高等院校双语教学规划教材

（英文版）

Advanced Mathematics (I)

高等数学（上）

东南大学大学数学教研室　编著

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·南京·

内 容 提 要

本书是为响应东南大学国际化需要，根据国家教育部非数学专业数学基础课教学指导分委员会制定的工科类本科数学基础课程教学基本要求，并结合东南大学数学系多年教学改革实践经验编写的全英文教材。全书分为上、下两册，内容包括极限、一元函数微分学、一元函数积分学、常微分方程、级数、向量代数与空间解析几何、多元函数微分学、多元函数积分学、向量场的积分、复变函数等十个章节。

本书可作为高等理工科院校非数学类专业本科生学习高等数学的英文教材，也可供其他专业选用和社会读者阅读。

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前　言

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本书对基本概念的叙述清晰准确，对基本理论的论述简明易懂。在内容处理上依据国内工科类本科数学基础课程教学基本要求，按照现行的国内微积分教材体系结构进行编排，比国外同类教材简洁，理论性更强。同时，本书还兼顾美国教材重视应用、便于自学的特点，例题和习题的选配典型多样，增加了应用内容与相关的实际问题，强调对基本运算能力及理论的实际应用能力的培养。

本教材的内容是工科学生必备大学数学知识，利用英文编写更有利于学生提高与国际同行专家交流的能力。本书可作为高等理工科院校非数学类专业本科生学习高等数学课程的英文教材，也可供其他专业选用和社会读者阅读。

本书上册共四章，其中第一、二章由陈文彦编写，第三章由范赟编写，第四章由马红铝编写，最后由陈文彦统稿。

本书在编写的过程中得到了东南大学教务处的大力支持，数学系的王栓宏教授、卢剑权教授对本教材的编写提出了许多有益的建议，在此一并对他们表示感谢。本书中缺点和错误在所难免，欢迎读者批评指正。

编　者

2014 年 5 月

CONTENTS

Chapter 1 Limits

The notion of limit is one of the fundamental ideas that distinguishes calculus from the other branches of mathematics. In fact, we define calculus this way: Calculus is the study of the limits.

§ 1.1 The Concept of Limits and its Properties

The concept of limit is central to many problems in the physical, engineering, and social science. In this section we introduce the concept of limits, and its properties. We start with the limit of sequences first and will study limit of functions later.

§ 1.1.1 Limits of Sequence

An **infinite sequence** (or **sequence**) of numbers is a function whose domain is the set of integers greater than or equal some integer n_0. Sequences are defined the way other functions are, some typical rules being

$$a_n=\sqrt{n}, \quad a_n=(-1)^{n+1}\frac{1}{n}, \quad a_n=\frac{n-1}{n}.$$

To indicate that the domains are sets of integers, we use a letter like n from the middle of the alphabet for the independent variable, instead of the x, y, z, and t used widely in other contexts. The number a_n is the ***n*th term** of the sequence, or the **term with index *n***.

The sequences $\left\{\frac{1}{n}\right\}$, $\left\{(-1)^{n+1}\frac{1}{n}\right\}$, and $\left\{\frac{n-1}{n}\right\}$ each seem to approach a single limiting value as n increases, and $\{3\}$ is at a limiting value from the very first. On the other hand, terms of $\left\{(-1)^{n+1}\frac{n-1}{n}\right\}$ seem to accumulate near two different values: -1 and 1, while the terms of $\{\sqrt{n}\}$ become increasingly large and do not accumulate anywhere.

To distinguish sequences that approach a unique limiting value a, as n

increases, from those that do not, we say that the former sequences converge, according to the following definition.

Definition 1 The sequence $\{a_n\}$ **converges** to the number a if to every positive number ε there corresponds an integer N such that for all n,

$$n>N \Rightarrow |a_n-a|<\varepsilon.$$

If no such number a exists, we say that $\{a_n\}$ **diverges.** If $\{a_n\}$ converges to a, we write $\lim_{n\to\infty} a_n=a$, or simply $a_n \to a$ $(n\to\infty)$, and call a the **limit** of the sequence.

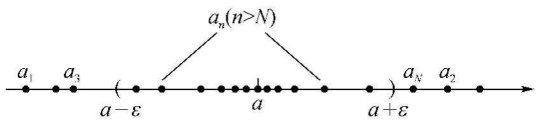

Figure 1. 1

In Figure 1. 1, all the a_n's after a_N lie within ε of a.

Example 1 Using the definition to show that $\lim_{n\to\infty}\frac{1}{n}=0$.

