

研究生前沿教材书系

Modern Foundations of Quantum Optics

现代量子光学基础

(英文影印版)

Vlatko Vedral

(University of Leeds, UK)




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Vlatko Vedral

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Vlatko Vedral

英国 Leeds 大学终身教授,量子信息科学研究室主任。

1971 年出生,英籍南斯拉夫裔科学家。1995 年取得 Imperial College of Science, Technology and Medicine 的理论物理学士学位,1998 年在导师 Peter L. Knight 爵士的指导下,取得该校物理学博士学位,论文题目是“纠缠态的量子信息论”。同年进牛津大学从事博士后研究。2001 年至 2003 年为牛津大学 Keble 学院的高级讲师、帝国理工学院高级讲师兼量子物理学科审稿人。主讲量子力学、光学、数学、热力学、统计力学、电磁学等多门课程,同时给博士后研究班开设《量子计算和量子光学》讲座。这期间还到奥地利的维也纳大学、加拿大的 Waterloo 大学、新加坡大学、维也纳的 Schrödinger 研究所、美国的普林斯顿大学做访问教授。他是 *Nature*, *Phys. Rev.*, *Proc. Roy. Soc.*, *J. Mod. Opt.* 等杂志的审稿人,也是英国皇家协会、奥地利科学基金会以及其他国际基金会的评审人,还是博士论文和研究人员职称资格的评审人。在短短的 6 年时间里,培养博士 10 名,发表量子光学、量子信息和量子物理的学术论文 90 多篇,单独出版教材 2 本,合作出版著作 3 本。到世界各地讲学 100 次,并经常接受媒体访谈,已成为英国学术界一颗耀眼的新星。

内 容 简 介

这本教材是作者根据自己从 2001 年到 2004 年连续三年给 Leeds 大学物理系高年级学生讲授量子光学的讲稿整理而成的。这也是迄今为止对量子光学阐述得最全面、最新颖的教科书。通过干涉现象阐述光的本性是全书的核心。全书由 11 章内容和 5 套练习及解答构成,分别就经典、旧量子论、半经典和完全量子力学四个层面阐述光与物质的相互作用、激光的原理和应用、场的量子化以及量子光学的最新进展等等。本书就激光冷却、玻色凝结、量子信息和传输等最新进展作了简单易懂的介绍,难怪剑桥大学的 Artur Ekert 教授作出了极高的评价:“我很难想到作者会用如此高超的办法,把量子光学的那些最新的有趣的研究领域融入到量子光学的概念、方法和应用之中。很显然,这是一本既透彻、又新鲜的教科书,这也是一本写得特别清晰、充满激情和细致认真的书。作者把自己深厚的学术功底、透彻的论证推理和通俗易懂的表述风格和创造性的演绎融合在一起。我相信《现代量子光学基础》一书,一定会引起包括初学者和专家在内的广大读者的关注。”

序 言

这本书是根据我于 2001 年到 2004 年三年间连续给伦敦帝国理工学院讲课的讲稿写成的。我已根据正式出版物的要求对讲稿作了必要的梳理,与此同时,我又根据三、四年级物理本科生选修课的要求,尽可能保持原讲稿的精神和风格不变。教程是由 26 次讲演及 3 次专题报告组成的。3 次专题讲演包括了量子光学最近期的进展和实际应用。我把重点放在腔体中量子电动力学(QED) Rabi 振荡实验、原子的玻色-爱因斯坦凝结的成就以及量子传输等专题上。这些新成就,有些已经颁发了最近几年的诺贝尔奖,这充分说明量子光学是一门极为重要而活跃的值得学习的学科。

读者将了解到,除了现代应用之外,我将尽量采用原汁原味的方式来讲述很多课题,并且时刻牢记现代的进展和理论解释。当然,我的讲稿也包括许多规范的理论推导,这些推导在很多其他的教科书中也可以找到,而且这些书中有些推导还相当仔细。我则有意既不写得过于细节,也不拘泥于学科的完整表述。专题的选取,多少反映了我个人的偏好以及我的研究兴趣和专长。例如,我选择了麦克斯韦妖这一课题,讨论光的波粒二象性如何违背热力学第二定律。我还讨论量子力学中的相位概念、动力学相与几何相的差别以及度规原理背后的某些极为基本的概念,以及如何从薛定谔方程导出电磁学等等。这些附加的专题,在通常的教科书中是不包括在内的,这说明量子光学并不是一门孤立的学科,同物理学的其他相关领域的联系是非常密切的。参插着这些专题,也有利于避免阐述过程中的单调乏味之感。我要向学生表明,即使对于导介性的教学而言,也能对学科做到生动而兴趣盎然的讲述,只有这样,方能使学生从一开始便能积极参与其中。

