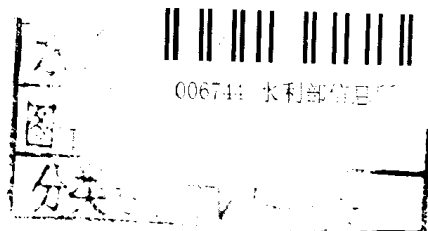
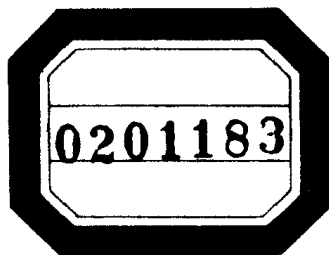


钱宁论文集





# 钱 宁 论 文 集

《钱宁论文集》编辑委员会

清华大学出版社

## 内 容 提 要

本论文集是我国著名的河流泥沙问题专家钱宁教授一生科研成果的精华，共收入钱宁教授自五十年代起在国内外公开发表的论文 61 篇。所选论文基本上分为两大类：(1)泥沙运动力学方面，如：“关于‘床沙质’和‘冲泻质’的概念的说明”，“推移质公式的比较”，“高含沙水流运动研究述评”等；(2)河床演变学方面，如：“钱塘江河口沙坎的近代过程”，“关于河流分类及成因问题的讨论”，“长江三峡枢纽工程的几个泥沙问题”等。这些研究成果及其渗透的学术思想，过去和现在都具有很高的学术价值和实用价值。为便于读者查阅和国际学术交流，书后附有钱宁著述目录。

本书可供水利、地理等专业的科技人员和高等院校师生参考。



## 钱 宁 论 文 集

《钱宁论文集》编辑委员会编

责任编辑 姚美瑞

☆

清华大学出版社出版

北京 清华园

中国科学院印刷厂印刷

新华书店总店科技发行所发行

☆

开本：787×1092 1/16 印张：59.5 字数：1523 千字

1990 年 4 月第 1 版 1990 年 4 月第 1 次印刷

印数：0001—1500

ISBN 7-302-00560-5/TV·8

定价：35.00 元

## 钱宁论文集编辑委员会

主任 林秉南  
副主任 董曾南 谢鉴衡 窦国仁 龙毓骞 戴定忠  
委员 丁联臻 张启舜 张仁 惠遇甲  
唐日长 麦乔威 华国祥 戴泽衡  
杜国翰 范家骅 严镜海 冯启德  
曹素滨 李保如 周耀庭 刘建民  
宋根培 李昌华 赵业安 万兆惠  
洪柔嘉 周志德 曹如轩 王兆印  
顾问 严恺 张瑞瑾 夏震寰 龚时昉 魏廷铮

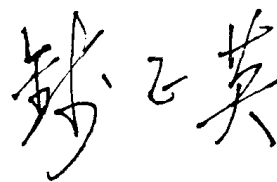
## 钱宁论文集编辑工作小组

清华大学 张仁 姚美瑞  
杨美卿 王兴奎 王士强  
水利水电科学研究院 万兆惠  
国际泥沙研究培训中心 周志德

## 序 言

钱宁同志 1955 年自美国回国。三十多年来，为了解决我国水利建设中的河流泥沙问题，他孜孜奉献，竭尽心力。他考察了许多河流，发表了大量的研究论文和报告，为我国黄河、长江等大江大河的治理提供了重要的依据。他所主持研究的“集中治理黄河中游粗沙来源区”的成果是治黄认识上的一个重大突破。在与病魔搏斗的七年里，他以惊人的毅力，完成了《泥沙运动力学》和《河床演变学》两部专著，共一百八十余万字。前者从微观的角度，研究了泥沙的冲刷、搬运和沉积的过程；后者则是以泥沙运动力学和地质、地理学方面的知识为基础，从宏观的角度，研究了河流的形成和演变。这两部专著极大地丰富了我国河流泥沙科学的内容。

为了纪念钱宁同志，并使钱宁同志的研究成果和学术思想能更好地发挥作用，除了他的专著外，我们现将他在国内外公开发表的论文，汇编成《钱宁论文集》出版。这部文集，包括了他在国外发表的十余篇论文和他的博士论文，但绝大部分文章是他回国后三十多年来的研究成果。它是我国泥沙科学的一份可贵的财富。这部论文集和他的两部专著一起，无疑将会对国内外河流泥沙科学的发展起到有力的促进作用。



