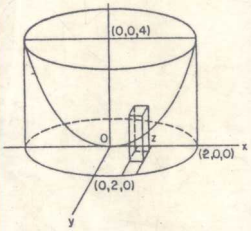
GRADUATE RECORD EXAMINATION

留美研究生數學考試

MATHEMATICS

Subject (Advanced) Test.



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托福出版社叢書之一〇二

目　錄

編者的話

▲爲什麼要考GRE、Sub.、MATH？

簡答之，就是要增加申請者的競爭力；因爲數學潛力高的研究生，適合在學術領域上求發展。

根據柏克萊加大的電機電腦系（EECS）的申請書，該系要求GRE Sub. Computer Science的成績在80％以上；但是可以用Engineering，Mathematics或physics三者之一代替。而對大多數中國考生言，編者相信，在Mathematics上得到80％以上的成績，是最有希望而最可行的。所以不僅是數學本科系的研究生，對有志攻讀電腦的同學，數學學科測驗也是一條捷徑。

▲是否只有數學系的畢業生，才能考好GRE、Sub.、MATH？

一般而言，數學學科測驗的範圍遍及數學系的大部份課程，但是考試的內容廣度要超過深度，我們試看以下的SAMPLE TEST QUESTION：

一、微積分及解析幾何（CALCULUS AND ANALYTICAL GEOMETRY）

1.

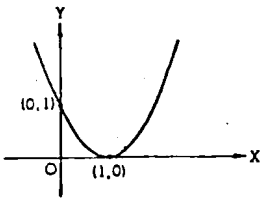

Figure 1

Which of the following could be an equation of the curve shown in Figure 1?

(A) $y = x^3 + 1$ (B) $y = (x + 1)^2$ (C) $y = |x - 1|$
(D) $y = (x - 1)^2$ (E) $y = |x| + x + 1$

答案：(D)。

2

If $f(x) = x|x|$ for all real numbers x, then $f'(x)$ is a real number for

(A) no real number x
(B) $x = 0$ only
(C) $x > 0$ only
(D) $x \neq 0$ only
(E) all real numbers x

答案：(E)。

3

If f is a function having derivatives of all orders and if $f'(x) = [f(x)]^2$, then the nth derivative of f at x is given by

(A) $n[f(x)]^n$ (B) $n![f(x)]^{n+1}$ (C) $(n+1)![f(x)]^{n+1}$
(D) $(n+1)[f(x)]^n$ (E) $n[f(x)]^{2n}$

答案：(B)。

4.

Let f be a continuous function such that for all $c > 0$,

(i) $f(c) > 0$ and
(ii) the region bounded by the line $x = c$, by the coordinate axes, and by the curve with equation $y = f(x)$ has area ce^c.

For all $x > 0$, $f(x) =$

(A) e^x (B) xe^x (C) $xe^x - e^x$ (D) $xe^x + e^x$ (E) $x^2e^x - xe^x$

答案：(D)。

5.

The interval of convergence of the series

$\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^2 2^{n+2}}$ is indicated by

(A) $-1 < x < 1$ (B) $-1 \leq x \leq 1$ (C) $-1 < x < 3$
(D) $-1 \leq x \leq 3$ (E) $0 \leq x \leq 2$

答案：(D)。

6.

Let f be a continuous function of x (f not identically zero) and let s and t be nonzero numbers. If $I = t\int_0^{\frac{1}{t}} f(s + tx)\,dx$, then the value of I

(A) varies with x

(B) depends on the ratio $\frac{s}{t}$

(C) depends on s, but is independent of t and x
(D) depends on t, but is independent of s and x
(E) is constant for all s, t, and x

答案：(C)。

7.

If f is infinitely differentiable everywhere, then

$$\lim_{k\to 0}\left[\lim_{h\to 0}\frac{f(p+k+h)-f(p+k)-f(p+h)+f(p)}{hk}\right] =$$

(A) $f'(p)$ (B) $f''(p)$ (C) $(f'(p))^2$
(D) $f'(f'(p))$ (E) $f'(h)f'(k)$

答案：(B)。

8.

If $f(x)=x^{\left(\frac{1}{x-1}\right)}$ for all positive $x \neq 1$ and if f is continuous at 1, then f(1) is

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) e

(E) none of the above

答案：(D)。

9.

If $f(x)=\begin{cases}\frac{x}{|x|} & \text{if } x \neq 0 \\ 1, & \text{if } x=0\end{cases}$, then $\int_{-1}^{3} f(x)\ dx =$

(A) -e (B) -ln 2 (C) 1 (D) 2 (E) e

答案：(D)。

10.