讲稿的写作次序大体上是循着历史发展的脉络,有时符合教学

法,但常常又不循规蹈矩。更多采用的写作次序不是遵循从简单到复杂的常规之道,因此常常表现出与历史发展不相一致。本书的逻辑展开遵循着我们理解的光与物质相互作用的不同等级——量子光学是我们所理解的最复杂的课题。粗略地讲,讲课笔记中分为四个层次:经典的、旧量子论、半经典的和完全量子力学的。我的意图是采用技术上和概念上带有挑战性的实例来阐述多少带有传统性的专题。例如,我在一开始就以单光子来介绍 Mach-Zehnder 干涉仪并以此不仅说明光子的行为既像粒子,同时又像波,而且也指出能够利用这种行为说明经典物理学难以想像的行动规则——比如说无相互作用的测量。本书中包括了 5 套习题和解答。这些题目来源于用来检查学生对所阐述内容的理解的 3 份考卷,通过解题对深刻理解一门学科常常是十分关键的。

本书终止于场论方法开始适用之处。这也许多多少少会使人产生一些误解,量子光学属于完全量子场论的最低一级近似。根据我的教学经验,学习量子光学首先是理解量子场论,而不是理解通常的二次量子化形式的最好方法。

最后,我感到在伦敦帝国理工学院同学生打交道十分有趣,他们不但教给我学科知识,还使我懂得如何教学。我希望你会像我乐于教学那样,乐于阅读我的这本讲课笔记。

V. Vedral

出版者的话

复旦大学出版社出版英文影印版《研究生前沿教材书系》，主要基于以下几点考虑。

1. (新加坡)世界科技出版公司以出版科技专著闻名于世,同我社已有 10 多年的友好交往。从 20 世纪 90 年代以来,尤其是 1995 年该公司并购了伦敦帝国学院出版社(Imperial College Press) 51% 的股份(近年已完成了 100% 的股份收购)之后,这两大出版机构在潘国驹教授的集中指挥下,充分发挥了编辑学术委员会的职能,使得出书范围不断拓宽,图书层次逐渐丰富,因此从中遴选影印图书的空间就更大了。再加上该公司在上海设有办事机构,相关工作人员工作细致,服务周到,给我们两个单位的合作交流带来极大的便利。

2. 研究生教育是创新人才培养的关键,教材建设直接关系到研究生科学水平和创新能力的培养。从 2003 年开始,我社陆续出版了 *Fudan Series in Graduate Textbooks* 这套丛书,国内的读者反响很好。但限于作者人力,这套丛书涵盖的学科和门类都较为不足。为此,我们想到再借助国外出版力量,引进一批图书作为硕士研究生的补充教材,(新加坡)世界科技出版公司与我社的合作,恰好提供了这样一个良好的机会。我们从该公司提供的大量近期书目中,遴选出 30 多本样书,经过专家审读后,最终确定了其中的 11 种作为首批《研究生前沿教材书系》影印出版。这 11 种图书的作者来自美、英、法、德、加拿大 5 个国家的 10 多所高校或研究部门,他们既是相关学科科研的领军人物,又是高年级本科生和研究生教学的杰出教授(详见各书的作者介绍)。各门教材既考虑到深入浅出的认知规律,又突出了前沿学科的具体应用,每本书都有充实的文献资料,有利于读者和研究人員深入探索。其中 6 本教材配有习题,

还包括一本具有物理背景的人员都需要了解的高级科普读物——《理解宇宙——从夸克到宇宙学》。

3. 为了有利于广大读者和图书管理、图书采购、图书销售人员的使用,特请龚少明编审为每本影印书编写出中文内容介绍和作者概况,并由他将 preface(序言)译成中文。序言是一本书的总纲,它涉及写作要旨、逻辑体系、内容特色和研读指导等等,我们将其译成中文至少有利于读者的浏览和选购,避免买书仓促带来的失误,毕竟英语是多数读者的第二种语言。

4. 原版书价格较贵,大大超出读者的购买能力,即使图书馆或大学资料室也会受到经费不足的制约。出版影印本的书价不到原价的十分之一,无疑会给需要这些书的研究生和图书馆带来真正的实惠,这也是(新加坡)世界科技出版公司与我们合作的目的之一。

5. 考虑到物理类图书是(新加坡)世界科技出版公司的第一品牌,我们首次引进的11本书,都属大物理的范畴。这一尝试如果得到读者和专家的认可,今后我们将陆续开辟其他学科的影印渠道。

欢迎读者批评指正,并提出有益的建议。

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I acknowledge Peter Knight, who proposed the first course on Quantum Optics at Imperial College London and whose syllabus I have modified only a bit here and there when I taught it myself.