1987 年 5 月 4 日

## 编者的话

钱宁教授是我国杰出的河流泥沙问题专家,早年在我国水利界名学者的指引下成长,后于1947年考取公费留美。他先在衣阿华大学从严师 H·饶斯等教授学习,在流体力学方面打下了坚实的基础,然后转入加州大学,长期受到泥沙工程名家 H·A·爱因斯坦的熏陶。他勤奋好学,更兼天然异禀,所以学习成绩十分优秀。在河流泥沙工程学方面,很早就做出成绩。有些在留学时期提出的论文和试验资料,至今还为人们所引用。

泥沙工程学不是一门理论科学,但它的发展又不能只靠试验和观测,而必须试验、观测和理论分析三结合。河流的形成与演变以及泥沙的来量和搬运,也受水文、地质、地理、气象、农业等多方面因素的影响和制约。对于这一点,钱宁教授早有认识,在五十年代初就已提出河流泥沙的研究是跨学科的,不单纯属于水力学的范畴。基于以上认识,他很早就选定了试验、观测和理论分析三结合,跨学科研究河流泥沙的途径。三十多年来,他孜孜不倦,身体力行,既博览群书,又大力组织试验,同时,更不辞劳苦、冒险犯难,长期在黄河两岸工作和查勘。他的著作有很大一部分是这样奋斗研究的结晶,因而具有很高的科学技术价值。我们遵照水利电力部和清华大学的决定,编辑了“钱宁论文集”,以便广大水利工作者参考和应用这些研究成果。本集共收入钱宁教授写作或参与写作并已公开发表的论文和报告共 61 篇。

学无止境。钱宁教授已对河流泥沙工程学作出了他的贡献,我们后人应继续前进,争取使这门复杂而广博的学科得到进一步的发展,使它更好地为水利工程和其它产业服务。为了达到这一目的,沿什么道路前进是至关重要的课题。编者以为钱宁教授走过的学术道路是值得人们步其后尘的。

《钱宁论文集》编辑委员会主任



# 钱宁同志生平



钱宁(1922—1986)

钱宁同志生前系清华大学教授，中国科学院学部委员，中国水利学会名誉理事，国际泥沙研究培训中心顾问委员会副主席，著名的水利科学家，中国共产党党员。1986年12月6日因患癌症在北京逝世。

钱宁同志1922年12月4日出生于杭州市，1943年毕业于中央大学土木系，1948年获美国衣阿华大学硕士学位，1951年在美国加利福尼亚州立大学获博士学位。回国后，先后任中国科学院水工研究室研究员，水利水电科学研究院河渠所副所长。1973年后任清华大学教授，泥沙研究室主任，中国水利学会常务理事，泥沙专业委员会副主任，《泥沙研究》、《国际泥沙研究》(英文版)主编等职，还担任过北京市第七届人大常委会委员。

钱宁同志从青年时代就立志报效祖国。1955年他放弃了在国外享有的地位和优裕的生活条件，与其他留美学者一道，冲破重重阻力，毅然回国参加社会主义建设。三十多年来，与祖国人民同甘共苦，共患难。十年浩劫中，虽遭受残酷迫害而矢志不移。1979年他不幸身患癌症，但仍以昂扬的乐观主义精神与惊人的毅力，进行了生命的最后拚搏，为祖国的水利事业做出了突出贡献。1985年他被表彰为北京市劳动模范，1986年被中共北京市委表彰为优秀共产党员，并获得中华全国总工会授予的“五一”劳动奖章。

钱宁同志是一位具有深湛理论修养与开拓精神的科学家。他继承与发展了H·A·爱因斯坦泥沙运动力学理论体系，倡导了高含沙水流运动机理的研究，为我国的河流动力学与地貌学结合研究河床演变的开拓起了重要作用，写成了具有较高理论价值的著作《泥沙运动力学》与《河床演变学》。前者获得全国优秀科技图书一等奖。他一生发表了大量论文，还博采众长，做了很多翻译介绍工作。

