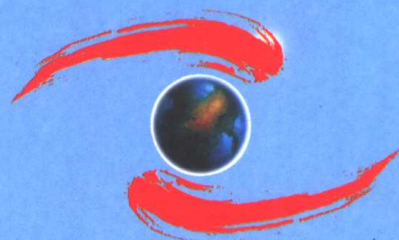


经济与金融高级研究丛书

Advanced Studies in Economics and Finance

丛书主编 邹恒甫

Editor in Chief Heng-fu Zou



经济动力学与控制

Contributions to Economic Dynamics and Control

邹至庄 著

Gregory Chow

北京大学出版社

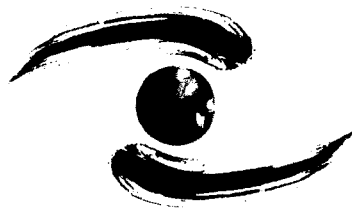
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出版前言

中国的经济学和金融学研究如何走向世界？这是一个值得探讨的问题。中国经济学者素以刻苦求知、真诚报国为荣。在国内，自改革开放以来经济学者的突出贡献已深得国人认同；在海外，中国的经济学者同样做出了可喜的成绩。但是，国内经济学者的学术成果得到国际上认可的为数寥寥，而海外中国经济学者所取得的学术成果在国内也鲜为人知。同时，国际经济学者的学术成果在国内的传播也很有限。凡此种种，原因当然是多方面的，其中之一是学术传播与交流上的障碍。这些障碍的存在造成彼不知我，我亦不知彼，国内经济学者的学术研究难以走向世界，国际经济学者和海外中国经济学者的学术研究难以走进中国这样一种尴尬的局面。不言而喻，在全球经济一体化趋势主导世界潮流的今天，这种状况不利于中国经济和中国经济学的发展。

随着改革开放的一步步深化，中国经济与世界经济日益接轨。世界各国经济学者对中国经济发展和中国经济研究的兴趣和热情有增无减。海内外中国经济学者的拳拳报国之心也日益高涨。科学无国界，学术交流也无国界。我们相信，学者们的热情与努力将冰释学术交流中的所有障碍。因此，在经济全球化的今天，在经济腾飞指日可待的中国，这套《经济与金融高级研究丛书》的出版是时代的要求，更是我们的历史使命。

本套丛书将尽可能全面地收录国际经济学者特别是中国经济学者在国际上已获得公认的学术成果。每部著作将基本保留其最初发表在国际刊物上的原貌(或其创作的原貌),由作者按研究专题编纂成书。此举一方面是为了让更多的国人了解这些学者的研究成果,或者至少感知一下国际经济学者和海内外中国经济学者在国际主流经济学发展进程中所迈出的坚实的步伐,从而激励更多的青年学子求知问道;另一方面也是为了使世界各国的经济学者对中国经济学者的研究成果有更多和更全面的了解,或者至少感知到中国的经济学研究并非固步自封置身世界之外,而是与世界同步与潮流并进的。知己知彼,互相交流,这对于繁荣学术是有百利而无一弊的。北京大学出版社真诚地希望更多的海内外学者向我们赐稿,并给我们批评、建议,以助于这项造福世人的学术文化传播事业。

北京大学出版社

经济和金融高级研究丛书

编者说明

本丛书收录世界各国经济学者特别是海内外中国经济学者从事当代经济学和金融学理论研究和实际研究的前沿成果。就某一专题或者多个专题,作者既可以把已经发表的论文收集成册,也可以编辑整理成一部或多部专著。收集成册的公开发表的论文一律保持其发表时的各刊物排版印制的原貌,以方便读者查寻援引;尚未公开发表的论文则一律保持其创作原貌,以供读者参考。

本丛书主编同时还与海内外众多学者合作主办英文学术刊物 *Annals of Economics and Finance*。此刊物出版尚未发表的至少具有一些原创性的经济学和金融学(英文)论文。如有兴趣借此刊物宣布自己学术思想的学人,敬请寄论文给:

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但愿此丛书和杂志能促进中国经济学者与世界各国经济学者的学术交流,促进中国经济学和金融学研究走向世界主流。

邹恒甫

于北京大学

Acknowledgments

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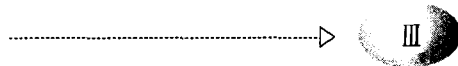
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Introduction

To introduce the papers in these two volumes, I will first describe their contents and then give some historical background of the research work behind them.