Very special thanks goes to Caroline Rogers for preparing the manuscript for the final submission to Imperial College Press. She has redrawn many of the figures, as well as corrected and clarified some parts of the book. Her hard work was essential for the final preparation, which otherwise may have taken a much longer time to complete.

My deepest gratitude goes to my family, Ivona and Michael, who provide a constant source of inspiration and joy.

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Chapter 1

From Geometry to the Quantum

According to one legend, Lucifer was God's favorite angel before stealing light from him and bringing it to mankind. For this, to us a generous act, Lucifer was expelled from heaven and subsequently became the top angel in hell. Most of us are not able to steal possessions from God, but we can at least admire his most marvellous creation — light. Quantum optics is the theory describing our most sophisticated understanding of light.

This book intends to acquaint you with the basic ideas of how physics describes the interaction of light and matter at three different levels: classical, semi-classical and quantum. You will be able to understand basic principles of laser operation leading to the ideas behind non-linear optics and multiphoton physics. You will also become familiar with the ideas of field quantization (not only the electromagnetic field, but also a more general one), nature of photons, and quantum fluctuations in light fields. These ideas will bring you to the forefront of current research. At the end of this book, I not only expect you to understand the basic methods in quantum optics, but also to be able to apply them in new situations — this is the key to true understanding. The notes contain five sets of problems, which are intended for your self-study. Being able to solve problems is definitely crucial for your understanding, and a great number of problems have been chosen from the past exam papers at Imperial College London set by me. I also hope — and this is I believe really very important — that the book will teach you to appreciate the way that science has developed within the last 100 years or so and the importance of the basic ideas in optics in relation to other ideas and concepts in science in general. The book contains a number of topics from thermodynamics, statistical mechanics and information theory that will illustrate that quantum optics is an integral part of a much larger body of scientific knowledge. I hope that at the end of it all, and this is really

my main motivation, you will appreciate how quantum description of light forms an important part of our cultural heritage.

Optics itself is an ancient subject. Like any other branch of science, its roots can be found in Ancient Greece, and its development has always been inextricably linked to technological progress. The ancient Greeks had some rudimentary knowledge of geometrical optics, and knew of the laws of reflection and refraction, although they didn't have the appropriate mathematical formalism (trigonometry) to express these laws concisely. Optics was seen as a very useful subject by the Greeks: Archimedes was, for example, hired by the military men of the state to use mirrors and lenses to defend Syracuse (Sicily) by directing the Sun's rays at enemy ships in order to burn their sails. And like most of human activity (apart from some forms of art and mathematics) the Greek knowledge was frozen throughout the Middle Ages only to awaken more than 10 centuries later in the Renaissance. At the beginning of the 15th century, Leonardo da Vinci designed a great number of machines using light and was apparently the first person to record the phenomenon of *interference* — now so fundamental to our understanding not only of light, but matter too (as we will see later in this book). However, the first proper treatment of optics had to wait for the genius of Fermat and Newton (and, slightly later, Huygens) who studied the subject, making full use of mathematical rigor. It was then, in the 16th and 17th centuries, that optics became a mature science and an integral part of physics.

If you could shake a little magnet 428 trillion times per second, it would start making red light. This is not because the magnet would be getting hotter — the magnet could be cold and situated in the vacuum (so that there is no friction). This is because the electromagnetic field would be oscillating back and forth around the magnet which produces red light. If you could wiggle the magnet a bit faster, say 550 trillion times per second, it would glow green, while at around 800 trillion times per second it would produce light that is no longer visible — faster still and it would become ultraviolet. In the same respect, we can think of atoms and molecules as little magnets producing light — and their behavior as they do so is the subject of quantum optics.