钱宁同志是国内外知名的泥沙专家，对黄河、长江的治理作出了重要贡献，是我国河流泥沙研究工作的卓越组织者之一。

他善于把理论和实践相结合，一贯重视我国水利建设中泥沙问题的研究，是我国研究黄河的著名专家。他所著《黄河下游河床演变》一书，阐述了黄河下游河道自然演变规律，为防洪治河实践提供了重要依据。他主持研究的“集中治理黄河中游粗沙来源区”的成果，是治黄认识上的一个重大突破，获得国家自然科学奖。他对制订治理黄河规划、解决钱塘江口及长江葛洲坝水利枢纽工程泥沙问题，都作出了重要贡献。在病重期间，钱宁同志仍积极关心和指导三峡工程泥沙问题的研究，抱病参加会议，发表了重要意见，并反复叮嘱一起工作的同志，要把三峡工程的科研工作做细，做好，为中央决策提供科学的、负责任的明确意见。他以艰苦创业的精神，先后积极领导和参加了中国科学院水工研究室、水利水电科学研究院河渠所和清华大学

泥沙研究室等科研机构的创建,并长期在黄河现场进行观测试验研究工作,踏遍黄河上下和中游水土流失地区,多次成功地组织了重大泥沙问题的科技攻关,在全国泥沙科研和工程技术人员中享有很高的威望。

钱宁同志始终不渝地为培养科研骨干力量和推动国际泥沙研究合作,做出了巨大的努力。

他一贯重视发现和培养人才。从六十年代起,就倡导组织了多种类型的泥沙专业培训,为全国各地培养了大批从事泥沙工作的骨干力量,晚年则更以巨大热忱,精心指导硕士、博士研究生。为了贯彻中央教育体制改革的决定,他积极写出了教育改革建议书,呼吁加强学生的基础理论教学,注意人才的培养。

为了促进我国泥沙科学事业的发展,钱宁同志十分注意加强国际学术交流,开展科技合作。他积极发起组织在我国召开河流泥沙国际学术讨论会。在他的倡导下,1984年在北京成立了国际泥沙研究培训中心,并创办了英文版《国际泥沙研究》杂志,进一步加强了同各国学者间的联系,促进了我国泥沙科研工作的发展,在较短的时期里跨进了国际科坛,扩大了我国的影响。

钱宁同志是一位有名望的科学家。他不但有严谨的好学风,刻苦勤奋的精神,又有出色的组织领导才能。更可贵的是他能密切联系群众,团结同志,平易近人,待人宽厚,关心他人比自己还重,因此深受同志们的敬佩。

钱宁同志无愧为我国知识分子的优秀代表,他所走过的道路和他的业绩将永远值得我们学习和怀念。



# 目 录

序言	iii
编者的话	v
钱宁同志生平	vii
论文	
Investigation of the Maximum Equilibrium Rate of Bed Load Movement	1
The Efficiency of Depth-Integrating Suspended-Sediment Sampling	44
Transport of Sediment Mixtures with Large Ranges of Grain Sizes	51
Can the Rate of Wash Load be predicted from the Bed Load Function?	88
Second Approximation to the Solution of the Suspended Load Theory	98
Ripple Tank Studies of Wave Refraction	118
Effects of Heavy Sediment Concentration Near the Bed on the Velocity and Sediment Distribution	120
The Present Status of Research on Sediment Transport	186
Similarity of Distorted River Models with Movable Beds	220
Graphic Design of Alluvial Channels	240
A Concept of the Regime Theory	258
灌溉引水的自动控制问题 (The Automatic Control Problems on Irrigational Diversion)	276
根据悬移质积点测量决定输沙率时所引起的误差问题 (On the Determination of the Total-Load Rate from Point-Integrating Suspended-Sediment Sampling)	297
关于“床沙质”和“冲泻质”的概念的说明 (On the Concepts of “Bed Material Load” and “Wash Load”)	307
灌溉渠首的防沙措施 (The Measures Preventing Sediment from Entering the Irrigation Head Works)	323
冲积河流稳定性指标的商榷 (On the Stability Criterion of Alluvial Streams)	331
水温对于泥沙运动的影响 (The Effect of Water Temperature on Sediment Motion)	346
浑水的粘性及流型 (The Viscosity and Stress-Flow Relation of Sediment-Laden Flow)	358
修建水库后下游河道重新建立平衡的过程 (The Fluvial Processes of Re-establishing Equilibrium below Impounding Reservoirs)	380

