# 路见可论文选集

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验据通过类量的基本方法。 歐洲外語自由主義

第十二、血臟率性精神 一条并工工工工并编制。他并得到是打出數學的主

医的物果基础。在17世间自动,是和广防自己性。

江苏工业学院图书馆 藏 书 章

武 汉 大 学 一九八九

# 1] 整合边值问题。武坝大学学说(自然)舞z版)。《麓4625 15 - 21

- 1.本集只选有关解析函数边值问题和奇异积分方程及其应用的论文(到1988年为止), 着和其它方面的论文不选,综述性论文也不选。
- 2. 论文按出于李邦列;有的论文采用转载的,但次序以最初出版年为准。
- 3. 有的论文用中、英文同时发表,这里只选用一种。则对别一的金母对11周30度
- 4. 论文中发现的个别错误已予改正。学人大海、顾问本基面平台单级无常能经带

3千不同弹性材料的平面促集问题,或汉大空关报(自然科学服)。3(196 2、30-36)"汉转魏于高等主控台供料器契据(数律以次学是联及

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[7] <sup>(4)</sup> 关于循环对称弹性平面中的数学问题。就设大学学报(自然科学版)。 (1964)、1954年(1964)、1954年(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964)(1964

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# Φ(z) should be continuous to L from the interior of D and satisfy

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MATHEMATICS

## ON COMPOUND BOUNDARY PROBLEMS\*

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### ABSTRACT

In this paper, the so-called Riemann-Hilbert compound problems are considered. Such problems may be formulated as follows. Let D be a Liapounoff region with the boundary L, and  $\Gamma$  be a finite set of non-intersecting smooth contours in D. Find a sectionally holomorphic function in D satisfying a Riemann boundary condition on  $\Gamma$  and a Hilbert boundary condition on L. The cases in which  $\Gamma$  consists of open arcs and the corresponding vector problems are also considered. The method of solution is to eliminate the "Riemann part" of the problem at first. The problem then reduces to a certain Hilbert one.

## NOITOUGHAL I. Lengurd to (2.2) temporarily

In [1], we have given a brief sketch of the so-called compound boundary problems and the process of solution by the method of elimination. Here we shall give these results in detail. We retain the notations in [2], and the Riemann problems and the Hilbert problems will be denoted briefly by the R-problems and the H-problems respectively, while the compound boundary problems, by the RH-problems.

#### II. THE RH-PROBLEMS FOR SIMPLY CONNECTED REGIONS

Let D be the region bounded by a Liapoùnoff curve L (i.e., a curve, the inclination of the tangent of which, considered as the function of its arc length, satisfies the Hölder condition), and  $\Gamma_1, \dots, \Gamma_n$  be a set of non-intersecting and mutually exclusive contours in D. Denote  $\Gamma = \Gamma_1 + \dots + \Gamma_n$ . Take the counter-clockwise sense along L as its positive sense, while along  $\Gamma_i$ , we take the clockwise sense as positive. By  $D_i^-$  we denote the inner region bounded by  $\Gamma_i$ , and by  $D_0^+$ , the region bounded by L and all the  $\Gamma_i$ .

The RH-problem for D may be formulated as follows. Find a sectionally holomorphic function  $\Phi(z)$  in D (i.e., regular everywhere in D except the points on  $\Gamma$  and continuous to  $\Gamma$  from both sides of it) subjected to the following two conditions: 1) in the neighbourhood of  $\Gamma$ ,  $\Phi(z)$  should satisfy the boundary condition

(2.1) with 
$$\mathcal{D}$$
 only in the  $\sigma \mathcal{T} \ni \tau$  ,  $(\tau)g + (\tau) \Phi^-(\tau) = G(\tau)\Phi$  in a R-problem is always

where  $G(\tau)$ ,  $g(\tau)$  are given on  $\Gamma$ , satisfying the Hölder conditions and  $G(\tau) \neq 0$ ; 2)

<sup>\*</sup> First published in Chinese in Acta Scientiarum Naturalium Scholarum Superiorum Sinensium, Pars Mathematica, Mechanica et Astronomica, No. 1, pp. 1-11, 1964.

 $\Phi(z)$  should be continuous to L from the interior of D and satisfy the boundary condition

Re 
$$[\overline{\lambda(t)}\Phi(t)] = c(t), \quad t \in L,$$
 (2.2)

where  $\lambda(t)$ , c(t) are given on L, also satisfying the Hölder conditions and  $\lambda(t) \neq 0$ (c(t)) being real of course).

ON COMPOUND BOUNDARY PROBLEM

Denote

$$\operatorname{Ind}_{\Gamma_{j}}G(\tau) = \frac{1}{2\pi} \left[ \operatorname{arg} G(\tau) \right]_{\Gamma_{j}} = \kappa_{j}, \quad \operatorname{Ind}_{\Gamma} G(\tau) = \kappa = \sum_{j=1}^{n} \kappa_{j},$$

$$\operatorname{Ind}_{L} \lambda(t) = k.$$

We shall call

$$K = \kappa + k$$

In this paper, the sa-collest Riemann Hilbert compound problems are considered. Such problems the index of the above RH-problem. Bonuppel a Lie D be a Liappunost a major and a solid as believed as follows.

