

环的同调维数

程耀长 葛 志 著



· 理科现代科技研究丛书 ·

环的同调维数

程福长 著
易 忠

广西师范大学出版社

图书在版编目(CIP)数据

环的同调维数 / 程福长, 易忠著. — 桂林: 广西师范大学出版社, 2000. 10

(理科现代科技研究丛书)

ISBN 7-5633-2909-9

I . 环… II . ①程… ②易… III . 环-同调维数
IV . 0153.3

中国版本图书馆 CIP 数据核字(1999)第 42029 号

广西师范大学出版社出版发行

(桂林市中华路 36 号 邮政编码: 541001)
(电子信箱: pressz@public. glptt. gx. cn)

出版人: 萧启明

全国新华书店经销

广西地质印刷厂印刷

(广西南宁市建政东路 邮政编码: 530023)

开本: 890 mm × 1 240 mm 1/32

印张: 9.875 字数: 284 千字

2000 年 10 月第 1 版 2000 年 10 月第 1 次印刷

印数: 0 001 ~ 1 000 定价: 18.00 元

如发现印装质量问题, 影响阅读, 请与印刷厂联系调换。

序 言

环的同调维数是近代环论一个重要研究领域,特别是80年代以来,关于凝聚环和非交换 Noether 环的同调维数的理论研究,极大地丰富和发展了环的同调维数的经典结果,它的理论和方法影响到代数学和其他数学学科.

本书在假定读者已经熟悉抽象代数和同调代数基本知识的基础上进一步叙述环的同调维数理论,包括这一领域的许多新的结果.第一章主要介绍环和模的基本知识.第二章介绍一般环和模的经典的同调维数,还包括了 Ng 维数.这两章有些内容仅给出结论以及相关文献,不再详细叙述.第三章专门介绍 Noether 环(不一定是交换的)的同调维数.第四章主要把 Noether 环一些经典的著名结果推广到凝聚环,还讨论了凝聚 FP-环和 $(a, 2, b)$ -环的结构性质和分类问题.第五章讨论了 π -凝聚环和 FGT-维数.第六章主要把局部环的结果推广到半局部环.第七章叙述了对偶模的同调特征.第八章主要刻划群环、斜群环、交叉积和群分次环的同调维数.第八章是由易忠教授撰写的.

本书可供数学专业的研究生和数学工作者参考,也可作为代数方向二年级研究生的教材.

本书在编写时,收集了有关的文献资料,参考了一些文献的结果,特向这些文献的作者致以诚挚的谢意.我还要感谢我的导师[张禾瑞]教授、周伯坝教授、刘绍学教授和王世强教授,是他们引导我学习环论和同调代数.本书的出版获得广西自然科学基金资助,还得到广西师范大学出版社的赞助和鼎力支持,余鑫晖教授一直关心此书的出版,研究生赵巨涛、黄寄洪和张积成在学习上述讲义的过程中提出了许多有益建议,也在此表示感谢.

由于水平有限,书中一定会有不足之处,敬请读者批评指正.

程 福 长

1998 年 2 月于广西师范大学

HOMOLOGICAL DIMENSION OF RINGS

Cheng Fuchang
Yi Zhong

ABSTRACT

The theory and application of homological dimension of rings are presented systematically in this book. There are 8 chapters. Its contents are as following: rings and modules, homological dimension, modules and their homological dimension over Noetherian rings, homological dimension of Coherent rings, π -Coherent rings and FGT-dimension, modules and their homological dimension over semi-local rings, homological properties of dual modules, homological dimension of group rings, skew group rings, crossed products and group graded rings.

This is a reference book for mathematicians and research students who have studied Abstract Algebra and have some elementary knowledge about Homological Algebra.

PREFACE

Homological dimension of rings is one of the main research field in ring theory. Since 1980's the research about non-commutative Noetherian rings and coherent rings largely developed and extended the classical results about homological dimensions of rings. The new results and methods have been applied to other mathematics subjects and other Algebra branches.