黄河下游的糙率问题 (Channel Roughness of Lower Yellow River).....	401
黄河下游河床的粗化问题 (Change of Bed material Composition in a Degraded bed and Its Effect on the Stabilization of Stream Channel) .....	417
从泥沙的角度来看黄河下游河道整治问题 (The Regulation Problems of the Lower Yellow River from the Aspect of Sedimentation) .....	425
The Braided Stream of the Lower Yellow River .....	436
黄河下游游荡性河道的特性及其成因分析 (The Characteristics and Genesis Analysis of the Braided Stream of the Lower Yellow River) .....	452
黄河下游的一些水面现象 (Some Water Surface Phenomena in the Lower Yellow River) .....	478
多沙河流上修建水库后下游来沙量的估计 (Estimation of Sediment Supply after the Construction of a Reservoir in the Upstream Basin) .....	484
近年来国外在泥沙输移方面研究的主要成就及发展方向 (The Main Achievement and Development Trend of the Recent Research Work on Sediment Transport Abroad).....	496
多沙河流上修建大型蓄水库后下游游荡性河道的演变趋势及治理 (The Regulation of the Braided Stream with Heavy Sediment Load) .....	509
钱塘江河口沙坎的近代过程 (The Fluvial Processes of the Big Sand Bar Inside the Qiantangjiang Estuary) .....	527
近底高含沙量流层对水流及泥沙运动影响的初步探讨 (The Effects of Sediment Concentration Gradient on the Characteristics of Flow and Sediment Motion) .....	544
从黄河下游的河床演变规律来看河道治理中的调水调沙问题 (The Regulation of Flow and Sediment Based on the Principles of Fluvial Processes for the Improvements of the Lower Yellow River).....	562
黄河的高含沙水流问题 (The flow with Heavy Sediment Concentration in the Yellow River Basin) ...	574
泥沙运动力学的发展与前瞻 (The Development and Prospect of the Mechanics of Sediment Transport)...	587
中国河流泥沙问题及其研究概况 (The Problem of River Sedimentation and the Present Status of Its Research in China) .....	602
黄河中游粗泥沙来源区对黄河下游冲淤的影响 (The Source of the Coarse Sediment in the Middle Reaches of the Yellow	

River and Its Effect on the Siltation of the Lower Yellow River) .....	615
西北地区高含沙水流运动机理的初步探讨 (Preliminary Study on the mechanism of the Hyperconcentrated Flow in Northwest China) .....	623
推移质公式的比较 (A Comparison of the Bed Load Formulas) .....	639
高含沙水流运动的几个问题 (Some Problems on the Motion of Hyperconcentrated Flow) .....	649
关于黄河中下游治理的意见 (Some Suggestions on the Problem of Harnessing the Yellow River) .....	661
黄河下游挟沙能力自动调整机理的初步探讨 (A Preliminary Study on the Mechanism of Self Regulation of Sediment Transport Capacity in the Lower Yellow River) .....	670
粒径分布和细颗粒含量对两相管流水力特性的影响 (The Effect of Sediment Composition and Content of Fine Particles on the Characteristics of Two-Phase Flow in Pipes) .....	683
Reservoir Sedimentation and Slope Stability Technical and Environmental Effects .....	698
黄土丘陵沟壑区高含沙水流的形成及汇流过程 (The Formation and Process of Confluence of the Flow at Hyperconcentra- tion in the Gullied-Hilly Loess Areas of the Yellow River Basin) .....	715
阳武河灌区处理泥沙经验在低水头枢纽引水防沙中的应用 (Application of the Experiences Obtained from the Yangwu Irrigation District in Dealing with the Sediment Problem of Low-Head Diversion Works) .....	726
高含沙水流长距离稳定输送条件的分析 (An Analysis on the Conditions of Transporting Flow with Hyperconcentra- tion over a Long Distance) .....	735
The Flood and River Sedimentation Problems of the Lower Yellow River in China .....	746
阳武河灌区分水渠首及沉沙池的模型试验和原型观测 (Model and Prototype Study of the Diversion Headworks and the Sediment Settling Basin of the Yangwu Irrigation District) .....	758
泥石流运动机理的初步探讨 (A Preliminary Study on the Mechanism of Debris Flows) .....	767
高浓度泥沙悬浮液物理特性的实验研究 (Experimental Study on the Physical Properties of Sediment Suspensions at Hyperconcentration) .....	777
粗颗粒高含沙两相紊流运动规律的实验研究 (Experiment Study of Two-phase Turbulent Flow with Hyperconcentration of Coarse Particles) .....	786