In each  $D_i^2$ , take an arbitrary point  $z_i$  and denote a drooms gainesistation to the similar state  $z_i$  and addition reaches a point  $z_i$  and addition reaches  $z_i$  and  $z_i$  and

satisfying a Niembrit boundary condition on 
$$f$$
 and a ranbert boundary condition on  $L$ , the case which  $f$  consists of open are and the corresponding vector problems are also considered. The method of solution is to eliminate the  $i^{*}(iz-z)\prod_{i=1}^{n}(z)H$  leads at these the problem than reduces to a certain Hilbert one.

We shall find first of all a function  $\Phi_1(z)$ , sectionally holomorphic in D and continuous to L, satisfying the condition (2.1), with no regard to (2.2) temporarily. The general solution of this problem (called the corresponding R-problem of the previous and the process of solution by the method of climination. Here we shall give see (meldong

(2.2) all. We recain the notation 
$$F(z) + F(z) = F(z) F$$

where X(z) is its canonical function; HR and the problem of the compound boundary problem is the compound by the X(z) is its canonical function.

$$X(z) = \begin{cases} X^{+}(z) = \Pi(z)^{-1}e^{r^{+}(z)} & \text{when } z \in D_{0}^{+}, \\ X(z) = \begin{cases} X^{-}(z) = e^{r^{-}(z)} & \text{when } z \in \sum_{j} D_{j}^{-}, \\ X^{-}(z) = e^{r^{-}(z)} & \text{when } z \in \sum_{j} D_{j}^{-}, \end{cases}$$
Let  $D$  be the region boundaries the function of its arr length, satisfies the top

condition), and 
$$\Gamma_{i}$$
 .  $\Gamma_{i}$  be a  $\int_{0}^{\infty} \frac{1}{|x|} \frac{1}{$ 

(any branch of the logarithm may be chosen);  $\Psi(z)$  may be taken as 0The RH problem for D may be formulated as follows. Find a sectionally holomorphic

The critical depth of 
$$\Psi(z) = \frac{1}{2\pi i} \int_{\Gamma} \frac{g(\tau)}{X^{+}(\tau)} \frac{d\tau}{\tau} \int_{\Gamma} \frac{g(\tau)}{X^{+}(\tau)} \frac{d\tau}{\tau}$$
 and contains the conditions of  $\frac{1}{2\pi i} \int_{\Gamma} \frac{g(\tau)}{X^{+}(\tau)} \frac{d\tau}{\tau} \frac{d\tau}{\tau}$  and  $\frac{1}{2\pi i} \int_{\Gamma} \frac{g(\tau)}{X^{+}(\tau)} \frac{d\tau}{\tau} \frac{d\tau}{\tau}$  and  $\frac{1}{2\pi i} \int_{\Gamma} \frac{g(\tau)}{X^{+}(\tau)} \frac{d\tau}{\tau} \frac{d\tau}{\tau} \frac{d\tau}{\tau}$  and  $\frac{1}{2\pi i} \int_{\Gamma} \frac{g(\tau)}{X^{+}(\tau)} \frac{d\tau}{\tau} \frac{d\tau}{\tau} \frac{d\tau}{\tau}$  and  $\frac{1}{2\pi i} \int_{\Gamma} \frac{g(\tau)}{X^{+}(\tau)} \frac{d\tau}{\tau} \frac{d\tau}$ 

while F(z) is an arbitrary function, holomorphic in D and continuous on D. As we deal with D only in the original RH-problem, so the corresponding R-problem is always solvable, and the arbitrary function F(z) is now in place of the polynomial presented in the general solution for the ordinary R-problem in the entire plane.

Note that, when  $g(\tau) = 0$ , X(z) itself is also a solution of (2.1):

$$X^+(\tau) = G(\tau)X^-(\tau), \quad \tau \in \Gamma$$

Transform the unknown function  $\Phi(z)$  to another one,  $\Phi_0(z)$ , by means of the expression

$$\Phi(z) = \Phi_1(z) + X(z)\Phi_0(z); \text{ulay elignized (1) The point (2.5)}$$

then it is evident that  $\Phi_0(z)$  is also sectionally holomorphic in D- and continuous to L. Besides, when  $\tau \in \Gamma$ , noting that  $\Phi_1(z)$  satisfies (2.1), we obtain, by (2.4),

$$\begin{split} \Phi^{+}(\tau) &= \Phi_{1}^{+}(\tau) + X_{-}^{+}(\tau)\Phi_{0}^{+}(\tau) = \quad \text{(8.2) in antivariations} \\ &= G(\tau)\Phi_{1}^{-}(\tau) + g(\tau) + G(\tau)X_{-}^{-}(\tau)\Phi_{0}^{+}(\tau) = \\ &= G(\tau)[\Phi_{1}^{-}(\tau) + X_{-}^{-}(\tau)\Phi_{0}^{+}(\tau)] + g(\tau). \end{split}$$

