高等学校教材

数学专业英语

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数学专业英语

English for mathematics

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Foreword

With fast economic globalization and advanced requirement for creative students, in order to satisfy the needs of senior undergraduate students and graduate students of mathematics major and other related specialties to improve their ability in reading technical English, we compile this textbook. Principles of material selection are:

1 Normativeness and pureness of language. All articles in this book have been carefully recommended by experts and experienced teachers both in technical field and language teaching field.

2 Extensiveness and advancedness of professional knowledge. Texts of this book mainly consist of articles and materials focusing on concept exposition and principle explanation. Contents cover all major technical areas and new development trends in mathematics and information sciences. Some theorems, new methods for proof are also included in the contents.

3 Proceeding in an orderly and step by step way is employed in sequence arrangement of the material. Basic matirials are arranged in the first units to make preparation for further study.

4 Systematicness and specialization. This book tries to give a systematic exposition of mathematics either in its theoretical level or in its application level. All materials focus on the above mentioned two levels. It maintains the topics in set theory, logarithm function, vector algebra, linear transformation, application to analytic geometry, eigenvalue and eigenvector, continuity of function, differentiation, differential equation, complex analysis, probability and statistics, measurable function, topological space, inner product space, introduction to numerical analysis, and information theory.

Through reading this book, students can familiarize themselves with large volume of vocabulary and terminology frequently used in mathematics and other related fields, significantly increase their anility in reading and understanding original scientific English literature and thus lay a solid foundation of reading scientific English books and journals in the future. Besides, by reading this book, studends can also deepen their understanding of some basic concepts in mathematics and other related fields and learn some new technology as well.

This book is compiled by Xie Fufen (Unit1 to Unit6) ,Qi Chunze (Unit 7 to Unit11) , Li Zhancun (Unit 12 to Unit 17) and planed as whole.

For lack of experience, there must be some inappropiateness in covering scope of contents and complexity of language. Therefore, we sincerely welcome the criticism and suggestions from users of this book, so that improvement and enrichment can be made in the future.

Brief Contents

Unit 1 SOME BASIC CONCEPTS OF THE THEORY OF SETS

The term set is frequently seen in mathematics. To understand what a set is and its related terminologies is basic to students who study mathematics and other mathematics-related subjects This section is going to introduce set theory, definition of set, relationships between two sets, and operations of sets.

Section A

Introduction to set theory and notation for designating sets

Introduction to set theory In discussing any branch of mathematics, be it analysis, algebra or geometry, it is helpful to use the notation and terminology of set theory. This subject, which was developed by Boole and Cantor in the latter part of the 19th century, has had profound influence on the development of mathematics in the 20th century. It has unified many seemingly disconnected ideas and has helped to reduce many mathematical concepts to their logical foundations in elegant and systematic way. A thorough treatment of the theory of sets would require a lengthy discussion which we regard as outside the scope of this book. Fortunately, the basic notations are few in number, and it is possible to develop a working knowledge of the methods and ideas of set theory through an informal discussion. Actually, we shall discuss not so much a new theory as an agreement about the precise terminology that we wish to apply to more or less familiar ideas.

In mathematics, the word "set" is used to represent a collection of objects viewed as a single entity. The collections called to mind by such nouns as "flock," "tribe," "crowd," "team," and "electorate" are all examples of sets. The individual objects in the collection are called elements or members of the sets, and they are said to belong to or to be contained in the set. The set, in turn, is said to contain or be composed of its elements.

We shall be interested primarily in sets of mathematics objects: sets of numbers, sets of the curves, sets of geometric figures, and so on. In many applications it is convenient to deal with sets in which nothing special is assumed about the nature of individual objects in the collection. These are called abstract sets. Abstract set theory has been developed to deal with such collections of arbitrary objects, and from this generality the theory derives its power.

Notation for designating sets Sets usually are denoted by capital letters: A, B, C, $\cdots$, X, Y, Z; elements are designated lower-case letters: a, b, c, $\cdots$, x, y, z. We use the special notation $x \in S$ to mean that "x is an element of S" or "x belongs to S." If x does not belong to S, we write $x \notin S$. When convenient, we shall designate sets by displaying the elements in braces; for example, the set of all positive even integers is displayed as $\{2, 4, 6, \cdots\}$, the three dots taking the place of "and so on." The dots are used only when the meaning of "and so on" is clear. The method of listing the numbers of a set within braces is sometimes referred to as the roster notation.