层移质运动规律的实验研究 (Experimental Study of Motion of Laminated Load) .....	793
关于河流分类及成因问题的讨论 (On the Classification and Causes of Formation of Different Channel Patterns) .....	800
高含沙水流运动研究述评 (Review of the Studies of Flow Movement at Hyperconcentration of Sediment) .....	809
Changes in River Regime after the Construction of Upstream Reservoirs .....	817
A Preliminary Investigation on the Mechanism of Hyperconcentrated Flow.....	835
The Hyperconcentrated Flow in the Main Stem and Tributaries of the Yellow River .....	848
细颗粒含量对粗颗粒两相高含沙水流流动特性的影响 (The Effect of Fine Particles on the Two-Phase Flow at Hyperconcentration of Coarse Grains).....	862
1855 年铜瓦厢决口以后黄河下游历史演变过程中的若干问题 (Problems Concerning Evolution of the Lower Yellow River Subsequent to Dykebreach in 1855 at Tongwaxiang) .....	871
紊动对细泥沙浆液絮凝结构的影响 (The Effect of Turbulence on the Flocculent Structure of the Slurry of Fine Grains) .....	879
Erosion and Transportation of Sediment in the Yellow River Basin.....	889
长江三峡枢纽工程的几个泥沙问题 (On Some Sedimentation Problems of the Three Gorges Project On the Yangtze River) .....	921
钱宁著述目录.....	936

# Investigation of the Maximum Equilibrium Rate of Bed Load Movement\*

## Chapter 1 Introduction

The movement of sediment along the bed of alluvial rivers has long been one of the most perplexing and challenging problems confronting the hydraulic engineer. In the past few decades, because of the rapid expansion of activity in both engineering and geology, the importance of this problem has been gravely felt and great stimulus has been given to the search for a general solution.

Since the classic experiments of Gilbert in 1914, numerous laboratory studies of sediment transportation have been conducted throughout the United States and the continent of Europe. Many sediment transportation formulas have been derived from these studies; among them are the more rational approaches of Professors Einstein and Kalinske.

An examination of these flume studies reveals that the experimental procedures generally consist of the following steps: (a) a sand bed with a certain slope is molded in the flume, then (b) water is discharged over the bed and the equilibrium condition is obtained by reintroducing the sediment collected at the downstream traps to the flow at the upper end until the movement of detritus becomes constant. In some cases where fine sand is used, the moving particles as well as the water are circulated continuously until the average concentration of sediment in the water no longer varies. Under these conditions, the moving particles are invariably eroded bed particles, and only enough sediment is scoured from the bed to satisfy the equilibrium. The rate of transport thus determined corresponds to a minimum equilibrium rate on the given bed, if an entire range of such equilibrium rates does exist.

If it is recalled that a natural alluvial river is defined as a river whose bed is formed by the deposited material carried down by the water, it becomes evident that, except at certain localities, (e. g., such as beyond a newly built dam), the channel bed is being built up in a very gradual manner. The case is, therefore, one of slow deposition. It is conceivable that since a lower or scour limit of equilibrium transport exists, there may exist also an upper limit of equilibrium transport, above which the surplus of sediment will settle out on the bed. This maximum equilibrium rate may be different from the minimum rate for the given bed and flow. Between these two limits, the rate of transport of sediment may assume any value without changing the bed, depending on the availability for transport of the sediment from the upstream watershed. The difference between the load and the minimum equilibrium rate actually represents what has been called wash load<sup>[1,2]</sup>. The latter is defined as that part of the load which is washed through the channel without any effect on the channel. Its rate is thus governed by its availability in the watershed only.

The purpose of this study is to find experimentally under which conditions the maximum equilibrium rate differs from the minimum equilibrium rate.

---

\* 本文为钱宁(Ning Chien)就读于美国加利福尼亚大学研究生院时发表的博士论文(Ph.D. Dissertation, University of California, Berkeley, California),发表的时间为1951年6月,由H.A. Einstein 副教授、B.A. Etcheverry 教授、J.W. Johnson 副教授负责评审,原文共96页——编者注(以下各篇\*均为编者注,不另加说明)。

## Chapter 2 Minimum Equilibrium Condition and the Einstein Bed Load Function

### 2.1 Mode of Transportation

Consider, first, a bed consisting of thoroughly mixed material. When water flows over the bed, a lifting force results because of the difference between the velocities of the flow above and beneath the particles resting on the bed surface<sup>[37]</sup>. The particles are picked up from the bed when the instantaneous lifting forces on the particles are larger than their weight under water. These particles roll or slide along the bed, sometimes jump in successive steps. When the turbulence of the flow is sufficiently large, these particles may be raised to and kept in suspension at higher elevations. Their movements are arranged sometimes in such a manner that the bed forms a system of bars and hollows. The transition from a smooth bed to a system of sand bars occurs rather suddenly, and the writer has succeeded in eliminating these sand bars in one case by changing the entrance condition. This leads one to suspect that their occurrence may be connected with the pulsations of the flow.