If  $\Phi(z)$  is the required solution, then it should satisfy (2.1) also, i.e.,

$$\Phi^{+}(\tau) = G(\tau)\Phi^{-}(\tau) + g(\tau) = G(\tau)[\Phi_{1}^{-}(\tau) + X^{-}(\tau)\Phi_{0}^{-}(\tau)] + g(\tau).$$

Comparing these and remembering that  $X^{-}(\tau) \neq 0$ , immediately we obtain

(and solution 
$$\Phi_0^+(\tau) = \Phi_0^-(\tau)$$
,  $\tau \in \Gamma$ ,

hence  $\Phi_0(z)$  is holomorphic in D and continuous on  $\overline{D}$ .

Conversely, if  $\Phi_0(z)$  is holomorphic in D and continuous on  $\overline{D}$ , then it is easy to prove that the sectionally holomorphic function  $\Phi(z)$  determined by (2.5) will satisfy (2.1) and will also be continuous to L.

Therefore, the proposed problem has been transformed to such one: to find a function  $\Phi_0(z)$  holomorphic in D and continuous on  $\overline{D}$  such that it should satisfy the corresponding condition transformed from (2.2). Substituting (2.5) into (2.2), we get readily this condition:

$$\operatorname{Re}\left[\overline{\lambda(t)}X(t)\Phi_{0}(t)\right] = c^{*}(t), \quad t \in L, \tag{2.6}$$

where

$$c^*(t) = c(t) - \operatorname{Re}\left[\overline{\lambda(t)}\phi_1(t)\right]. \tag{2.7}$$

Thus, the original problem has been reduced to the H-problem (2.6) for D, while the condition (2.1) and all the  $\Gamma_i$  have been eliminated. That is why the name "method of elimination" is suggested.

The choice of the arbitrary function F(z) in (2.3) is indifferent. For, if a function f(z) is added to F(z), then a term X(z) f(z) will be added to  $\Phi_1(z)$ , and from (2.6),  $\Phi(z)$  will remain invariant if we subtract f(z) from the required function  $\Phi_0(z)$ ; this is equivalent to a subtraction of the term  $\text{Re}[\bar{\lambda}X/]$  from both sides of (2.6), so that  $\Phi_0(z)$  is invariant also. Hence, without loss of generality, we may always take  $F(z) \equiv 0$  in (2.3).

For the H-problem (2.6) satisfied by  $\Phi_0(z)$ , the index is

must be added; when 
$$(t)X(t)X(t)X(t)X(t)X(t)X(t)X(t)X(t)X(t)$$
 when and only when  $(2.8)$ 

Noting that, when  $t \in L$ ,

formed to a homogeneous H-problem again, When 
$$K \ge 0$$
, the general solution is the same as (2.11), but a term  $t = (t) + T = (t) + X = (t) + X = (t) + X = (t) + X = 0$ ,

we have

Transform the anknown function 
$$\Phi(z)$$
 to another one,  $\Phi_{n}(z)$ , by Dression  $\Phi(z)$ ,  $e^{r+(z)}$ .

Since  $\Gamma^+(t)$  is single-valued and continuous on L, so

then it is evident that 
$$\mathcal{C}(z) = \frac{1}{2\pi i} \lim_{t \to 0} \frac{1}{1} \lim$$

substituting in (2.8) and (2.9), we finally get (1) (1)

$$\operatorname{Ind}_{L}\left[\lambda(t)\overline{X(t)}\right] = k + \kappa = K. \tag{2.10}$$

That is to say, the index of the transformed H-problem is just that of the original RH-problem.

Therefore, applying the usual theory about the number of solutions and the conditions of solubility of H-problems, we can discuss RH-problems similarly:

1) Suppose  $g \equiv 0$ ,  $c \equiv 0$  (the homogeneous case). If  $K \ge 0$ , the original problem has 2K + 1 linearly independent (with respect to real coefficients) solutions:

$$\Phi(z) = X(z) \sum_{i=1}^{2K+1} c_i \Phi_{0i}(z), \qquad (2.11)$$

where  $\Phi_{0r}(z)$   $(s=1, \dots, 2K+1)$  are the complete system of solutions of the corresponding H-problem (2.6) (now  $c^* \equiv 0$ ), and  $c_i$  are arbitrary real constants (now  $\Psi(z) \equiv 0$ ).

If K < 0, the corresponding H-problem has the trivial solution  $\Phi_0(z) \equiv 0$  only, consequently the original problem has also the trivial solution only.

2) Suppose  $g \equiv 0$ ,  $c \not\equiv 0$ . Now we may take  $\Phi_1(z) \equiv 0$ , so that  $\Phi(z) = X(z) \Phi_0(z)$ , and  $\Phi_0(z)$  is the solution of the non-homogeneous H-problem on L:

Re 
$$[\overline{\lambda(t)}X(t)\Phi_0(t)] = c(t), t \in L.$$

In this case, if  $K \ge 0$ , the problem is solvable. Let  $\Phi_{\infty}(z)$  be a particular solution; then the general solution of the original problem is

$$\Phi(z) = X(z) \left[ \Phi_{00}(z) + \sum_{k=1}^{2K+1} c_k \Phi_{0k}(z) \right].$$
 and a contamination of the property of the pro

If K < 0, the problem has a unique solution when and only when c(t) satisfies -2K-1 conditions.