Volume 1 begins with Section I on econometrics. The first paper provides a test of equality between subsets of coefficients in two linear regressions. The test of equality of two entire sets of coefficients is later known as the “Chow test.” The paper points out that in the case of testing the entire sets of coefficients when there are sufficient observations in the second sample to estimate the regression separately, the result was known in the statistics literature. It is in the cases of having insufficient observations in the sample to estimate the second regression and of testing subsets of coefficients that the paper provides new results. In the case with known result, the paper did bring to the attention of the economics profession the importance of testing for temporal stability of economic relationships. This is a subject of active research today and probably will remain so in the future. Paper 2 provides a test to decide which of two sets of variables in two alternative multiple regressions has a better explanatory power for the same dependent variable. It generalizes a test of Hotelling applicable to the case of choosing between two variables, rather than two sets of variables, for the prediction of the same dependent variable.

Many of the papers that follow deal with the estimation of a system of simultaneous stochastic equations. Other topics are treated in paper 4 on multivariate regression, papers 7 and 13 on best linear unbiased interpolation, extrapolation and estimation of missing observations, paper 11 on multi-period predictions by Bayesian methods, 12 on regression residuals, 14, 15 and 16 on the information criterion for selecting econometric models, 17 on models with random and time-varying coefficients, 18 on the maximum likelihood estimation of misspecified models, and 19 on estimation of simultaneous equations with unit roots.

Section II of volume 1 includes empirical or theoretical studies of the US economy

covering four areas. The areas are demand for durable goods, business cycles, demand for money and the stock market as categorized in four subsections. Section III includes studies of the Chinese economy, covering economic planning in paper 32, national income determination in 33, price level determination in 34, capital formation and economic growth in 35, estimating economic effects of the Great Leap Forward Movement and the Cultural Revolution in 36, rates of return in schooling in 37, challenges of China's economic system for economic theory in 38, Hong Kong stock prices in 39 and Shanghai stock prices in 40.

Volume 2 begins with a paper which extends to the case of stochastic models the classic paper of Samuelson showing that, for a deterministic model, the interaction of a Keynesian consumption function and an investment equation based on the acceleration principle can generate complex roots and therefore oscillations in national income. It introduces spectral methods to study stochastic models and shows that the acceleration principle in the investment equation is necessary to generate complex roots in a linear stochastic model which explains a number of consumption expenditures by distributed lags of income. Paper 2 derives spectral properties for nonstationary linear stochastic systems with roots larger than unity. Paper 3 initiates the use of optimal control techniques for the determination of macroeconomic policies and obtains the optimal policy rules (also known as feedback control equations) by using the elementary method of Lagrange multipliers. A series of paper on the methods and applications of optimal control to economic policy formulation follow, from papers 4 to 17.

A new element is introduced in papers 18 to 24, with the exception of 22. It is the hypothesis of rational expectations. Up to the middle 1970s, the prevalent hypothesis on the formation of expectations was adaptive expectations. Under this hypothesis, an economic agent is assumed to change his expectation in the current period by a fraction of the difference between his expectation and the actual value of the economic variable in the last period. Instead, rational expectations assume that the psychological expectation of the economic agent equals to the conditional mathematical expectation of the variable generated by the model used by the econometrician. Which hypothesis is better under what circumstances is still an open question. This question is also discussed in papers 31 and 39 of volume 1.