On the other hand, the moving particles will settle back to the bed due to their own weight, and stay in the bed whenever the local flow condition is unfavorable to their moving again. Based on the statistical analysis of data from experimental studies of sediment transportation of coarse particles, Einstein<sup>[41]</sup> found that the average distance travelled by a certain particle between successive points of deposition in the bed is a constant independent of the hydraulic conditions or the composition of the bed. This distance may be assumed to be  $\lambda D$ , where  $D$  is the diameter of the particle and  $\lambda$  is a constant which depends only upon the shape of the particle; the latter has a value of 100 for sediment grain of average sphericity.

If steady conditions prevail, there will be an equilibrium between (a) the rate of deposition on, and the rate of scour from, the bed surface, and (b) the rate of upward transport due to turbulence and the rate of fall due to the weight of the particles at every elevation above the bed.

### 2.2 Segregation of Bed Material

The non-moving particles, if any exist, and the slow moving particles are accumulated in the bed during the transport if the material used is well graded and if sufficient time is allowed for the segregation to develop<sup>[57]</sup>. The top layer of the bed (layer 2) is the active layer of exchange with the load. It consists prevalently of medium and fine grains with few very coarse particles, which project not only above the rest of the bed, but also above the laminar sublayer attached to the bed surface in the case of a smooth bed. These large particles shield to some extent the smaller grains from the flow. A clearly defined coarse layer (layer 3) is accumulated below layer 2. Beneath this layer is the original material (layer 4) undisturbed by the flow. A typical set of mechanical analyses of the materials from these different layers is shown in Fig. 1\*. Plate 1 shows the surface and cross-section of such a bed.

The segregation of bed material makes the sediment study even more complicated. As will be demonstrated later the interpretation of the experiments depends to some degree on the choice of what is considered as the bed material. Theoretically speaking, layer 2 should be selected as the

---

\* 图中符号“”表示英尺(呎),下同.

bed material, since the particles in that layer are active in the exchanging roles between moving and resting. In the field, however, it is almost impossible to acquire such a thin, undisturbed sample at the bed surface over which the water is flowing continuously. The problem is complicated further when one attempts to interpret the data from flume studies with regard to the analogous situation in the river. In other words, it is a question of whether the formation of the bed layer in the flume bears the same relation to the flow as that which prevails in the river. In ordinary laboratory practice a sand bed is molded before introducing the flow; a thick bed sample will usually include some material which has never been touched by the flow (layer 4 of Fig. 1). Obviously the same relation never exists in an aggraded natural river.