If f is a continuous real-valued function with domain a closed interval [a, b] such that $f'(x)=0$ for one and only one number x between a and b, then f

(A) might not have a maximum on [a, b]
(B) cannot have an even number of extrema on [a, b]
(C) cannot have a maximum at one end point and a minimum at the other
(D) might be monotone increasing
(E) might be unbounded

答案：(D)。

1.

Which of the following correctly describe the graph G in the XY-plane of $y=\frac{1}{x}\sin\frac{1}{x}$?

I. G does not intersect any line $y=c$ where $c<0$.
II. G contains infinitely many points $(x,0)$ with $0<x<1$.
III. G contains no points $(x,0)$ with $1<x$.

(A) I only (B) II only (C) III only
(D) I and III (E) II and III

答案：(E)。

12.

If f is a continuous, decreasing function defined on the positive reals and its graph is concave upward, then the graph of the inverse function f^{-1} is

(A) decreasing and concave upward
(B) decreasing and concave downward
(C) increasing and concave upward
(D) increasing and concave downward
(E) not necessarily any of the above

答案：(A)。

- 以上的十二個試題，是典型的試題，仔細的讀者不難發現微積分的基礎相當重要，考題的分佈也徹底而普遍，從基本的函數、反函數觀念，求導函數，利用導數判斷函數圖形的變化，到級數的收斂半徑，及應用題等，幾乎是鉅細靡遺。

二、線性代數（LINEAR ALGEBRA）及矩陣運算（MATRIX MANIPULATION）

1.

The solution set for the equation $\begin{vmatrix} 2 & 3 & x \\ 2 & 1 & x^2 \\ 6 & 7 & 3 \end{vmatrix} = 0$ is

(A) empty (B) {0} (C) {1} (D) {1, -3} (E) $\{\sqrt{3}, -\sqrt{3}\}$

答案：(D)。

2.

If T is the linear transformation mapping the vectors (1,0,0), (0,1,0). and (0,0,1) to the vectors (2,2,1), (1,1,2), and (0,0,1), respectively, which of the following is the image of the vector (2,3,4) under T?

(A) (7,7,12) (B) (7,7,11) (C) (7,7,8) (D) (6,6,11) (E) (3,3,4)

答案：(A)。

3.

Which of the following is equal to the product $P^{-1}\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}P$ for some invertible 2×2 matrix P?

(A) $\begin{pmatrix}3 & 4\\ 1 & 2\end{pmatrix}$ (B) $\begin{pmatrix}2 & 2\\ 3 & 2\end{pmatrix}$ (C) $\begin{pmatrix}2 & 0\\ 0 & 0\end{pmatrix}$ (D) $\begin{pmatrix}4 & 3\\ 2 & 1\end{pmatrix}$ (E) $\begin{pmatrix}2 & 1\\ 4 & 3\end{pmatrix}$

答案：(D)。

4.

The order of the element $\sigma=\begin{pmatrix}1 & 2 & 3 & 4 & 5\\ 1 & 1 & 1 & 1 & 1\\ 4 & 5 & 1 & 3 & 2\end{pmatrix}$ of the symmetric group S, is

(A) 2 (B) 3 (C) 6 (D) 8 (E) 12

答案：(C)。

5.

If $A=\begin{pmatrix}2 & 1\\ 3 & -1\end{pmatrix}$ and $I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, which of the following matric polynomials is zero?

(A) A^2-A-5I (B) A^2+A-5I (C) A^2+A-I
(D) A^2-4I (E) $A^2-3A+5I$

答案：(A)。

- 以上的五個試題，包括了線性代數中的主要課題：行列式的運算，線性映射、反矩陣、倒置數、單位矩陣等。值得一提的是線性代數在電腦及工程方面都扮演著相當重要的角色，必須切實了解其基本內容。

三、微分方程式（DIFFERENTIAL EQUATIONS）

1.

Which of the following is an equation of the curve through the origin which intersects at right angles all the curves satisfying the differential equation $\frac{dy}{dx} = x + 1$?

(A) $e^{-y} = x + 1$ (B) $e^y + x - 1 = 0$ (C) $e^y = x + 1$
(D) $2y = x^2 + 2x$ (E) $y(x^2 + 2x) = -2$

答案：(A)。

- 在本書的試題中，還有一些微分方程式的求解問題；希望讀者要能熟悉常微分方程式的各種解法。

四、數論（NUMBER THEORY）

1.

Let x and y be integers such that $9x + 5y$ is divisible by 11. For which of the following values of k must $10x + ky$ be divisible by 11?

(A) 0 (B) 1 (C) 3 (D) 7 (E) 8

答案：(E)。

2.

How many different solutions, modulo 24, does the congruence $18x \equiv 12 \pmod{24}$ have?