In this book we focus on homological dimensions of rings and we suppose that the readers are familiar with Abstract Algebra and have some knowledge about Homological Algebra. Many new results are included in this book. Chapter one is an introduction about rings and modules. Chapter two states many classical results about homological dimensions of rings and modules, and the Ng dimension is also studied. In these two chapters, for many theorems we state the results and omit the proofs, but relevant references are pointed out. Chapter three is specifically for homological dimensions of Noetherian rings. In chapter four we generalize some classical results about Noetherian rings to coherent rings, coherent FP-rings are discussed and the structure and classification of $(a, 2, b)$ -rings are studied too. In chapter five π -coherent rings and FGT-dimension are studied. Chapter six is designed to generalize the results of local rings to semilocal rings. Chapter seven is about the homological character of dual modules. Chapter eight is devoted to the study of homological dimensions of group rings, skew group rings, crossed products and group graded rings. This chapter is written by Professor Yi Zhong.

This book is a reference book for mathematicians and mathematics students. It can also be used as a textbook for research students in their second year.

In this book we collected many new results and referred to many books and papers. We are very grateful to the original authors. I would also like to express my gratitude to my supervisors, Professors Zhang Herui, Zhou Boxun, Liu Shaoxue and Wang Shiqiang. It is them who guided me to study ring theory and homological Algebra. I also hope to thank Guangxi Teachers' University Press and Guangxi Natural Science Foundation for their support for the publication of this book. I am also indebted to Professor Yu Xinhui, who has been of great assistance in the production of this book, and to Zhao Juntao, Huang Jihong, Zhang Jicheng, who have used parts of the earlier text in a course of a term, and have called my attention to many places where improvements in the exposition could be made.

Finally, please excuse the shortcomings or errors appeared in this book. I would be glad to hear some suggestions and comments.

Cheng Fuchang

Guangxi Teachers University, Guilin

February, 1998.

目 录

第一章 环和模	(1)
1.1 投射模和生成元, 内射模和余生成元	(1)
1.2 平坦模	(9)
1.3 凝聚环	(18)
1.4 半遗传环、遗传环、Von Neumann 正则环	(26)
1.5 半单环	(35)
1.6 局部环和半局部环	(41)
1.7 半完全环和完全环	(51)
第二章 同调维数	(63)
2.1 模的投射维数和内射维数	(63)
2.2 模的平坦维数	(70)
2.3 环的整体维数和弱整体维数	(76)
2.4 Ng 维数	(82)
2.5 FP -内射维数和 f.p.gl.dim 维数	(90)
第三章 Noether 环上的模及其同调维数	(103)
3.1 Noether 环上的模	(103)
3.2 Noether 环的整体维数	(107)
3.3 Noether 拟局部环上的模	(110)
3.4 余维数	(114)
3.5 正则局部环	(119)
第四章 凝聚环的同调维数	(126)
4.1 凝聚环上的模	(126)
4.2 凝聚拟局部环上的模	(133)
4.3 凝聚局部环的余维数	(137)

4.4	凝聚 GCD 整环的同调特征	(141)
4.5	凝聚 FP-环的结构	(149)
4.6	$(a, 2, b)$ -FP 环的分类	(151)
4.7	$\text{Ng. dim } \Lambda = 2$ 的凝聚环	(156)
第五章	π -凝聚环和 FGT-维数	(160)
5.1	模范畴的等价性和对偶性	(160)
5.2	π -凝聚环的定义及基本性质	(165)
5.3	π -凝聚环上的模	(169)
5.4	FGT-投射维数	(174)
5.5	FGT-内射维数	(179)
5.6	FGT-平坦维数	(187)
第六章	半局部环上的模及其同调性质	(199)
6.1	Noether 半局部环的同调性质	(199)
6.2	半局部环的全维数	(207)
6.3	凝聚半局部环的同调维数	(212)
6.4	不可分凝聚半局部环	(219)
6.5	半完全环和完全环的同调维数	(223)
6.6	弱半局部环	(228)
第七章	对偶模的同调性质	(233)
7.1	自反模与环的自内射维数	(233)
7.2	对偶模的同调维数	(238)
7.3	特殊模的对偶模	(242)
7.4	半局部环上 G-Matlis 对偶模	(249)
7.5	关于内射余生成元的对偶模	(255)
第八章	群环、斜群环、交叉积和群分次环的同调维数	(262)
8.1	基本概念和基本结果	(263)
8.2	群环的同调维数	(267)
8.3	群分次环的同调维数	(273)
8.4	交叉积和斜群环的同调维数	(284)
	参考文献	(296)