Vocabulary and Phrases

notation	*n*.	符号	terminology	*n*.	术语
collection	*n*.	类,集合	entity	*n*.	实体,实物
belong to	*v*.	属于	be contained in	*v*.	含于
curve	*n*.	曲线	geometric figure	*n*.	几何图形
be denoted by		以…标注	be designated		以…指定
roster notation	*n*.	枚举表示法	analysis	*n*.	分析
algebra	*n*.	代数	geometry	*n*.	几何

Notes

1. It has unified many seemingly disconnected ideas and has helped to reduce many mathematical concepts to their logical foundations in elegant and systematic way 它整合了许多看上去毫无联系的想法,并以优雅且系统的方式把许多数学概念简化到它们的逻辑基础之上.

2. In mathematics, the word "set" is used to represent a collection of objects viewed as a single entity. 在数学中,集合用以表示被视为单一实体的一些事物

的类.

Pronunciation of common mathematical notations

a^2	*a* square / *a* squared
b^3	*b* cube / *b* cubed
c^4	*c* raised to the fourth (power)
d^{-1}	*d* to the minus one
10^5	the fifth power of ten

Exercises

Read the following paragraph

We adopt, as most mathematicians do, the naïve point of view regarding set theory. We shall assume that what is meant by a set of objects is intuitively clear, and we shall proceed on that basis without analyzing the concept further. Such an analysis properly belongs to the foundations of mathematics and to mathematical logic, and it is not our purpose to initiate the study of those fields.

Logicians have analyzed set theory in great detail, and they have formulated axioms for the subject. Each of their axioms expresses a property of sets that mathematicians commonly accept, and collectively the axioms provide a foundation broad enough and strong enough that the rest of mathematics can be built on them.

Section B

Relations and operations of set theory

The first basic concept that relates one set to another is equality of sets:

Definition of set equality Two sets A and B are said to be equal (or identical) if they consist of exactly the same elements, in which case we write $A = B$. If one of the sets contains an element not in the other, we say the sets are unequal and we write $A \neq B$.

Example 1 According to this definition, the two sets $\{2, 4, 6, 8\}$ and $\{2, 8, 6, 4\}$ are equal since they both consist of the four elements 2, 4, 6 and 8. Thus when we use the roster notation to describe a set, the order in which the elements appear is irrelevant.

Example 2 The sets {2, 4, 6, 8} and {2, 2, 4, 4, 6, 8, } are equal even though, in the second set each elements 2 and 4 is listed twice. Both sets contain the four elements 2, 4, 6, 8 and no others; therefore, the definition requires that we call these sets equal. This example shows that we do not insist that objects listed in the roster notation be distinct. A similar example is the set of letters in the word Mississippi, which is equal to the set {M, i, s, p}, consisting of the four distinct letters M, i, s, and p.

Subsets From a given set S we may form new sets, called subsets of S. For example, the set consisting of those positive integers less than 10 which are divisible by 4 (the set {4, 8}) is a subset of the set of all even integers less than 10. In general, we have the following definition.

Definition of subset *A* set *A* is said to be a subset of a set B, and we write

$$A \subseteq B,$$

whenever every element of *A* also belongs to *B*. We also say that *A* is contained in B or that *B* contains *A*. The relation $\subseteq$ is referred to as set inclusion.

The statement $A \subseteq B$ does not rule out the possibility that $B \subseteq A$. In fact, we may have both $A \subseteq B$ and $B \subseteq A$. But this happens only if A and B have the same elements. In other words,

$$A = B \text{ if and only if } A \subseteq B \text{ and } B \subseteq A.$$

This theorem is an immediate consequence of the foregoing definitions of equality and inclusion. If $A \subseteq B$ but $A \neq B$, then we say that A is a proper subset of *B*; we indicate this by writing $A \subseteq B$.

In all our applications of set theory, we have a fixed set *S* given in advance, and we are concerned only with subsets of this given set. The underlying set S may vary from one application to another; it will be referred to as the universal set of each particular discourse. The notation

$$\{x \mid x \in S \text{ and } x \text{ satisfies } P\}$$

will designate the set of all elements x in which satisfy the property *P*. When the universal set to which are referring is understood, we omit the reference to *S* and write simply $\{x \mid x \text{ satisfies } P\}$. This is read "the set of all x such that x satisfies P." Sets designated in this way are said to be described by a defining property. For example, the set of all positive real numbers could be designated as $\{x \mid x > 0\}$; the universal set S in this case understood to be the set of all real numbers. Similarly, the set of all even positive integers $\{2, 4, 6, \cdots\}$ can be designated as $\{x \mid x$ is a positive even integer$\}$. Of course, the letter x is a dummy and may be replaced by any other convenient symbol. Thus, we may write

$$\{x \mid x > 0\} = \{y \mid y > 0\} = \{t \mid t > 0\}$$

and so on.