(A) None (B) One (C) Three (D) Six (E) Twelve

答案：(D)。

- 在數論的試題中，同餘問題是最常出現的。

五、機率及統計（ PROBABILITY AND STATISTICS ）

1.

Alternately a coin is tossed and a die is thrown, beginning with the coin. What is the probability that the coin will register a "head" before the die registers a "5" or "6"?

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{3}{5}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

答案：(E)。

- 機率及統計的試題較常見的有直觀機率及最小二乘法等，在本書的試題中均有出現。

六、電腦程式（ COMPUTER PROGRAMMING ）

1.

If the binary operation * is defined for all integers by $a * b = a + b - ab$, which of the following are true?

I. * is commutative.
II. * is associative.
III. There exists some integer which is an identity for *.

(A) I only (B) III only (C) I and II only
(D) I and III only (E) I, II, and III

答案：(E)。

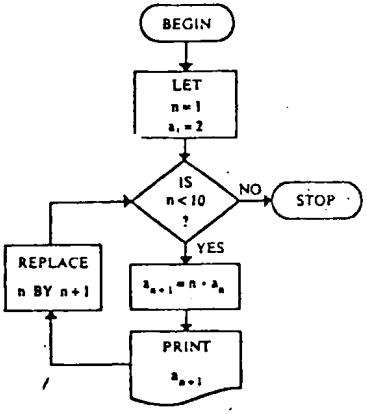

Figure 2

For the flowchart in Figure 2, the *last* number printed is

(A) $9 \cdot 2^9$ (B) $10 \cdot 2^{10}$ (C) $2 \cdot 9!$ (D) $2 \cdot 10!$ (E) $2^9 \cdot 9!$

答案：(C)。

- 此部份應注意二進位運算及 BASIC 語言，還應對 BCC（Basic Computer Concepts）有所了解。

七、拓樸學（TOPOLOGY）

1.

Let A and B be topological spaces, let f be a mapping from A to B, and let f^{-1} be the inverse of f. Under which of the following conditions must f be continuous?

(A) The image under f of any open set in A is an open set in B.
(B) The image under f of any closed set in A is a closed set in B.
(C) The image under f of any bounded set in A is a bounded set in B.
(D) The image under f^{-1} of any open set in B is an open set in A.
(E) The image under f^{-1} of any discrete set of points in B is a discrete set in A.

答案：(D)。

- 拓樸學的定義以及基本觀念是常考的，這一部分應參考基

本的教科書。（如：ELEMENTARY TOPOLOGY 2nd by GEMIGNANI 開發圖書公司翻印，該書之優點在於讀者只須具備微積分及解析幾何的基礎，即可了解）。

八、複變分析（COMPLEX ANALYSIS）

1.

If $f(z)=\frac{2z+1}{z-1}$ for all complex numbers $z \neq 1$, then f maps the point (1,1) to the point

(A) (-3,2) (B) (-2,3) (C) (2,-3) (D) (3,2) (E) at infinity

答案：(C)。

2.

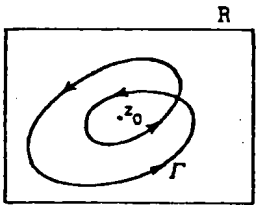

Figure 3

Let f be analytic in a region R with the single exception of point z_0. If the residue of f at z_0 is 1, what is the value of $\int_{\Gamma} f(z)\,dz$, where Γ is the path shown in Figure 3?

(A) 0 (B) 1 (C) 2 (D) $2\pi i$ (E) $4\pi i$

答案：(E)。

九、方程式論（EQUATION THEORY）

1.

Let p and q be constants. If $f(x) = p \sin x + qx \cos x + x^2$ for all real numbers x and if $f(2) = 3$, then $f(-2)$ is

(A) -3 (B) -1 (C) 1 (D) 5
(E) not uniquely determined by the information given

答案：(D)。

2.

Of the following equations, which has the greatest number of roots between 100 and 1,000?

(A) $\sin x = 0$ (B) $\sin (x^2) = 0$ (C) $\sin \sqrt{|x|} = 0$
(D) $\sin (x^3) = 0$ (E) $\sin \sqrt[3]{x} = 0$

答案：(D)。

十、數學應用（MATHEMATICAL APPLICATIONS）

1.

What is the least upper bound of the set of all numbers A such that a polygon with area A can be inscribed in a semicircular region of radius 1?

(A) $\frac{4}{5}$ (B) $\frac{2}{\sqrt{5}}$ (C) 1 (D) $\frac{\pi}{2}$ (E) 2

答案：(E)。

2.