From our modern perspective, optics can be divided into three distinct areas which are in order of increasing complexity and accuracy (they also follow the historical development):

- *Geometrical optics* is the kind of optics you would have done in your sixth form and the first year of university,

prior to learning that light is an electromagnetic wave. Despite the fact that this is the lowest approximation of treating light, we can still derive some pretty fancy results with it — how lenses work, for instance, or why we see rainbows. I will assume that you are fully familiar with geometrical optics.

- *Physical optics* is based on the fact that light is an electromagnetic wave and, loosely speaking, contains geometrical optics as an approximation when the wavelength of light can be neglected ($\lambda \rightarrow 0$). Behavior of light as described by physical optics can be entirely deduced from Maxwell's equations, and it is this level of sophistication that we will investigate at the very beginning of the book.
- *Quantum optics* takes into account the fact that light is quantized in chunks of energy (called photons), and this theory is the most accurate way of treating light known to us today. It contains physical optics (and hence geometrical optics) as an approximation when the Planck constant can be neglected ($\hbar \rightarrow 0$). This treatment will be the core of the book.

Geometrical optics can be summarized in a small number of fundamental principles. For those of you interested in the colorful history of optics, I mention Huygens' *Treatise on Optics* as a good place to read about the early understanding of light. Here are the three basic principles that completely characterize all the phenomena in geometrical optics:

- (1) In a homogeneous and uniform medium, light travels in a straight line.
- (2) The angle of incidence is the same as that of reflection.
- (3) The law of refraction is governed by the law of sines — to be detailed below (see Figure 1.1).

Are these laws independent of each other or can they be derived from a more fundamental principle? It turns out that they can be summarized in a very beautiful statement due to Fermat.

Fermat's principle of least time. Light travels such that the time of travel is extremized (i.e. minimized or maximized).

All the above three laws can be derived from Fermat's principle. We will now briefly demonstrate this. The fact that in a homogeneous and uniform medium light travels in a straight line is simple, as the speed of light is the same everywhere in such a medium (by

Geometrical Optics Principles

Fermat's Principle

definition of the medium), and therefore a straight line, being the shortest path between two points, also leads to the shortest time of travel. The same reasoning applies for the incidence and reflection angles. The law of sines is a bit more complicated to derive, but I will now show you how to do so in a few lines. Suppose that light is going from a medium of refractive index 1 to a medium of refractive index n as shown in Figure 1.1.

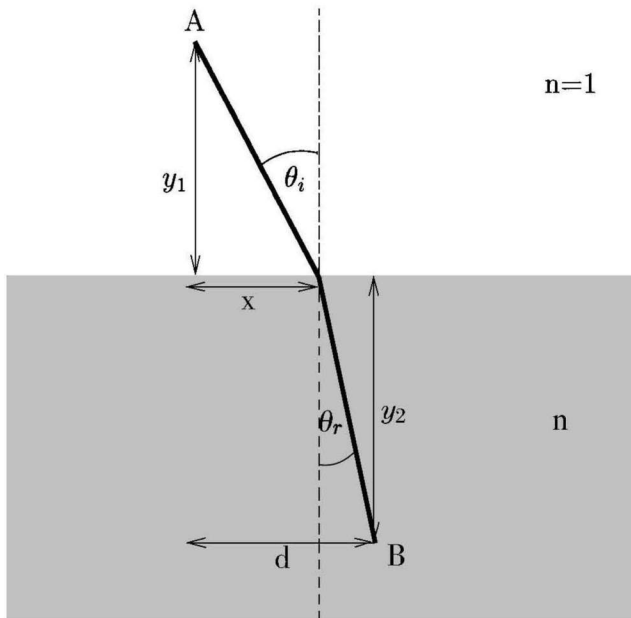


Fig. 1.1 The law of sines can be derived from Fermat's principle of least time. The full derivation is in the notes.

The total time taken from the point A to the point B is

$$t \propto \sqrt{x^2 + y_1^2} + n\sqrt{y_2^2 + (d-x)^2} \quad (1.1)$$

Note that the second term is multiplied by n , as the speed of light is smaller in the medium of refractive index n , being equal to c/n where c is the speed of light in vacuum. Now, Fermat's principle requires that the time taken is extremized, leading to

$$\frac{dt}{dx} \propto \frac{x}{\sqrt{x^2 + y_1^2}} - \frac{(d-x)}{n\sqrt{y_2^2 + (d-x)^2}} = 0 \quad (1.2)$$