3) Suppose  $g \not\equiv 0$ ; then  $\Phi_1(z) \not\equiv 0$ . We now have a classical H-problem by noting that  $c^*(t)$  also satisfies the Hölder condition on L. Two subcases are possible now:

(i) If  $c^*(t) \not\equiv 0$ , i.e.,  $c(t) \not\equiv \text{Re}[\lambda(t) \Phi_1(t)]$ , then it is the same as the previous case 2). When  $K \ge 0$ , to the right-hand side of the general solution (2.12), a term  $\Phi_1(z)$  must be added; when K < 0, then there is a unique solution when and only when -2K-1 conditions are satisfied among  $c, \lambda, G, g$ .

(ii) If  $c^*(t) \equiv 0$ , i.e.,  $c(t) \equiv \text{Re}[\overline{\lambda(t)}\Phi_1(t)]$ , then the problem will be transformed to a homogeneous H-problem again. When  $K \geq 0$ , the general solution is the same as (2.11), but a term  $\Phi_1(z)$  must be added to the right-hand side; when K < 0,

the H-problem has the trivial solution  $\Phi_0(z) \equiv 0$  only, hence the original problem has the unique (non-trivial) solution  $\Phi(z) = \Phi_1(z)$ .

and (i) of 3) the proper non-homogeneous problems, while the cases 2)

The condition of the quasi-homogeneity of the problem is  $c(t) = \text{Re}[\overline{\lambda(t)}\Phi_1(t)]$ , where  $\Phi_1(t) = X(t)\Psi(t)$ . If we take the particular solution of the corresponding R-problem as (2.3) at the very beginning, where F(z) is a certain function holomorphic in D and continuous on D (satisfying the Hölder condition on L), then the quasi-homogeneous RH-problem may be reduced to a non-homogeneous H-problem for  $\Phi_0(z)$ :

$$\operatorname{Re}\left[\overline{\lambda(t)}X(t)\Phi_0(t)\right] = -\operatorname{Re}\left[\overline{\lambda(t)}X(t)F(t)\right].$$

When  $K \ge 0$ , the general solution of the problem is

Let now 
$$\Phi(z) = \Phi_1(z) + X(z) \left[ \Phi_{00}(z) + \sum_{i=1}^{K+1} c_i \Phi_{0i}(z) \right],$$
 where

where  $\Phi_{00}(z)$  is a particular solution of this H-problem. When K < 0, the H-problem possibly has one solution at most, but  $\Phi_0(z) = -F(z)$  is its solution evidently, hence  $\Phi(z) = \Phi_1(z) - F(z) = \Phi^*(z)$  is the unique (non-trivial) solution of the original problem. This result is exactly the same as that mentioned above.

Therefore, the general condition for an RH-problem to be quasi-homogeneous is (besides  $g \not\equiv 0$ ): there exists a function  $\Phi_1(z)$  (expressed as (2.3)), sectionally holomorphic in D, continuous to L, and satisfying the condition (2.1), such that

Re 
$$\left[\overline{\lambda(t)}\Phi_1(t)\right] = c(t), \quad t \in L.$$
 (2.13)

From the above discussion, we conclude:

**Theorem 1.** For simply connected regions, when  $K = \kappa + k > 0$ , in the general solution of the RH-problem there exist 2K + 1 arbitrary real constants; when K < 0, 1) there is only the trivial solution for the homogeneous case  $(c \equiv 0, g \equiv 0), 2$  there is a unique non-trivial solution for the quasi-homogeneous case, 3) there is a unique solution if and only if the given coefficients of the problem satisfy -2K - 1 conditions for the proper non-homogeneous case.

When K < 0, there is only one solution (trivial or non-trivial) for the homogeneous or quasi-homogeneous case.

The quasi-homogeneous problem may be considered as a particular case of the proper non-homogeneous problem. When  $K \ge 0$ , this conclusion has been brought out in Theorem 1. Suppose now K < 0. Since there is only one solution for the quasi-homogeneous problem, its coefficients should also satisfy -2K-1 conditions, similar to those of the non-homogeneous case. Or, we may obtain them alternatively: we know that (cf. [2], §§ 28, 29), if p(s) is the factor of regularization for  $\lambda(s)$ , and if we put

$$e^{-\omega_1(s)} = p(s)|\lambda(s)|,$$

then these -2K-1 conditions may be written as

$$\int_{0}^{2\pi} e^{\omega_{1}(s)} c^{*}(s) e^{-ihs} ds = 0 \quad (h = 1, \dots, -2K - 1).$$

In the quasi-homogeneous case, we may choose  $\Phi_1(z)$  suitably in order that (2.13) is satisfied; from (2.7),  $c^*(s) \equiv 0$ , therefore these conditions are certainly satisfied, one

If we consider the problem stated at the beginning for the exterior region bounded by L ( $\Gamma_i$  lying also in this exterior region) and supplementarily require  $\Phi(\infty)$  to be bounded, it is obvious that we may obtain the results analogous to Theorem 1. Or, by inversion, it may be transformed to a problem for an inner region and then treated as before. We shall not go to details. We have shown in [1] that the RH-problems for simply connected regions may be

solved by using N. I. Muskhelishville's method (cf. [3], § 42), but this method is unavailable for multiply connected regions, with which we shall deal in the next section.