There was a gap of seven years between the publication of papers 24 and 25. During these years I spent much time in promoting modern economics education in China, advising the Chinese government on economic reform and studying the Chinese

economy. Studies in the Chinese economy were reported in *The Chinese Economy* (Harper and Row, 1985; Chinese translation, Nankai University Press, 1985), *Understanding China's Economy* (World Scientific, 1994) and papers included in section III of volume 1. When my interest in optimum control was resumed in 1991 I discovered that the method of Lagrange multipliers which I applied in paper 3 to solve a control problem with a linear stochastic model and a quadratic objective function is applicable to more general objective functions and nonlinear models. This discovery was reported in papers 25 and 26. Applications of the Lagrange methods and its further development are reported in papers 27 to 30. In *Dynamic Economics: Optimization by the Lagrange Method* (Oxford University Press, 1997) I discuss the method further in comparison with the method of dynamic programming and provide applications in the study of economic growth, general equilibrium, business cycles, finance and investment. This is still one of my major research interests today. Some of the topics in econometrics and dynamic optimization which have interested me and some of their applications to the formulation and analysis of economic policy in which I have participated are discussed in a paper "Econometrics and Economic Policy," forthcoming in *Statistica Sinica*, 2001.

On historical background, I entered Lingnan University in Guangzhou, China in 1947 majoring in political science at the suggestion of my mother. China did not have a stable government at the time and my mother thought it needed good people to serve in the government. In 1948 I transferred to Cornell University still majoring in political science. In 1949 the Chinese government changed hands and I decided to stay in the United States. A degree in political science was not very useful for me in the United States. Several economics teachers including Alfred Kahn stimulated my interest in economics and I graduated with a double major in both subjects. In 1950 I became interested in mathematical and quantitative analysis of economic problems. With very limited knowledge I tried to read one or two mathematically simplest papers in Econometrica and joined the Econometrics Society in 1950 when its membership was about 80.

The University of Chicago was the obvious place for my graduate study in 1951 because it had the Cowles Commission for Research in Economics and it was the only university offering a graduate course entitled "econometrics." At Chicago I was extremely fortunate to learn from its very distinguished faculty, including Gerald Debreu, Milton Friedman, Arnold Harberger, Clifford Hildreth, Hendrick Houthakker,

D. Gale Johnson, Frank Knight, Tjalling Koopmans, William Kruskal, Jacob Marschak, L. Jimmie Savage, Theodore W. Schultz, George Tolley and W. Alan Wallis. My fellow graduate students included Gary Becker, Zvi Griliches, Marc Nerlove, Roy Radner and Lester Telser, among others.

My PhD thesis deals with a theory of demand for durable goods and its application to the demand for automobiles in the United States. My thesis committee consisted of A. C. Harberger (chairman), Jacob Marschak and Alan Wallis. Milton Friedman provided valuable comments when drafts of the thesis were presented before his money workshop. The thesis was published as *Demand for Automobiles in the United States: A Study in Consumer Durables* (Amsterdam: North Holland Publishing Company, 1957). In 1958, Harberger decided to publish several Chicago theses on the demand for durable goods. Since mine was already published I wrote paper 20 of volume 1 to update my thesis and to test whether the demand function previously estimated remained valid for the 4 years 1954 – 1957. The method employed was published in paper 1 of the first volume. Thus my interest in econometric method was stimulated by the need to solve an applied problem. The second paper of volume 1 was also motivated by an applied problem of which a special case had already been treated in my thesis. Once the interest in econometrics was stimulated, a series of papers from 3 to 19 followed through the years.

My interest in economic dynamics and optimal control was also motivated by applied problems. Having estimated a small econometric model for the US reported in paper 22 of volume 1, I wished to study its dynamic properties. This led to papers 1 and 2 of volume 2. The interest in using this model to formulate economic policies led to the methodological paper 3. Then followed a series of papers from number 4 on. Dynamic optimization and its applications remain my current research interest.

I have been very fortunate to know how to do research in economics. Research has given me much intellectual challenge and enjoyment. It gives me great joy to share some of the results with the readers of these two volumes. I would like to express my sincere thanks to Zou Heng-fu who decided to publish these papers in two volumes and has spent his valuable time and effort in guiding their publication.

Gregory Chow
Princeton University
October 19, 2000

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THE ACCELERATION PRINCIPLE AND THE NATURE OF BUSINESS CYCLES *

GREGORY C. CHOW

I. Introduction, 403.—II. Demand equations and evidence for acceleration, 404.—III. The nonstochastic case: no oscillations without acceleration, 406.—IV. Autocovariances of a linear stochastic system, 408.—V. Spectral densities of a linear stochastic system, 411.—VI. Conclusions, 417.