CONTENTS

Chapter 1	Rings and Modules	(1)
1.1	Projective modules and generators, injective modules and cogenerators	(1)
1.2	Flat modules	(9)
1.3	Coherent rings	(18)
1.4	Semihereditary rings, hereditary rings and Von Neumann regular rings	(26)
1.5	Semisimple rings	(35)
1.6	Local rings and semilocal rings	(41)
1.7	Semi-perfect rings and perfect rings	(51)
Chapter 2	Homological Dimensions	(63)
2.1	Projective dimensions and injective dimensions of modules	(63)
2.2	Flat dimensions of modules	(70)
2.3	Global dimensions and weak global dimensions of modules	(76)
2.4	Ng dimensions	(82)
2.5	FP-injective dimensions and f. p. global dimensions	(90)
Chapter 3	Modules and homological dimensions over Noetherian rings	(103)
3.1	Modules over Noetherian rings	(103)
3.2	Global dimensions of Noetherian rings	(107)
3.3	Modules over Noetherian quasi-local rings	(110)

3.4	Codimensions	(114)
3.5	Regular local rings	(119)
Chapter 4 Homological Dimensions of Coherent Rings		(126)
4.1	Modules over coherent rings	(126)
4.2	Modules over coherent quasi-local rings	(133)
4.3	Codimensions of coherent local rings	(137)
4.4	Homological character of coherent GCD rings	(141)
4.5	Structure of coherent FP-rings	(149)
4.6	Classification of (a, 2, b) -FP rings	(151)
4.7	Coherent rings of Ng. dimension two	(156)
Chapter 5 π -Coherent Rings and FGT-Dimensions		(160)
5.1	Equivalence and duality of module categories	(160)
5.2	Definitions and basic properties of π -coherent rings	(165)
5.3	Modules over π -coherent rings	(169)
5.4	FGT-projective dimensions	(174)
5.5	FGT-injective dimensions	(179)
5.6	FGT-flat dimensions	(187)
Chapter 6 Modules and Their Homological Properties over Semi-local Rings		(199)
6.1	Homological properties of Noetherian semi-local rings	(199)
6.2	Total dimensions of semi-local rings	(207)
6.3	Homological dimensions of coherent semi-local rings	(212)
6.4	Indecomposable coherent semi-local rings	(219)

6.5	Homological dimensions of semi-perfect and perfect rings	(223)
6.6	Weak semi-local rings	(228)
Chapter 7	Homological Properties of Dual Modules	(233)
7.1	Reflexive modules and self-injective dimensions of rings	(233)
7.2	Homological dimensions of dual modules	(238)
7.3	The dual modules of some specific modules	(242)
7.4	G-Matlis dual modules over semi-local rings	(249)
7.5	Dual modules of injective cogenerators	(255)
Chapter 8	Homological Dimensions of Group Rings, Skew Group Rings, Crossed Products and Group Graded Rings	(262)
8.1	Concepts and elementary results	(263)
8.2	Homological dimensions of group rings	(267)
8.3	Homological dimensions of group graded rings	(273)
8.4	Homological dimensions of crossed products and skew group rings	(284)
References		(296)

第一章

环和模

在这一章里,我们主要介绍环和模的基本概念,如投射模、内射模、平坦模、凝聚环、半遗传环与遗传环、Von Neumann 正则环、半单环、局部环和半局部环、半完全环和完全环等重要环类.我们假定读者具有抽象代数和同调代数的基本知识,在此前提下,有些内容仅给出结论以及相关的文献,不再详细证明.