## THE RH-PROBLEMS FOR MULTIPLY CONNECTED REGIONS

Let now D be a multiply connected region bounded by a set of Liapounoff curves  $L_0, L_1, \dots, L_m$ , in which  $L_0$  surrounds all the others ( $L_0$  may be absent). Denote  $L = \sum_{p=0}^{\infty} L_p^{1}$ . In the interior of D, there are a set of curves  $\Gamma_{\mathbf{b}} \cdots \Gamma_n$  as before, but some of  $\Gamma_i$  may surround some of  $L_p$ . For  $L_0$ , we take the counter-clockwise sense as its positive sense, and for the others, the clockwise senses. (3) 0 = (5) 1 - (5) 0 = (5)

At this time, the RH-problem for D may be stated as follows. Find a function  $\Phi(z)$ , sectionally holomorphic in D, continuous to L, satisfying the conditions (2.1) and (2.2). The conditions subjected to all the given coefficients remain unchanged. If  $L_0$  is absent, then we supplementarily require  $\Phi(\infty)$  to be bounded. It is a supplementarily require  $\Phi(\infty)$ 

Note that k and k are defined as before, but now

$$k = \sum_{p=0}^{m} k_p, \quad k_p = \operatorname{Ind}_{L_p} \lambda(t);$$

K=k+k is called the index of the RH-problem again.

The method of solution is similar to that in the case of simply connected region. Take an arbitrary  $z_i$  in the interior of  $\Gamma_i$ . When  $\Gamma_i$  surrounds some  $L_p$ ,  $z_i$  may be taken interior to, exterior to, or on L,. Find at first the solution (2.3) for the R-part of the problem (we may again take  $F(z) \equiv 0$ ), then through (2.5), the unknown function is transformed to  $\Phi_0(z)$ , and the problem is reduced to H-problem (2.6) for D. The index of the latter is

Ind<sub>L</sub> [
$$\lambda(t)\overline{X(t)}$$
] = Ind<sub>L</sub>  $\lambda(t)$  - Ind<sub>L</sub>  $X(t)$  =  $k$  -  $\sum_{p=0}^{p=0}$  Ind<sub>Lp</sub>  $X(t)$ . The quasi-

homogeneous problem, its coefficients should also satisfy - 2K - 1 conditions, similar for

chose of the non-nonogeneous, 
$$X(t) = \operatorname{Ind}_{L_0} X(t) = \operatorname{Ind}_{L_0} X^+(t) = -\kappa$$
, we know that

which is the same as before. For  $L_p(p \ge 1)$ , if no  $\Gamma_i$  surrounds it, we then have  $\operatorname{Ind}_{L_p} X(t) = \operatorname{Ind}_{L_p} X^+(t) = \operatorname{Ind}_{L_p} \Pi(t) e^{\Gamma^+(t)} = 0$ 

$$\operatorname{Ind}_{L_p} X(t) = \operatorname{Ind}_{L_p} X^+(t) = \operatorname{Ind}_{L_p} \Pi(t) e^{\Gamma^+(t)} = 0$$

<sup>1)</sup> In case of the absense of  $L_0$ , the summation begins from p=1. This will be always the same in the following.

since all the  $z_i$  are situated in the exterior of  $L_p$ . Hence  $\operatorname{Ind}_{L_p}\Pi(t)=0$ , and we have already seen that  $\operatorname{Ind}_{L_p}e^{r^+(t)}=0$ . If  $L_p$  is surrounded by some  $\Gamma_i$ , then

$$\operatorname{Ind}_{L_p} X(t) = \operatorname{Ind}_{L_p} X^{-}(t) = \operatorname{Ind}_{L_p} e^{\Gamma^{-}(t)} = 0.$$

Therefore, in any case, the index of the transformed H-problem is  $K = \kappa + k$  too.

Thus, using the known results about the H-problems for multiply connected regions, we obtain immediately

Theorem 2. For the RH-problem for the (m+1)-ply connected regions, if its index K > m-1, then it is always solvable and there are 2K-m+1 independent constants in its general solution; if K < 0, then the homogeneous problem has the trivial solution only, the quasi-homogeneous problem has a unique solution (non-trivial), and the proper non-homogeneous problem has a unique solution when and only when the given coefficients satisfy -2K+m-1 conditions.

By the quasi-homogeneous problem, we mean here that the condition (2.13) is fulfilled.