Solution Let $\varepsilon>0$ be given. We must show that there exists an integer N such that

$$n>N \Rightarrow \left|\frac{1}{n}-0\right|<\varepsilon.$$

This implication will hold if $\frac{1}{n}<\varepsilon$ or $n>\frac{1}{\varepsilon}$. If we choose $N=\left[\frac{1}{\varepsilon}\right]$, the implication will hold for all $n>N$.

Example 2 Prove that $\lim_{n\to\infty} q^n=0$ $(|q|<1)$.

Solution If $q=0$, then $q^n=0$ $(n=1,2,\cdots)$, and the limit is 0 obviously. If $0<|q|<1$, let $0<\varepsilon<1$ be given. We want to find N such that

$$n>N \Rightarrow |q^n-0|=|q|^n<\varepsilon.$$

Consider the inequality,

$$|q|^n<\varepsilon \Leftrightarrow n\ln|q|<\ln\varepsilon \Leftrightarrow n>\frac{\ln\varepsilon}{\ln|q|}.$$

If we take $N=\left[\frac{\ln\varepsilon}{\ln|q|}\right]$, the implication will hold for all $n>N$.

Example 3 Prove that $\lim_{n\to\infty}\frac{2n+1}{3n-1}=\frac{2}{3}$.

Solution Let $\varepsilon>0$ be given. Choose $N=\left[\frac{1}{\varepsilon}\right]$. Then $n>N$ implies that

$$\left|\frac{2n+1}{3n-1}-\frac{2}{3}\right|=\frac{6n+3-6n+2}{3(3n-1)}=\frac{5}{3(3n-1)}<\frac{6}{3\cdot 2n}=\frac{1}{n}<\varepsilon.$$

We have given the definition of divergence above, now we present the definition more precisely.

Definition 2 If there exists $\varepsilon_0>0$, for any $N\in\mathbf{N}^*$, there exists $n_0>N$, such that $|x_{n_0}-a|\geqslant\varepsilon_0$, then we say that $\{a_n\}$ **diverges.**

Example 4 Prove that $\{(-1)^n\}$ diverges.

Proof Take $\varepsilon_0=1$. If $a\geqslant 0$, then for any N, there exists an odd number $n_0>N$, such that

$$|(-1)^{n_0}-a|=|-1-a|\geqslant 1=\varepsilon_0.$$

If $a<0$, then for every N, there exists an even number $n_0>N$, such that

$$|(-1)^{n_0}-a|=|1-a|>1=\varepsilon_0.$$

This implies that $\{(-1)^n\}$ diverges.

§ 1.1.2 Limits of Functions

Consider the functions

$$f(x)=x^2,\quad g(x)=\frac{1}{x},\quad h(x)=\sin\frac{1}{x}.$$

When x approaches to 0, $f(x)=x^2$ also approaches to 0, while the absolute value of $g(x)=\frac{1}{x}$ gets larger and does not approach a fixed finite number. Moreover, the value of h swings, such as

$$h\left(\frac{1}{2n\pi}\right)=0,\quad h\left[\frac{1}{\left(2n+\frac{1}{2}\right)\pi}\right]=1,$$

therefore $h(x)$ does not approach any one specific finite number.

We saw similar behaviors for sequences when n approaches infinity. The function $f(x)$, $g(x)$, $h(x)$ are compare to $\left\{\frac{1}{n^2}\right\}$, $\{(-1)^n n\}$, $\{(-1)^n\}$. Only the first converges. This leads to the similar definition of limit for functions.

Definition 3 Let f be defined on an open interval about x_0, except possibly at x_0 itself. We say that $f(x)$ approaches the limit a as x approaches x_0, and write

$$\lim_{x \to x_0} f(x) = a,$$

if, for every number $\varepsilon > 0$, there exists a corresponding number δ such that for all x

$$0 < |x - x_0| < \delta \Rightarrow |f(x) - a| < \varepsilon.$$

The definition $\lim\limits_{x \to x_0} f(x) = a$ means that $f(x)$ goes close to a when x gets sufficiently close to (but is different from) x_0 (Figure 1.2).

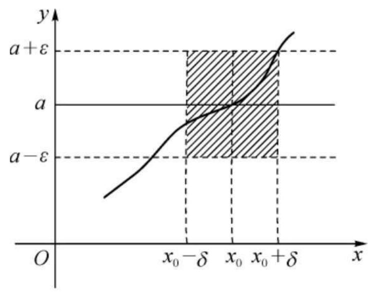

Figure 1.2

Example 5 Use the definition to prove that $\lim\limits_{x \to 0} a^x = 1$, where $a > 1$.