A certain procedure requires 2^{40} calculations. A programmer decides that, to be useful on his computer, the procedure must be modified so as to require only 2^{30} calculations. If the procedure is modified each day in such a manner that the number of calculations required is exactly one-half that required by the procedure on the previous day, what is the total number of days that will be required to obtain a modification using only 2^{30} calculations?

(A) One (B) Two (C) Three (D) Thirty (E) Forty

答案：(E)。

3.

In the process of constructing a highway across a certain region in which there are many hills and valleys, the engineer will be certain that

{there is some level in between the elevations of the highest hill and lowest valley at which he can lay the surface of the highway using the tops of the hills as fill material for the valleys and such that *no* additional fill dirt need be brought in from another region and none will be left over to be hauled away.}

To build a mathematical model of this situation, let S be a long, narrow rectangular region (the roadbed) bounded by the lines $x = a$, $x = b$, $y = c$, and $y = d$; let f be a continuous function on S with M and m being, respectively, the maximum and minimum values of f on S. If the graph of f is identified with the surface of the land, then, of the following, which best corresponds to the assertio set off in braces above?

(A) There exists a point p in S such that $m \leq f(p) \leq M$.

(B) There exists a value q of f such that $M - m = q$.

(C) There exists a point p in S such that $\iint_S f = f(p) \cdot (\text{Area of S})$.

(D) $\int_a^b \left(\int_c^d f(x,y)dy \right) dx = \frac{\partial f}{\partial x}\Big|_{(a,b)} + \frac{\partial f}{\partial y}\Big|_{(c,d)}$

(E) There exists a value of q of f such that $\frac{q}{(\text{Area of S})} = \frac{\partial f}{\partial x}\Big|_{(a,b)} + \frac{\partial f}{\partial y}\Big|_{(c,d)}$

答案：(C)。

4.

Two planes P and Q intersect at an angle of 30°. Two *distinct* rays, one in P and one in Q, form an angle of x° with its vertex in the intersection of P and Q. What is the least possible value of x?

(A) 30 (B) 36 (C) 45 (D) 90 (E) There is no least possible value of x.

答案：(E)。

- 解這一部分的試題需要清楚的觀念和靈活的頭腦，有時還會用到簡單的物理觀念，如反射定律等。

十一、其　他

1

If x and y are integers such that $x \geq 3$ and $x - y \geq 9$, then the minimum possible value of $3x + 7y$ is

(A) −33 (B) 0 (C) 9 (D) 27 (E) nonexistent

答案：(E)。

2

Suppose that H is a nonempty subset of a multiplicative group G and that H is closed under the group operation. Which of the following conditions is NOT sufficient is insure that H is a subgroup of G?

(A) The identity element of G is in H.
(B) Every element of H has a unique inverse in H.
(C) If $x, y \in H$, then $xy^{-1} \in H$.
(D) If $x, y \in H$, then $x^{-1}y^{-1} \in H$.
(E) If $y \notin H$, then $y^{-1} \notin H$.

答案：(A)。

3

Each of two sets of data, D_1 and D_2, is divided into categories, C_1 and C_2, and the following table is devised to record the number of data in each category.

	C_1	C_2
D_1		
D_2		

Furthermore there are two systems of weights, W_1 and W_2, each of which assigns a score for each datum in each category, and the following table is devised to show the score assigned to each category under each system of weights.

	W_1	W_2
C_1		
C_2		

Let x_{ij} be the number of elements of D_i that are put into category C_j. Let y_{ij} be the weight assigned to C_i under the system W_j. If $X=\begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix}$ and $Y=\begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix}$, which would be the matrix of entries showing the total score of set D_i under W_j?

(A) $X+Y$ (B) XY (C) YX (D) XYX^{-1} (E) YXY^{-1}

答案：(B)。

4.

Let f be a function with domain $[-1,1]$ such that the coordinates of each point (x,y) of its graph satisfy $x^2+y^2=1$. The total number of points at which f is necessarily continuous is

(A) zero (B) one (C) two (D) four (E) infinite

答案：(C)。

- 以上的三十六個題目，可以說是一份具體而微的試題，當然，我們可以從而得知準備的方向以及深度，我們希望讀者在這幾個領域做了相當的準備之後，再將本書中的四回模擬試題按照自我測驗，試題檢討，題型推廣，這三個部驟，反覆演練，以求融會貫通，切實了解。

  最後，編者要感謝美加叢主任爲這一系列的GRE SUBJECT參考書所做的指導，更希望這一系列的書籍能幫助莘莘學子在日後的考試中，創造輝煌的戰果。

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SAMPLE TEST 1

SAMPLE TEST 2

SAMPLE TEST 3

SAMPLE TEST 4