For the singular cases  $0 \le K \le m-1$ , we shall have results analogous to those due to F. D. Gahov<sup>[4]</sup> and B. V. Boiarsky<sup>[5]</sup>, which we shall omit here.

## IV. THE RH-PROBLEMS FOR OPEN ARCS

Let D be as before (simply or multiply connected), and let  $\Gamma_{1}, \dots, \Gamma_{n}$  be open smooth Jordan arcs:  $\Gamma_{i} = \widehat{a_{i}b_{i}}(j=1,\dots,n)$ . Take the positive sense of  $\Gamma_{i}$  as that from  $a_{i}$  to  $b_{i}^{(1)}$ . The formulation of the RH-problem now is the same as before, the conditions subjected to the coefficients of (2.1) and (2.2) remain unchanged, but in the neighbourhoods of the end-points of all the  $\Gamma_{i}$ , we shall require  $\Phi(z)$  to be bounded or integrably unbounded. We shall observe that, in the neighbourhoods of some of them, we can only require  $\Phi(z)$  to be almost bounded (i.e., for any  $\varepsilon > 0$ ,  $|z - c| \Phi(z) \to 0$  when  $z \to c$ ). Such end-points will be called special ones for the RH-problem. It will be proved in the following that they are just the special end-points  $c_{r+1}, \dots, c_{2n}$  of the corresponding R-problem (2.1). (For the definition of them, see, for example, [2], § 42.) For the remaining non-special end-points, we may, for example, require  $\Phi(z)$  to be bounded near  $c_{1}, \ldots, c_{n}$ , to be integrably unbounded at most near  $c_{n+1}, \ldots, c_{n}$ . According to the notations in [3], such solutions are said to belong to the class  $h(c_{1}, \ldots, c_{n})$ .

Here, the previous method of elimination remains valid. We show this briefly as follows: 1 and 2 and 2

On each  $\Gamma_i$ , we take a definite branch of  $\ln G(\tau)$  such that

$$\alpha_i = \operatorname{Re} \gamma_i = \operatorname{Re} \left[ -\frac{\ln G(a_i)}{2\pi i} \right]$$

satisfies the condition:

$$\begin{cases} 0 \leq \alpha_{i} < 1 & \text{when } a_{i} \in \{c_{1}, \dots, c_{q}, c_{r+1}, \dots, c_{2n}\}, \\ -1 < \alpha_{i} < 0 & \text{when } a_{i} \in \{c_{q+1}, \dots, c_{r}\}, \end{cases}$$

<sup>1)</sup> If some of the  $\Gamma_j$  are closed and the others are open, the discussion may be similarly carried out.

when  $a_i \in \{c_{r+1}, \dots, c_{2n}\}$ , we have precisely  $\alpha_i = 0$ . On putting

$$\alpha'_{i} = \operatorname{Re} \gamma'_{i} = \operatorname{Re} \frac{\ln G(b_{i})}{2\pi i},$$

we choose integers  $\kappa_i$  such that

become 
$$\{0 \le \alpha'_i - \kappa_i < 1 \}$$
 when  $b_i \in \{c_1, \cdots, c_q, c_{r+1}, \cdots, c_{2n}\}$ , we then have precisely  $\alpha'_i = 1 \}$ . We then have precisely  $\alpha'_i = 1 \}$ .

when  $b_i \in \{c_{r+1}, \dots, c_{2n}\}$ , we then have precisely  $\alpha'_i - \kappa_i = 0$ . Hence

constants in its general solution; if 
$$K < (n \text{ lie n the homogeneous problem has the trivial solution only, the quasi-homogeneous  $p_i(x) = \frac{1}{2} \pi_i x$  unique solution (non-trivial), and the$$

is the index of the corresponding R-problem with respect to the class  $h(c_1, \dots, c_q)$ . Denoting also  $k = \text{Ind } L\lambda(t)$ , we call  $K = \kappa + k$  the index of the RH-problem with respect to the class  $h(c_1, \dots, c_q)$ .

The canonical function of the corresponding R-problem is seem and any activated

In the second with the second 
$$X(z) = \prod_{j=1}^{n} (z - b_j)^{-s_j} e^{\Gamma(z)},$$
 (4.1)

where -

product near that 
$$r(z) = \frac{1}{2\pi i} \int_{\Gamma} \frac{\ln G(\tau)}{\tau - z} d\tau$$
. The second second (4.2)

Solve first the corresponding R-problem for D and find out a particular solution (the arbitrary function F(z) may be taken to be identical with zero):

arbitrary function 
$$F(z)$$
 may be taken to be identical with zero): standard and or best and decision as below  $\Phi_1(z) = \frac{X(z)}{2\pi i} \int_{\Gamma} \frac{g(\tau)}{X^+(\tau)} \frac{d\tau}{\tau - z}$ . Add links a will be bound (4.3)

Using the method of elimination, we put sol sand hards believed live saniog-bas double

$$\Phi(z) = \Phi_1(z) + X(z)\Phi_0(z),$$
where  $\Phi(z) = \Phi_1(z) + X(z)\Phi_0(z)$  and  $\Phi(z) = \Phi_1(z) + \Phi_1(z)$ 

where  $\Phi_0(z)$  is the new unknown function.