I. INTRODUCTION

Half a century ago, J. M. Clark stressed the importance of the acceleration principle in business cycle theory.¹ Almost thirty years ago P. A. Samuelson pointed out that the acceleration relation can interact with the multiplier to generate oscillations in national income.² This paper presents a formulation of the acceleration principle and some strong supporting evidence. It attempts to ascertain whether the acceleration principle is *necessary* for the generation of oscillations in national output. It also extends the discussion of the nature of economic fluctuations from the non-stochastic to the stochastic case.

Recently I constructed a simple macroeconomic model of the United States economy in the form of a system of linear stochastic difference equations in which the acceleration principle plays an important role.³ The present paper is partly the outcome of an analytical study of the dynamic properties of such a linear stochastic system. The tools employed are the autocovariances and the spectral densities of the time series so generated. The presentation of results on these matters will be elementary and self-contained.

* A preliminary version of this paper was presented before the Joint Faculty Colloquium on Mathematical Economics and Econometrics of Columbia, Johns Hopkins, University of Pennsylvania and Princeton, held at Columbia University in May 1966. Parts of this paper were written while the author was visiting the Harvard Economics Department and were presented to the Research Seminar on Quantitative and Mathematical Economics there, and to the Econometrics Seminar at MIT in May 1967. All comments are gratefully acknowledged. The author has also benefited greatly from the suggestions of T. W. Anderson, Alan J. Hoffman, and Richard E. Levitan.

1. J. M. Clark, "Business Acceleration and the Law of Demand: A Technical Factor in Economic Cycles," *Journal of Political Economy*, XXV (Mar. 1917), 217-35.

2. P. A. Samuelson, "Interactions between the Multiplier Analysis and the Principle of Acceleration," *Review of Economic Statistics*, XXI (May 1939), 75-78.

3. G. C. Chow, "Multiplier, Accelerator, and Liquidity Preference in the Determination of National Income in the United States," *Review of Economics and Statistics*, XLIX (Feb. 1967), 1-15.

This paper begins with a formulation of the acceleration principle and its supporting evidence in Section II. Section III shows that, without an acceleration equation, a system of nonstochastic linear demand equations for the components of national output cannot produce oscillations in the variables. This amounts to showing that the characteristic roots of the system are all real and positive. The discussion is generalized to allow for random disturbances in the equations. In Sections IV and V, respectively, the autocovariance matrix and the spectral density matrix will be derived in an elementary fashion. It is shown that random shocks, when introduced into an otherwise nonoscillatory system, can generate cycles. Thus the acceleration principle is no longer necessary. While Sections IV and V are confined to the stationary case (with all roots less than one in absolute value), Section VI contains brief comments on the nonstationary case as well as some concluding remarks on the nature of business cycles.

II. DEMAND EQUATIONS AND EVIDENCE FOR ACCELERATION

To ascertain whether a system of nonstochastic economic equations can produce oscillations without acceleration, one obviously has to specify the nature of the equations. It seems relevant to restrict the equations in a macroeconomic model to demand equations for different components y_t of national product $Y = \sum y_t$, which obey simple distributed lags:⁴

$$(1) \quad y_{it} = a_i Y_t + b_i y_{i,t-1}.$$

Of course, demand equations for investment goods and for some consumer durables are not of this type because of acceleration, but our purpose is precisely to find out whether there could be oscillations if all demand equations were of the form (1). In other words, while Samuelson⁵ demonstrated that the interaction of the multiplier and the accelerator can produce oscillations, we ask whether the latter is necessary, in the context of a system of equations.

For investment goods and possibly consumer durables, the demand functions for *total stock* s_i are assumed to be of the same form:

$$(2) \quad s_{it} = a_i Y_t + b_i s_{i,t-1}.$$

The flow of net investment y_{it}^* is $\Delta s_{it} = s_{it} - s_{i,t-1}$:

4. The intercept will be omitted, as exogenous variables are omitted in this paper which treats business cycles as the result of interactions of endogenous forces, holding the exogenous variables constant.

5. Samuelson, *op. cit.*