It is possible for a set to contain no elements whatever. This set is called the empty set or the void set, and will be denoted by the symbol ø. We will consider ø to be a subset of every set. Some people find it helpful to think of a set as analogous to a container (such as a bag or a box) containing certain objects, its elements. The empty set is then analogous to an empty container.

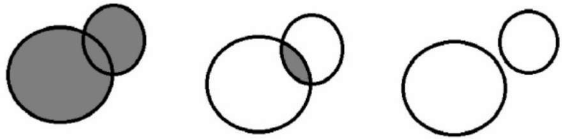
To avoid logical difficulties, we must distinguish between the element x and the set {x} whose only element is x. (A box with a hat in it is conceptually distinct from the hat itself.) In particular, the empty set Ø is not the same as the set {Ø} In fact, the empty set Ø contains no elements, whereas the set {Ø} has one element Ø. (A box which contains an empty box is not empty.) Sets consisting of exactly one element are sometimes called one – element sets.

Diagrams often help us visualize relations between sets. For example, we may think of a set S as a region in the plane and each of its elements as a point. Subsets of S may then be thought of as collections of points within S. For example, in Figure 1.6(b) the shaded portion is a subset of A and also a subset of B. Visual aids of this type, called Venn diagrams, are useful for testing the validity of theorems in set theory or for suggesting methods to prove them. Of course, the proofs themselves must rely only on the definitions of the concepts and not on the diagrams.

Unions, intersections, complements From two given sets A and B, we can form a new set called the union of A and B. This new set is denoted by the symbol

$$A \cup B \text{ (read "}A\text{ union }B\text{")}$$

and is defined as the set of those elements which are in A, in B. or in both. That is



(a) $A \cup B$ (b) $A \cap B$ (c) $A \cap B = \emptyset$

Figure 1.6 Unions and intersections

to say, $A \cup B$ is the set of all elements which belong to at least one of the sets A, B. An example is illustrated in Figure 1.6 (a), where the shaded porting represents $A \cup B$.

Similarly, the intersection of A and B, denoted by

$A \cap B$ (read "A intersection B"),

is defined as the set of those elements common to both A and B. This is illustrated by the shaded portion of Figure 1.6 (b). In Figure 1.6 (c), the two sets A and B have no elements in common; in this case, their intersection is the empty set $\emptyset$. Two sets A and B are said to be disjoint if $A \cap B = \emptyset$.

If A and B are sets, the difference $A - B$ (also called the complement of B relative to A) is defined to be the set of all elements of A which are not in B. Thus, by definition,

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}.$$

If Figure 1.6 (b) the unshaded portion of A represents $A - B$; the unshaded portion of B represents $B - A$.

The operations of union and intersection have many formal similarities to (as well as differences from) ordinary addition and multiplication of real numbers. For example, since there is no question of order involved in the definitions of union and intersection, it follows that $A \cup B = B \cup A$ and that $A \cap B = B \cap A$. That is to say, union and intersection are commutative operations. The definitions are also phrased in such away that the operations are associative:

$$(A \cup B) C = A \cup (B \cup C) \text{ and } (A \cap B) \cap C = A \cap (B \cap C).$$

These and other theorems related to the "algebra of sets" are listed as Exercises. One of the best ways for the reader to become familiar with the terminology and notations introduced above is to carry out the proofs of each of these laws. A sample of the type of argument that is needed appears immediately after the Exercises.

The operations of union and intersection can be extended to finite or infinite collections of sets as follows: Let F be a nonempty class of sets. The union of all the sets in F is defined as the set of those elements which belong to at least one of the sets in F and is denoted by the symbol

$$\bigcup_{A \in F} A.$$

If F is a finite collection of sets, says F = { $A_1, A_2, \cdots, A_n$ } we write

$$\bigcup_{A \in F} A = \bigcup_{k=1}^{n} A_k = A_1 \cup A_2 \cup \cdots \cup A_n d.$$

Similarly, the intersection of all the sets in F is defined to be the set of those elements which belong to every one of the sets in F; it is denoted by the symbol

$$\bigcap_{A \in F} A.$$

For finite collections (as above) , we write

$$\bigcap_{A \in F} A = \bigcap_{k=1}^{n} A_k = A_1 \cap A_2 \cap \cdots \cup A_n.$$