Proof Let $\varepsilon > 0$ be given, without loss of generality, we may assume that $0 < \varepsilon < 1$. We want to find $\delta > 0$ such that

$$0 < |x| < \delta \Rightarrow |a^x - 1| < \varepsilon.$$

Now

$$|a^x - 1| < \varepsilon \Leftrightarrow 1 - \varepsilon < a^x < 1 + \varepsilon \Leftrightarrow \log_a(1 - \varepsilon) < x < \log_a(1 + \varepsilon).$$

If we choose $\delta = \min\{-\log_a(1 - \varepsilon), \log_a(1 + \varepsilon)\}$, the implication will hold for $0 < |x| < \delta$.

Example 6 Prove that if $x_0 > 0$, then $\lim\limits_{x \to x_0} \sqrt{x} = \sqrt{x_0}$.

Proof For any $\varepsilon > 0$, we are looking for δ such that

$$0 < |x| < \delta \Rightarrow |\sqrt{x} - \sqrt{x_0}| < \varepsilon.$$

We should first insist that $\delta < x_0$, for then $(x_0 - \delta, x_0 + \delta) \subseteq [0, +\infty)$ implies that $x > 0$, so that $\sqrt{x}$ is defined. Now

$$|\sqrt{x} - \sqrt{x_0}| = \left| \frac{x - x_0}{\sqrt{x} + \sqrt{x_0}} \right| < \frac{|x - x_0|}{\sqrt{x_0}} < \varepsilon.$$

To make the latter less than ε requires that $|x - x_0| < \varepsilon \sqrt{x_0}$. Choose $\delta =$

$\min\{\sqrt{x_0}\,\varepsilon, x_0\}$, then the implication will hold for all $0<|x-x_0|<\delta$.

Example 7 Use the definition to prove that $\lim\limits_{x\to 2}\frac{x-3}{x+1}=-\frac{1}{3}$.

Proof Let $\varepsilon>0$ be given. We must find δ such that

$$0<|x-2|<\delta \Rightarrow \left|\frac{x-3}{x+1}+\frac{1}{3}\right|<\varepsilon.$$

Now

$$\left|\frac{x-3}{x+1}+\frac{1}{3}\right|=\left|\frac{3x-9+x+1}{3(x+1)}\right|=\left|\frac{4x-8}{3(x+1)}\right|=\frac{4}{3}\,\frac{|x-2|}{|x+1|}.$$

The factor $\frac{1}{x+1}$ is troublesome especially if x is near to -1. We can bounded this factor if we keep x away from -1. To this end, note that

$$|x+1|=|x-2+3|\geqslant|3-|x-2||.$$

Thus, if we choose $\delta<1$, we succeed in making $|x+1|\geqslant 2$. Finally, if we also require $\delta<\varepsilon$, then

$$\left|\frac{x-3}{x+1}+\frac{1}{3}\right|<\frac{4}{3}\,\frac{|x-2|}{|x+1|}<\frac{4}{3}\cdot\frac{1}{2}|x-2|<|x-2|<\varepsilon.$$

Choose $\delta=\min\{1,\varepsilon\}$, the implication will hold for $0<|x-2|<\delta$.

It is possible for a function to approach a limiting value as x approaches from only one side, either from the right or from the left. In this case we say that f has a **one-sided** (either right-hand or left-hand) limit at x_0.

Definition 4 A function $f(x)$ converges to a finite limit a on the right side of $x=x_0$ if $f(x)$ approaches a as $x>x_0$ and approaches x_0. In this case, we say that f has **right-hand limit** a at x_0, and denote

$$\lim_{x\to x_0^+} f(x)=a \text{ or } f(x_0+0)=a.$$

The left-hand limit at x_0 is defined similarly, denoted as

$$\lim_{x\to x_0^-} f(x)=a \text{ or } f(x_0-0)=a.$$

By the definition of the right-hand and left-hand limit, it is easily to see the following theorem.