Similar to § II, it is easy to prove that, at the inner points of each  $\Gamma_i$ ,  $\Phi_0^+(\tau) = \Phi_0^-(\tau)$ , so that  $\phi(z)$  is single-valued, analytic in D, possibly except all the end-points  $a_i$ ,  $b_i$ . We shall discuss the properties of  $\Phi_0(z)$  near all these points.

Before going on, we recollect some properties of X(z). From (4.2), we have

$$\Gamma(z) = \begin{cases} \gamma_i \ln(z - a_i) + \Gamma_i(z) & \text{when } z \text{ lies near } a_i, \\ \gamma_i' \ln(z - b_i) + \Gamma_i'(z) & \text{when } z \text{ lies near } b_i, \end{cases}$$

where  $\Gamma_i(z)$  and  $\Gamma'_i(z)$  are bounded near  $a_i$  and  $b_i$  respectively. Therefore,

$$e^{\Gamma(z)} = \begin{cases} (z - a_i)^{\gamma_i} e^{\Gamma_i(z)} & \text{when } z \text{ lies near } a_i, \\ (z - b_i)^{\gamma_j'} e^{\Gamma_j'(z)} & \text{when } z \text{ lies near } b_i. \end{cases}$$

<sup>1)</sup> The logarithm is to be understood as follows: after cutting the plane along  $\Gamma_i$  and extending the cut to infinity, we take an arbitrary branch of it. nogo are troduc and base based are A and to amos II (I

We then have

$$X(z) = \begin{cases} (z - a_i)^{r_i} e^{\Gamma_i^*(z)} & \text{when } z \text{ lies near } a_i, \\ (z - b_i)^{r_i' - \kappa_i} e^{\Gamma_i^{*'}(z)} & \text{when } z \text{ lies near } b_i, \end{cases}$$

or, near any end-point z = c, we may write

If an teding it personnel to 
$$\log X(z) = (z-c)^{r_c} e^{r_c(z)}$$
, and variated on that (4.5)

where  $\Gamma_c(z)$  is bounded near c, and  $x = (z) \otimes z \otimes z \otimes z$ 

$$\begin{cases} 0 \leq \operatorname{Re} \gamma_{c} < 1 & \text{when } c \in \{c_{1}, \cdots, c_{q}, c_{r+1}, \cdots, c_{2n}\}, \\ -1 < \operatorname{Re} \gamma_{c} < 0 & \text{when } c \in \{c_{q+1}, \cdots, c_{r}\}, \end{cases}$$

$$\begin{cases} 0 \leq \operatorname{Re} \gamma_{c} < 1 & \text{when } c \in \{c_{q+1}, \cdots, c_{r}\}, \\ -1 < \operatorname{Re} \gamma_{c} < 0 & \text{when } c \in \{c_{q+1}, \cdots, c_{r}\}, \end{cases}$$

where c is a special end-point,  $\operatorname{Re} \gamma_c = 0$ . To not ulos testinomiag a stro both next.

According to the well-known properties of integrals of the Cauchy type, from (4.2), we know that the particular solution  $\Phi_1(z)$  of the R-problem belongs to the class  $h(c_1, \dots, c_q)$ . From the requirements subjected to the unknown function  $\Phi(z)$ , we know that  $X(z)\Phi_0(z)$  must be bounded near the end-points  $c_1, \dots, c_q$  and integrably unbounded at most near the remaining end-points.

Let c be a special end-point. In order to insure that  $\Phi(z)$  is integrably unbounded near c,  $X(z)\Phi_0(z)$  must follow suit. But now since  $\operatorname{Re} \gamma_c = 0$ , so X(z) is bounded near z = c and  $\neq 0$ ; since c is an isolated singular point of  $\Phi_0(z)$  and  $X(z)\Phi_0(z)$  is integrable, so c is actually an ordinary point of  $\Phi_0(z)$  and then  $X(z)\Phi_0(z)$  is bounded near c. Eventually  $\Phi(z)$  is almost bounded near c. Thus, near the special end-points,  $\Phi(z)$  must be almost bounded.

If  $c \in \{c_1, \dots, c_q\}$ , then, in order that  $X(z)\Phi_0(z)$  is bounded near c,  $\Phi_0(z)$  must take c as an ordinary point since  $0 < \operatorname{Re} \gamma_c < 1$ . If  $c \in \{c_{q+1}, \dots, c_r\}$ , then, in order that  $X(z)\Phi_0(z)$  is integrably unbounded near c, c can only be an ordinary point of  $\Phi_0(z)$  also since  $-1 < \operatorname{Re} \gamma_c < 0$ .