这一章所指的环均是有单位元 1 的结合环,所有的模为酉模. f, g 表示有限生成, $\mathcal{C}_A^L$ 和 $\mathcal{C}_A^R$ 分别表示左 A -模的范畴和右 A -模的范畴.

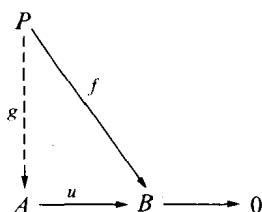
1.1 投射模和生成元,内射模和余生成元

投射模和内射模是模论及同调代数中最重要的两个基本概念.

定义 设 A 是环, P 是左 A -模,若函子 $\text{Hom}_A(P, -)$ 正合,则称 P 为投射模.

定理 1.1.1 设 A 是环, P 是左 A -模,则下列陈述是等价的:

- (i) P 是投射模;
 - (ii) 对 $\mathcal{C}_A^L$ 内任一如下的图,其中行正合,则存在一个 A -同态 $g: P \rightarrow A$,使下图交换,即 $f = ug$;
 - (iii) 对每个 A -满同态 $u: A \rightarrow B$,则 $\text{Hom}_A(P, u): \text{Hom}_A(P, A) \rightarrow \text{Hom}_A(P, B)$ 是满的;
 - (iv) 在 $\mathcal{C}_A^L$ 内任何短正合列
- $$0 \longrightarrow A \longrightarrow B \xrightarrow{u} P \longrightarrow 0$$
- 是分裂正合的;



(v) P 是自由模的直和项;

(vi) $\text{Ext}_\Lambda^1(P, A) = 0, \forall$ 左 Λ -模 A ;

(vii) $\text{Ext}_\Lambda^n(P, A) = 0, \forall$ 左 Λ -模 A 和任意整数 $n > 0$.

定理 1.1.1 的证明见文献[75].

由定理 1.1.1 得以下的推论.

推论 1.1.2 一个左 Λ -模 P 是 f, g -投射模当且仅当 $P \oplus P' \cong \Lambda^{(n)}$, 这里 n 是一个正整数, P' 是一个左 Λ -模.

投射模具有以下性质:

左 Λ -模 $A = \bigoplus_{i \in I} A_i$ 是投射模当且仅当每个 A_i 是投射模. 每个左 Λ -模必是投射模的同态象.

引理 1.1.3 (Schanuel) 设左 Λ -模序列

$$0 \longrightarrow A_1 \xrightarrow{f_1} P_1 \xrightarrow{g_1} B \longrightarrow 0 \text{ 和 } 0 \longrightarrow A_2 \xrightarrow{f_2} P_2 \xrightarrow{g_2} B \longrightarrow 0$$

正合, 这里 P_1, P_2 是投射模, 则 $A_1 \oplus P_2 \cong A_2 \oplus P_1$.

证明 考虑下图

$$\begin{array}{ccccccc} 0 & \longrightarrow & A_1 & \xrightarrow{f_1} & P_1 & \xrightarrow{g_1} & B \longrightarrow 0 \\ & & \downarrow \beta & & \downarrow \alpha & & \downarrow 1_B \\ 0 & \longrightarrow & A_2 & \xrightarrow{f_2} & P_2 & \xrightarrow{g_2} & B \longrightarrow 0 \end{array}$$

其中 1_B 是恒等映射. 因 P_1 是投射模, 故由定理 1.1.1 知, 存在 Λ -同态 $\alpha: P_1 \rightarrow P_2$, 使上图右边方块是交换的. 令 $\beta: A_1 \rightarrow A_2$, 使得 $\beta(a_1) = a_2$, 这里 $a_1 \in A_1, a_2 \in A_2$ 且 $f_2(a_2) = (\alpha f_1)(a_1)$. 若 $a'_2 \in A_2$, 适合 $(\alpha f_1)(a_1) = f_2(a'_2)$, 则得 $f_2(a_2 - a'_2) = 0$, 但 f_2 是单同态, 故 $a_2 = a'_2$. 因此 β 是一个映射. 不难验证, β 是一个 Λ -同态, 并且使得上图左