Theorem 1 (**One-sided vs. Two-sided Limits**) A function $f(x)$ has a limit as x approaches x_0 if and only if it has left-hand and right-hand limits there, and these one-sided limits are equal:

$$\lim_{x\to x_0} f(x)=a \Leftrightarrow \lim_{x\to x_0^-} f(x)=\lim_{x\to x_0^+} f(x)=a.$$

The sign function, which is also called a step function,

$$\operatorname{sgn}x=\begin{cases}1 & (x>0);\\ 0 & (x=0);\\ -1 & (x<0).\end{cases}$$

We have

$$\lim_{x\to 0^-} f(x)=-1,\quad \lim_{x\to 0^+} f(x)=1.$$

Since the two sided limits are not equal, $\lim_{x\to 0} f(x)$ does not exist.

Now we will give a new kind of limits. In analogy with our ε-δ definition for ordinary limits, we make the following definitions for limits as x approach ∞. The definition of $x\to+\infty$ and $x\to-\infty$ are similar, we omit here.

Definition 5 Let f be defined on $(-\infty,+\infty)$. We say that $\lim_{x\to\infty} f(x)=a$ if for $\varepsilon>0$ there is a corresponding number X such that

$$|x|>X \Rightarrow |f(x)-a|<\varepsilon.$$

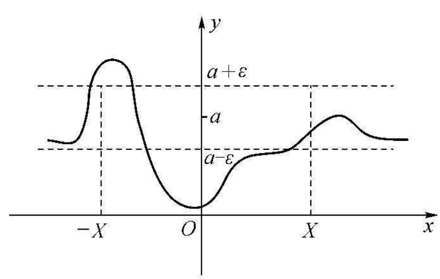

Figure 1.3

You will note that X can, and usually does, depend on ε. In general, the smaller ε is, the larger X will have to be. The graph in Figure 1.3 may help you to understand what we are saying.

Example 8 Show that $\lim_{x\to\infty}\frac{\sin x}{x}=0$.

Proof Let $\varepsilon>0$ be given, we are looking for M such that

$$\left|\frac{\sin x}{x}-0\right|=\left|\frac{\sin x}{x}\right|\leqslant\frac{1}{|x|}<\varepsilon,$$

we may choose $M=\frac{1}{\varepsilon}$. Then $|x|>M$ implies that $\left|\frac{\sin x}{x}\right| \leqslant \frac{1}{|x|}<\varepsilon$.

§ 1.1.3 Properties of Limits

The limit of a function has similar properties as the limit of a sequence, so we only state the properties of limit of a function.

Proposition 1 (**Uniqueness**) If $\lim\limits_{x \to x_0} f(x)$ exists, then it must be unique.

Proof Suppose that $\lim\limits_{x \to x_0} f(x)=a$ and $\lim\limits_{x \to x_0} f(x)=b$, $a \neq b$. Let $\varepsilon=\frac{|b-a|}{2}$. Since $\lim\limits_{x \to x_0} f(x)=a$, there exists $\delta_1>0$, such that

$$0<|x-x_0|<\delta_1 \Rightarrow |f(x)-a|<\varepsilon.$$

Since $\lim\limits_{x \to x_0} f(x)=b$, there exists $\delta_2>0$, such that

$$0<|x-x_0|<\delta_2 \Rightarrow |f(x)-b|<\varepsilon.$$

We may choose $\delta=\min\{\delta_1, \delta_2\}$, then

$$0<|x-x_0|<\delta \Rightarrow |b-a| \leqslant |f(x)-a|+|f(x)-b|<2\varepsilon=|b-a|.$$

Which is a contradiction. Thus $a=b$.

Proposition 2 (**Boundedness**) If $\lim\limits_{x \to x_0} f(x)$ exists, then f is bounded on some deleted interval about x_0.

Proof Let $\varepsilon=1$. Since $\lim\limits_{x \to x_0} f(x)=a$, there exists $\delta>0$, such that

$$0<|x-x_0|<\delta \Rightarrow |f(x)-a|<1.$$

That is, for $0<|x-x_0|<\delta$, we have

$$|f(x)| \leqslant |f(x)-a|+|a|<1+|a|.$$

Proposition 3 (**Order Rule**) Suppose $\lim\limits_{x \to x_0} f(x)=a$, $\lim\limits_{x \to x_0} g(x)=b$ and $a<b$, then $f(x)<g(x)$ for all x in some deleted interval about x_0.

