displays the cluster size frequency, $N(s, p; L)$.

1.1.2 *Quantities of interest*

We shall introduce the quantities of interest on a two-dimensional square lattice. For a fixed lattice size L measured in units of the lattice spacing, there is only one parameter in the problem, namely the occupation probability p.

For $p = 0$ the lattice is empty of clusters, while for $p = 1$ there is only one cluster of size L^2. For intermediate values $0 < p < 1$, each realisation will, in general, be different. But qualitatively, we expect the size of the largest cluster to increase with p. Figure 1.2 supports this trend, showing six realisations of increasing p on a lattice of size $L = 150$.