边方块是交换的. 对任意 $a_1 \in A_1$ 和 $(a_2, p_1) \in A_2 \oplus P_1$, 令 $\varphi: a_1 \mapsto \varphi(a_1) = (\beta(a_1), f_1(a_1))$ 和 $\psi: (a_2, p_1) \mapsto \psi(a_2, p_1) = \alpha(p_1) - f_2(a_2)$, 易知 φ 和 ψ 是 Δ -同态. 若 $\varphi(a_1) = 0$, 则 $f_1(a_1) = 0$. 但 f_1 是单同态, 故 $a_1 = 0$, 亦即 φ 是单同态. 其次, 若 $p_2 \in P_2$, 则存在 $p'_1 \in P_1$, 使得 $g_1(p'_1) = g_2(p_2)$. 因而 $g_2(\alpha(p'_1) - p_2) = 0$. 于是存在 $a'_2 \in A_2$, 满足 $f_2(a'_2) = \alpha(p'_1) - p_2$, 且 $\psi(a'_2, p'_1) = p_2$. 故 ψ 是满同态. 再证 $\ker \psi = \text{Im } \varphi$. 若 $(a_2, p_1) \in \text{Ker } \psi$, 则 $\alpha(p_1) - f_2(a_2) = 0$. 故 $g_1(p_1) = g_2\alpha(p_1) = g_2f_2(a_2) = 0$. 因此存在 $a_1 \in A$, 使得 $f_1(a_1) = p_1$, 且 $\varphi(a_1) = (a_2, p_1)$. 于是得 $\text{Ker } \psi \subseteq \text{Im } \varphi$. $\text{Im } \varphi \subseteq \text{Ker } \psi$ 是显然的, 故 $\text{Ker } \psi = \text{Im } \varphi$.

综合以上讨论, 序列 $0 \longrightarrow A_1 \xrightarrow{\varphi} A_2 \oplus P_1 \xrightarrow{\psi} P_2 \longrightarrow 0$ 正合. 因 P_2 是投射模, 故由定理 1.1.1 得, $A_2 \oplus P_1 \cong A_1 \oplus P_2$.

定义 设 A 是左 Δ -模, B 是 A 的一个子模, 若对任意子模 C , 当 $B + C = A \Rightarrow C = A$,

则称 B 为 A 的多余子模, 记作 $B \ll A$.

设 $g: A \rightarrow A'$ 是 Δ -满同态, 若 $\text{Ker } g \ll A$, 则称 g 是多余的.

定义 设 A 是左 Δ -模, 则 A 的投射复盖是指一个对 (P, ψ) , 它满足以下条件:

- (i) P 是一个左 Δ -投射模;
- (ii) $\psi: P \rightarrow A$ 是多余 Δ -满同态.

一个模不一定有投射复盖. 若一个模有投射复盖, 则在同构意义下是自然唯一的. 更一般地, 我们有以下结论:

设 $\psi: P \rightarrow A$ 是 Δ -模 A 的一个投射复盖. 若 Q 是 Δ -投射模, 且 $\varphi: Q \rightarrow A$ 是 Δ -满同态, 则 $Q = P' \oplus P''$, 使得

- (i) $P' \cong P$;
- (ii) P'' 是 $\text{Ker } \varphi$ 的子模;
- (iii) $(\varphi|P'): P' \rightarrow A$ 是 A 的投射复盖.

并且, 若 $f: A_1 \rightarrow A_2$ 是 Δ -同构, 而 $\psi_1: P_1 \rightarrow A_1$ 和 $\psi_2: P_2 \rightarrow A_2$ 是投射复盖, 则有 Δ -同构 $\bar{f}: P_1 \rightarrow P_2$, 使得 $\psi_2 \bar{f} = f \psi_1$.

内射模是投射模的对偶概念.