Unions and intersections have been defined in such a way that the associative laws for these operations are automatically satisfied. Hence, there is no ambiguity when we write

$$A_1 \cup A_2 \cup \cdots \cup A_n \ or \ A_1 \cap A_2 \cap \cdots \cap A_n$$

Furthermore, if F is a infinite collection of sets, says F = { A_1, A_2, A_3, ⋯, A_n ⋯} , we write

unions $$\bigcup_{A \in F} A = \bigcup_{k=1}^{\infty} A_k = A_1 \cup A_2 \cup \cdots \cup A_n,$$

and intersections $$\bigcap_{A \in F} A = \bigcup_{k=1}^{\infty} A_k = A_1 \cup A_2 \cup \cdots \cup A_n,$$

Vocabulary and Phrases

subset	n.	子集	universal set	n.	全集
empty set	n.	空集	void set	n.	空集
even	a.	偶的	odd	a.	奇的
region	n.	区域	Venn diagram	n.	文氏图
dummy	a.	哑的	operation	n.	运算
commutative	a.	交换的	associative	a.	结合的
union	n.	并	intersection	n.	交
difference	n.	差	complement	n.	补

Notes

1. The underlying set S may vary form one application to another; it will be referred to as the universal set of each particular discourse. 所列出的集合 S，它随应用的不同而变化，在每一具体的过程中被规定为全集.

2. For example, the set of all positive real numbers could be designated as { x | x }; the universal set S in this case understood to be the set of all real numbers 例如，正整数的集合可以用{ x | x }；表示，这种情形下的全集 S 应该理解为全体实数.

Pronunciation of common mathematical notations

$x \in A$	*x* is in A / *x* is a member of A / *x* is an element of A / *x* belongs to A
$x \notin A$	*x* is not in A / *x* is not a member of A /*x* dose not belong to *A*
$A \subseteq B$, $A \subset B$	*A* is a subset of *B* / *A* is contained in *B* / *B* contains *A*. / *A* is included in *B*
$A \cup B$	*A* union *B*
$A \cap B$	*A* intersection *B*
$A - B$	the difference *A* and *B* / *A* minus *B*
A^C	the complement of *A*

Exercises

1. Translate the following passage into Chinese.

(1) We have already defined what we mean by the union and the intersection of two sets. There is no reason to limit ourselves to just two sets, for we can just as well form the union and intersection of arbitrarily many sets.

Given a collection Q of sets, the union of the elements of Q is defined by the equation

$$\bigcup_{A \in Q} A = \{ x \mid x \in A \text{ for at least on } A \in Q \}$$

The intersection of the elements of Q is defined by the equation

$$\bigcap_{A \in Q} A = \{ x \mid x \in A \text{ every } A \in Q \}$$

(2) There is yet another way of forming new sets form old ones; it involves the notion of an " ordered pair " of objects. When you studied analytic geometry, the

first thing you did was convince yourself that after one has chosen an x－axis and a y－axis in the plane, every point in the plane can be made to correspond to a unique ordered pair (x, y) of real numbers. (In more sophisticated treatment of geometry, the plane is more likely to be defined as the set of all ordered pairs of real numbers!) The notion of ordered pair carries over to general sets. Given sets A and B, we define their Cartesian product $A \times B$ to be the set of all ordered pairs (a, b) for which a is an element of A and b is an element of B, Formally, $A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$.

2. Translate the following sentences into English.

(1) 由小于 15 且能被 3 整除的正整数组成的集合.

(2) 我们有时通过在括号中列举元素的办法来表示集合.

(3) 符号⊆表示集合间的包含关系.

(4) 基础集可根据使用场合不同而改变.

(5) 两个集合 A 与 B 相等当且仅当 A⊆B 且 B⊆A.

(6) 元素 x 与仅含一个元素的集合 {x} 是有区别的.

3. Proof of A∩B⊂A. in English

Example　Proof of commutative law $A \cup B = B \cup A$. Let $X = A \cup B$, $Y = B \cup A$. To prove that $X = Y$ we prove that $X \subset Y$ and $Y \subset X$. Suppose that $x \in X$. Then x is in at least one of A or B. Hence, x is in at least one of B or A; so $x \in Y$ Thus, every element of X is also in Y. Similarly, we find that $Y \subset X$, so $X = Y$.

Section C

Axiomatic method

In this section we are going to state the axioms of set theory, and we are going to show that our theorems are consequences of those axioms. The great advantage of the axiomatic method is that it takes totally explicit just what our initial assumptions are.

It is sometimes said that "mathematics can be embedded in set theory." This means that mathematical objects (such as numbers and differentiable functions) can be defined to be certain sets. And the theorems of mathematics (such as the fundamental theorem of calculus) then can be viewed as statements about sets. Further-