模糊数学及其应用丛书

模糊随机过程论

THE THEORY OF FUZZY STOCHASTIC PROCESSES

王光远 张 跃 著

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国家自然科学基金资助项目
中国煤炭科学基金资助项目

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黔新登（90）03号

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贵州科技出版社出版发行
（贵阳市中华北路 289 号　邮政编码 550001）
*
核工业中南 306 印刷厂印刷　贵州省新华书店经销
787×1092 毫米　32 开本　13.75 印张　290 千字
1994 年 9 月第 1 版　1994 年 9 月第 1 次印刷
印数：1—2000

ISBN 7－80584－293－0/O・010　定价：20.70 元

《模糊数学及其应用丛书》
资 助 单 位

贵州科技出版社
海南省粮油贸易公司
北京爱普亚太电子有限公司
中国人民解放军国防科学技术大学
贵州师范大学
中外合资贵州永兴电子仪表公司
贵阳大光五金站 吴石川
西南交通大学智能控制开发中心

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前　言

自从美国扎德(L. A. Zadeh)教授于1965年建立模糊集合论以来，由于它在处理广泛存在的一种不确定性——模糊性方面的成功，它在处理复杂系统方面的简捷与有力，在某种程度上弥补了经典数学与统计数学的不足，越来越受到欢迎。在这种背景下，随着模糊工程的开发和应用，模糊技术产品的广泛利用，日本于1990年将本田(Honda)奖授予了扎德教授，以表彰这一新方法论的成功。

20多年来，这一新的数学方法从理论到应用，从软技术到硬技术，都有了很大的发展，得到了越来越多的人的关心和支持，他们迫切希望了解这一新方法的研究与进展。在贵州科技出版社等单位的大力支持下，国际模糊系统协会中国分会(China Chapter of IFSA)和全国模糊数学与模糊系统学会组织编辑了《模糊数学及其应用丛书》。

这套丛书选编了一批学术性较强、应用性较好的模糊数学及其应用的专著，这些专著基本上反映了当前国际和国内水平。这些专著均是执笔者多年研究的成果，反映了当前国际同行的动态，其中多数属国家自然科学基金资助项目和国家863高技术计划项目。

我们相信这套丛书的出版，将对国内外模糊数学及其应用的研究与发展起到很好的推动作用。

刘应明

1991. 9

内容提要

本书用测度论的观点，系统地论述了作者近年来在模糊随机变量理论和模糊随机过程论方面的研究成果。在前四章中，为了建立模糊随机变量理论和模糊随机过程论的需要，分别介绍了研究思想、复模糊集合与复模糊数、模糊复分析以及模糊微分方程的基本内容，后六章分别研究了模糊随机变量理论、模糊随机过程的一般理论、正态模糊随机过程、平稳模糊随机过程、模糊随机系统和模糊随机微分方程基本理论。

读者对象为高等院校学生、研究生、教师以及有关专业科技工作者。

THE THEORY OF FUZZY STOCHASTIC PROCESSES

Abstract

Fuzzy stochastic phenomena as well as fuzzy random vibration engineering problems exist widely in reality, and far exceed the field that the classical probability methods can be applied. Therefore, it is a urgent subjects to establish the theory describing the developing procedure of fuzzy stochastic phenomena and construct the effective methods solving the problems of fuzzy random systems.

This present book is an introduction to the theory of complex fuzzy sets, fuzzy complex analysis, fuzzy differential equations, the theory of fuzzy random variables, the general theory of fuzzy stochastic processes, normal fuzzy stochastic processes, stationary fuzzy stochastic differential equations. Below is a brief outline of its contents.

In Chapter 2, the definitions of fuzzy complex set and fuzzy complex number are given, and some properties of them are discussed. It is the first time that the basic concepts of complex fuzzy set and complex fuzzy number are introduced, and closure under the basic operations of complex fuzzy sets, complex fuzzy numbers and that their basic operations may be performed in terms of a level complex sets are proved. The decomposition theorem and representation theorem of complex

fuzzy sets proved successfully in the chapter constructs a theoretical bridge between complex fuzzy sets and classical complex sets. This chapter investigates the representation forms of the extension and multi-variable extension of complex fuzzy sets, and provides a general method for extending non-fuzzy mathematical concepts to treat complex fuzzy quantity. Some fundamental properties of four arithmetic operations, intersection and union operation, supremum, infimum as well as superior limit, inferior limit and limit operations of bounded closcd complex fuzzy numbers are studied.

In Chapter 3, the methods of fuzzy complex analysis is brought forward. The concepts of dynamic fuzzy set and fuzzy set-valued function are extended to dynamic complex fuzzy set and complex fuzzy set-valued function, in addition, some properties of them are discussed. On the basis of extending the concepts of the derived function, primitive function and integration of interval-valued function to complex interval-valued function and using the decomposition theorem of complex fuzzy set, the derived function, primitive function and integration of complex fuzzy set-valed function and some properties of them are proved. The fuzzy complex analysis method advanced by the book provides a effective tool for establishing the theories of fuzzy stochastic processes, fuzzy stochastic dynamic systems and fuzzy stochastic systems.

In Chapter 4, time domain and frequency domain solving processes for fuzzy differential equations are advanced. The solving precesses of time domain and frequency domain for n-order linear fuzzy differential equations are given. The solving processes of time domain and frequency domain for system of one order non-homogeneous fuzzy differential equations are brought forward. The works in the chapter provides a indispensable mathematical tool for setting up the response analysis theory of dynamic systems to fuzzy interference or fuzzy stochastic interference and fuzzy differential equations.