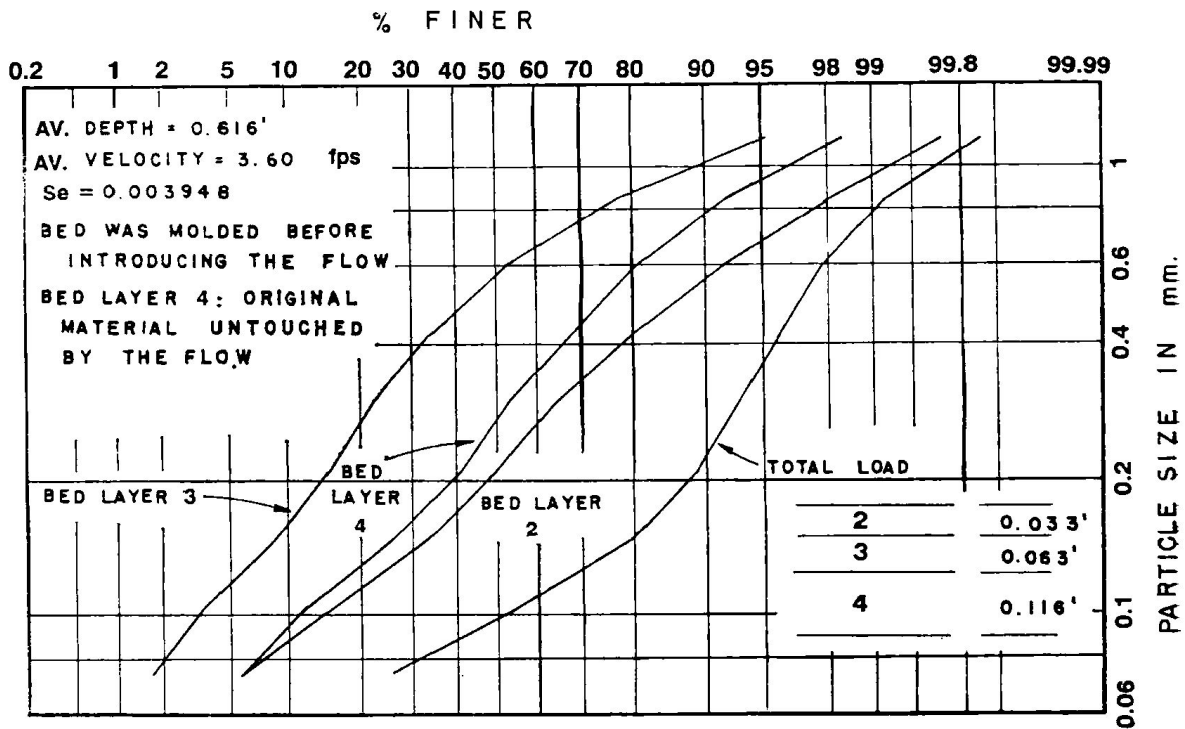


Fig.1 Segregation of bed material



Plate 1 Cross section and surface of the bed  
(Notice the segregation of the bed material)

### 2.3 The Einstein Bed Load Function

H. A. Einstein is believed to be the first one who has succeeded in presenting a complete theory which will permit the calculation of the minimum equilibrium rate at which various discharges will transport the different grain sizes of the bed material in a given channel<sup>[6]</sup>. He first concluded on the basis of observation that a given size of particle moves in a series of steps of definite length and frequency, and that the rate of transport depends upon the duration of the intermediate periods of the rest. The probability of a particle being eroded from the bed can be expressed in terms of the rate of transport and the size and relative weight of the particles. The same probability may be expressed also as the probability of the ratio of dynamic lift on the particle to the weight of the particle under water to be smaller than unity. By using the fact that the pressure fluctuations due to turbulence follow in their duration the normal error law, and by equating the two forms of probability relationships, the final bed load equation is obtained:

$$1 - \frac{1}{\sqrt{\pi}} \int_{-B_*\phi_* - \frac{1}{\eta_0}}^{B_*\phi_* - \frac{1}{\eta_0}} e^{-t^2} dt = \frac{A_*\phi_*}{1 - A_*\phi_*} \quad (1)$$

in which  $\eta_0$ ,  $A_*$  and  $B_*$  are universal constants. The equation is represented graphically by a single curve between the flow intensity  $\phi_*$  and the intensity of bed load transport  $\phi_*$ . Thus

$$\phi_* = \frac{i_B}{i_b} \phi = \frac{i_B}{i_b} \frac{q_B}{\rho_s g} \left( \frac{\rho_s}{\rho_s - \rho_f} \right)^{\frac{1}{2}} \left( \frac{1}{gD^3} \right)^{\frac{1}{2}} \quad (2)$$

$$\phi_* = \xi Y \left( \frac{\beta}{\beta_X} \right)^2 \phi = \xi Y \left[ \frac{\log 10.6}{\log (10.6X)/\Delta} \right]^2 \frac{\rho_s - \rho_f}{\rho_f} \frac{D}{R'_B S_c} \quad (3)$$

- where  $i_B$  = fraction of bed load in a given grain size  
 $i_b$  = fraction of bed material in a given grain size  
 $q_B$  = bed load rate in weight per unit of time and width  
 $\rho_s, \rho_f$  = density of the fluid and solids respectively  
 $D$  = grain size  
 $R'_B$  = hydraulic radius with respect to the grain  
 $S_c$  = energy slope  
 $\Delta$  = the apparent roughness diameter  
 $X$  = characteristic grain size of mixture  
 $Y$  = pressure correction in transition smooth-rough  
 $\xi$  = "hiding factor" of grains in a mixture

The correction factors  $\xi$  and  $Y$  are introduced for nonuniform materials; their significance requires further explanation.