Proof Let $\varepsilon=\frac{b-a}{2}$. Since $\lim\limits_{x \to x_0} f(x)=a$, there exists $\delta_1>0$, such that

$$0<|x-x_0|<\delta_1 \Rightarrow |f(x)-a|<\frac{b-a}{2},$$

then

$$f(x)<a+\frac{b-a}{2}=\frac{a+b}{2}.$$

Since $\lim\limits_{x \to x_0} g(x) = b$, there exists $\delta_2 > 0$, such that

$$0 < |x - x_0| < \delta_2 \Rightarrow |g(x) - b| < \frac{b-a}{2},$$

then

$$g(x) > b - \frac{b-a}{2} = \frac{a+b}{2}.$$

We may choose $\delta = \min\{\delta_1, \delta_2\}$. Thus for $0 < |x - x_0| < \delta$, we have

$$f(x) < \frac{b-a}{2} < g(x).$$

Corollary 1 If $\lim\limits_{x \to x_0} f(x) < 0$, then $f(x) < 0$ for all x in some deleted interval about x_0.

Corollary 2 If $\lim\limits_{x \to x_0} f(x) = a$, $\lim\limits_{x \to x_0} g(x) = b$ and $f(x) \leqslant g(x)$ for all x in some deleted interval about x_0, then $a \leqslant b$.

We saw some limits of functions inspired by the similar limits of sequences. This suggests that there is a relationship between the two kinds of limits.

Theorem 2 (**Heine's Theorem**) $\lim\limits_{x \to x_0} f(x) = a$ if and only if for any sequence $\{a_n\}$ satisfying $a_n \to x_0 \, (a_n \neq x_0)$, we have $\lim\limits_{n \to \infty} f(a_n) = a$.

Proof Let $\lim\limits_{x \to x_0} f(x) = a$. For every $\varepsilon > 0$, there exists $\delta > 0$, such that

$$0 < |x - x_0| < \delta \Rightarrow |f(x) - a| < \varepsilon.$$

Since $a_n \to x_0$, $a_n \neq x_0$, for the above $\delta > 0$, there exists $N \in \mathbf{N}^*$, such that

$$n > N \Rightarrow 0 < |a_n - x_0| < \delta,$$

so we conclude that, for every $\varepsilon > 0$, there exists $N \in \mathbf{N}^*$, such that $n > N$ implies that

$$|f(a_n) - a| < \varepsilon.$$

Thus the necessity of the theorem is proved. Next we will show the sufficiency by the method of contradiction.

Suppose that $\lim\limits_{x \to x_0} f(x) \neq a$, then there exists $\varepsilon_0 > 0$, for every $n \in \mathbf{N}^*$, there exists a_n which satisfy $0 < |a_n - x_0| < \frac{1}{n}$, but

$$|f(a_n) - a| \geqslant \varepsilon_0.$$

So we get a sequence $\{a_n\}$, $a_n \neq x_0$ and $a_n \to x_0$, but $\lim\limits_{n\to\infty} f(a_n) \neq a$. It is a contradiction.

The theorem is usually used to prove the divergence of a function.

Example 9 Prove that $\lim\limits_{x\to 0}\sin\dfrac{1}{x}$ diverges.

Proof

$$f(x_n)=\sin\left(2n\pi+\frac{\pi}{2}\right)=1,$$

for the sequence $x_n=\dfrac{1}{2n\pi+\frac{\pi}{2}}$ that converges to 0.

$$f(x_n')=\sin n\pi=0$$

for the sequence $x_n'=\dfrac{1}{n\pi}$ converges to 0.

Thus $\lim\limits_{x\to 0}\sin\dfrac{1}{x}$ diverges by Heine's Theorem.

Consider the graph $y=\sin\dfrac{1}{x}$. In any neighborhood of the origin, the graph wiggles up and down between -1 and 1 infinitely many times (Figure 1.4). Clearly, $\sin\dfrac{1}{x}$ is not near a single number.

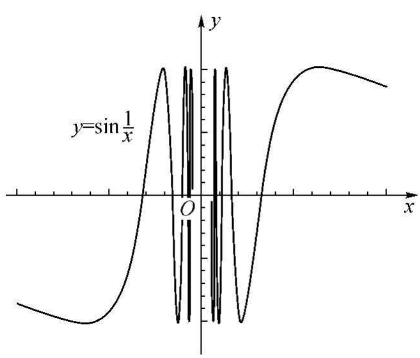

Figure 1.4

Exercise 1.1

1. Which of the following are equivalent to the definition of limit?

(1) For some $\varepsilon>0$ and every $\delta>0$, $0<|x-x_0|<\delta \Rightarrow |f(x)-a|<\varepsilon$.

(2) For every $\delta>0$, there is a corresponding $\varepsilon>0$ such that $0<|x-x_0|<\varepsilon \Rightarrow |f(x)-a|<\delta$.