In a word, whatever end-point c may be it must be an ordinary point of  $\Phi_0(z)$ . Hence  $\Phi_0(z)$  is holomorphic in D. Since both  $\Phi_1(z)$  and X(z) are continuous to L and  $X(z) \neq 0$  in the neighbourhood of L, so  $\Phi_0(z)$  must be continuous to L also. Now, the condition to be satisfied by  $\Phi_0(z)$  is the same as that in (2.6). Thus, the problem again reduces to an H-problem for  $\Phi_0(z)$ . From (4.1), we get readily

L is its counter-clockwise sense, where that of its its clockwise sense). Thus (5.1) rue. From this, we read to obtain 
$$\frac{1}{1-1}$$
 the covariation of the reduced H-problem is

therefore the index of this H-problem again is  $ab_{1}ba_{2}=[(1)X(1)X(1)X]$  the shall

If we can calculate 
$$N(x)$$
 effectively, then  $(i)X(i)X_{1}$  but of this id-problem are known, in rectore the original RH-problem can be reduced to a given definite H-problem, and we

The remaining discussion is then similar to that in § II or § III.

Evidently, for the cases of discontinuous coefficients, we may treat the problem similarly.

## V. THE RH-PROBLEMS FOR SEVERAL UNKNOWN FUNCTIONS

Notations L,  $\Gamma$  are the same as those in § II. Let the required  $\Phi(z)$  be an N-dimensional vector, holomorphic in D, continuous to L, and satisfying (2.1) and (2.2), where  $G(\tau)(\tau \in \Gamma)$ ,  $\lambda(t)(t \in L)$  are given non-degenerate matrices of order N,  $g(\tau)$ , c(t) are given N-dimensional vectors, and all of them satisfy the Hölder conditions. Thus, we have an RH-problem for N unknown functions or an N-dimensional vector.

This problem may be solved again by the method of elimination described in § II.

$$\operatorname{Ind}_{\Gamma} \det G(\tau) = \kappa$$
,  $\operatorname{Ind}_{L} \det \lambda(t) = k$ ,

and call  $K = \kappa + k$  the total index of the previous RH-problem.

On solving it, we first erect the canonical matrix of the corresponding R-problem (2.1) (the definition and the general method of construction of it may be referred to [6]), then find out a particular solution of this R-problem:

According to the well-control 
$$(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-1}(z)^{-$$

and at last, according to (2.5), change the unknown vector  $\Phi(z)$  to  $\Phi_0(z)$ . Noting that, in the present case, the reasoning from (2.5) on remains valid, we obtain finally  $\Phi_0^+(\tau) = \Phi_0^-(\tau)$ , i.e.,  $\Phi_0(z)$  is an N-dimensional vector, holomorphic in D and continuous on  $\overline{D}$ . Thus, the problem is reduced to the H-problem (2.6) once again.

The total index of this H-problem may be shown to be exactly equal to K. (According to the definition of N. P. Vekua, it is 2K, cf. [7], p. 179.) To this end, we first prove that

Ind<sub>L</sub> det 
$$X(t) = -\kappa$$
. Exhaust be almost bounded as  $K(t) = -\kappa$ . (5.1)

It is well known that (cf. [7], p. 41) > 7 SA > 0 some union variable as a sales

that 
$$X(z)\Phi_0(z)$$
 is integrably unbounded occar of can only be also since  $-1 < \text{Re } \tau$ ,  $\pi \in G(\tau)$ 

but between  $\Gamma$  and L the canonical function  $\det X(z) = \det X^+(z)$  is holomorphic and has no zero point, so that

the condition to 
$$(\tau)^+X$$
 tell  $(\tau)^+X$  and  $(\tau)^+X$  tell  $(\tau)^+X$  tell

(the presence of the minus sign on the right-hand side is due to that the positive sense of L is its counter-clockwise sense, whereas that of  $\Gamma$  is its clockwise sense). Thus (5.1) is true. From this, we readily obtain that the total index of the reduced H-problem is

$$\operatorname{Ind}_L \det \left[\lambda(t) \overline{X(t)}\right] = \operatorname{Ind}_L \det \lambda(t) + \operatorname{Ind}_L \overline{\det X(t)} = k + k = K.$$

If we can calculate X(z) effectively, then the coefficients of this H-problem are known. Therefore the original RH-problem can be reduced to a given definite H-problem, and we may solve it forwards (cf. [7], Chap. III, Part 1). For instance, this is the case when  $G(\tau)$  may be written as the product of two factors, which are the boundary values of two meromorphic matrices in the interior and exterior of  $\Gamma$  respectively (cf. [6], § 5).

For the cases when  $\Gamma$  consists of open arcs or when the coefficients have discontinuities, using the results of N. P. Vekua (cf. [7], Chap. II) and the method described in § IV, we may solve it analogously.

At last, we note that, as we have pointed out in [1], the method of elimination proposed here may be applied generally to solve an R-problem compounded with a boundary problem of some other type, and may also be employed to retreat in a simple manner the R-problems for open arcs, with discontinuous coefficients, or for complex boundaries.

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系加能效,仍成立、又流工。是整截、立一、四、一、一、一、一、光光、显微时、本等所设件等高能正常,也完全成立。

2、是一个时間曲线;实际上,以 12、是一副者前个对图曲器时,以下的前题具要