The factor,  $Y$ , is used for describing the change of the lift coefficient in mixtures, and is a function of  $K_s/\delta$ , (or, clearly, of the Reynolds number of the flow at the bed surface). The length  $K_s$  is the roughness diameter and  $\delta$  is the thickness of laminar sublayer.

The factor,  $X$ , is defined as the largest particle size that will be subjected to the shielding effect. This effect comes into action whenever the particle is hidden between other protruding coarse particles or in the laminar sublayer. The lift for these particles ( $D < X$ ), then, must be corrected by division with a parameter  $\xi$  which is itself a function of  $D/X$ . The parameter,  $\xi$ , remains unity until  $D/X$  is greater than one; after a short transition the curve  $\xi = \xi(D/X)$  fol-



flows almost a straight line with a slope of 2 in a logarithmic graph (see Fig. 2). The lift force,  $L$ , may be expressed as

$$L = C_L \rho_f \frac{1}{2} u^2 A_1 D^2 \quad (4)$$

where  $C_L$  = coefficient of lift

$A_1$  = a constant which depends on the shape of the particle

$u$  = the velocity acting on the particle.

For all particles of a mixture this velocity is a constant and is calculated from the logarithmic friction formula by using  $y=0.35X$ . The same consequence will result if, instead of applying a parameter  $\xi$ , an effective velocity of flow proportional to the grain size for each particle in the mixture is introduced to replace the constant velocity of eq. (4). This seems to explain the manner in which the shielding effect of the laminar sublayer is taken into consideration by the correction factor  $\xi$ , since within the sublayer the velocity distribution follows a straight line and is proportional to the distance from the theoretical bed. It is still not clear, however, how the same parameter  $\xi$  is able to define also the shielding effect due to the protruding particles on the bed surface

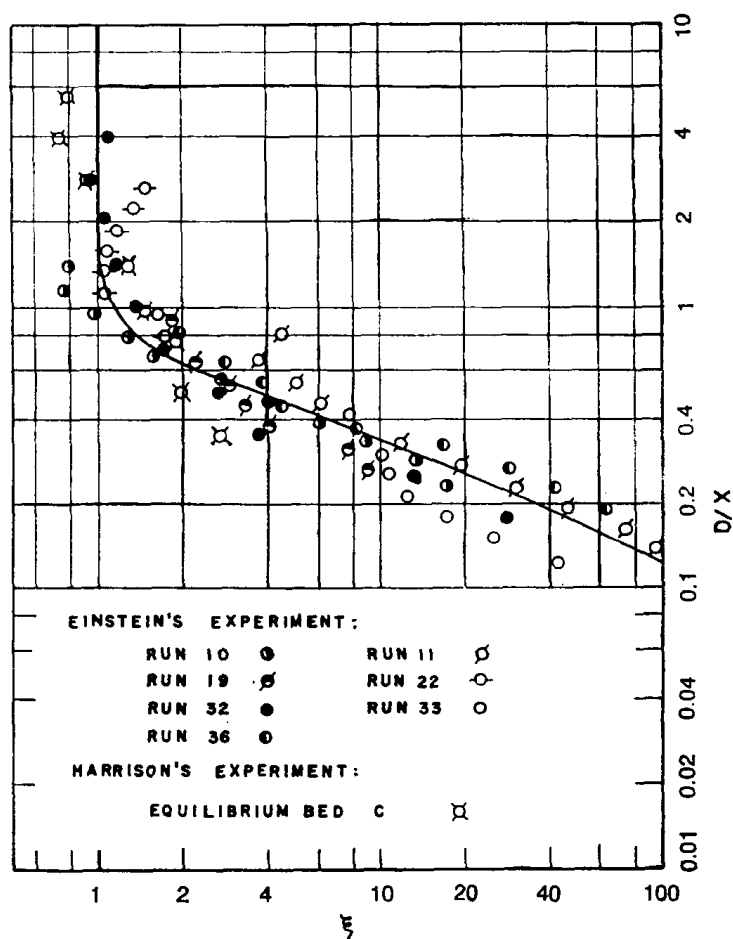


Fig. 2 Einstein's  $\xi$  curve.

The value  $X$  was found to be equal to  $0.77\Delta$  for a rough bed  $\left(\frac{\Delta}{\delta} > 1.80\right)$  and  $1.39\delta$  for