In Chapter 5, the theory of complex fuzzy random variables are constructed. From the point of view of measure theory, this chapter gives the unified definition of measurable real (or complex) fuzzy set-valued function and n-dimensional measurable real (or complex) fuzzy set-valued function, and discusses the fundamental properties of them and their inverse fuzzy set-valued function by combining with fuzzy σ-algebra. By regarding fuzzy random variables as the particular cases of measurable fuzzy set-valued functions and systematizing the achieved results, opening further up the study, the chapter introduces the concepts of complex fuzzy random variable, the distribution function and fall-shadow distribution function of complex fuzzy random variable, and investigates their fundamental properties. The basic properties and computational methods of fuzzy probability characteristic such as the expectation and variance of real (or complex) fuzzy random variables are studies, and some properties of the independence and analysis operations of real (or complex) fuzzy random variables are discussed. The concepts of expectation, covariance and independence of real (or complex) fuzzy random vectors are put forward, and their properties are proved. As metioned above, the construction of complex fuzzy random variable theory lays a foundation for the further research on fuzzy stochastic processes.

In Chapter 6, the theory of fuzzy stochastic processes is founded. Based on real (or complex) fuzzy random variables and vectors, the chapter introduces the concepts of real (or complex) fuzzy stochastic function and n-dimensional joint distribution function, joint fall-shadow distribution function of real (or complex) fuzzy stochastic function as well as real (or complex) fuzzy stochastic process vector and discusses their properties. Some fundamental properties of fuzzy probability characterictic, such as the fuzzy mean value function, fuzzy covariance function and fuzzy variance function of fuzzy stochastic processes are proved. Solving the basic problems of fuzzy stochastic processes successfully by introducing the fundamental concepts of the separability and measurability as well as the continuity of fuzzy sample functions of fuzzy stochastic processes, the chapter founds the general theory of fuzzy stochastic processes.

In Chapter 7, 8, the applications of normal fuzzy stochastic processes and stationary fuzzy stochastic processes to engineering are very attractive, these two kinds of fuzzy stochastic processes and their properties are studies, and the spectral decomposition theory of fuzzy correlation functions of stationary fuzzy stochastic processes are advanced and that normal fuzzy stochastic processes can be determined by their two order fuzzy statistical quantity and their linear operations are closure is proved.

The setting up of the theory of fuzzy stochastic proceses in Chapter 6, 7, 8 provides the theoretical basis for going further into fuzzy stochastic systems and fuzzy random systems and fuzzy stochastic systems.

In Chapter 9, the theoretical frame of fuzzy stochastic system is constructed. The common characteristic and connection of deterministic system, stochastic system and fuzzy system are discussed and analysed. The general definition of fuzzy stochastic system is given and that deterministic system, stochastic system and fuzzy system are the particular cases of fuzzy stochastic system are proved. The theory of discrete fuzzy stochastic system is set up. The response analysis theory of system input fuzzy stochastic processes and the analysis theory of controlled fuzzy stochastic systems are put forward. The basic problems such as the state space models of fuzzy stochastic systems and the relations of the input and output of fuzzy stochastic state space are researched. The theory of fuzzy stochastic system is the theoretical foundation of the fuzzy random dynamic systems.

In Chapter 10, time domain and frequency domain solving processes for fuzzy stochastic differential equations are advanced. The solving processes of time domain and frequency domain for n-order linear fuzzy stochastic differential equations are given. The solving processes of time domain and frequency domain for system of one order nonhomogeneous fuzzy stochastic differential equations are brought forward.

序 言

在现实世界中，一切已存在或已发生的具体事物，不管它们如何复杂甚至不能用具体尺度来计量，都有其自在性和规定性，因而都是确定性的，没有任何含混的余地。

但是在人们进行决策和规划时，处理的对象往往是未来的事物，而决策中又离不开人们的认识和经验，因而常常存在一些不确定性的因素和信息。

未来事物受很多偶然因素的影响，因果关系很复杂，有时根据现有信息不能完全确知未来事物的结果和进程，这种不确定性称为随机性。当事物一出现（试验完成），即转化为确定性事物。统计数学（概率论、数理统计和随机过程理论）给予了我们考虑事物随机性的强有力的工具。

前面已经说到，客观世界中已存在的和已发生的事物都是确定性事物，但是，对某些确定性的事物，人脑所把握的信息却可能是模糊的。例如，人的面孔本身没有任何不确定性，但是，即使在他爱人的头脑里储存的有关信息也是模糊的，不存在任何定量的确定性的数据。可是，人们却可以根据这种模糊信息明确无误地识别出自己所认识的人。

目前可以数学处理的模糊性事物是比较简单的，概括起来说，目前人们所考虑的事物的模糊性，主要是指由于不可能给某些事物以明确的定义和评定标准而形成的不确定性。这时人们考虑的对象往往可以表现为某些论域上的模糊集合。

1965 年 L. A. Zadeh 提出的模糊集合是对经典集合的拓

广，由于它触动了近代数学的基石——集合的概念，所以模糊数学必将拓广数学的所有分枝。

1990 年我们研究了另一种不确定性信息及其数学处理方法。但是因为在长期的研究工作中，我们发现有些决策问题所研究和处理的某些因素和信息可能既无随机性又无模糊性，但是决策者纯粹由于条件的限制而对它认识不清，也就是说，所掌握的信息不足以确定事物的真实状态和数量关系。我们称这种纯主观的，认识上的不确定性信息为"未确知信息"。未确知信息一般可以采用主观隶属度分布估计和主观概率分布估计来处理。

主观概率分布和主观隶属度分布虽然在概念上有所区别，但它们在本质上却非常接近，概括来说，它们都是人们对某一未确知事件和各种可能情况或可能取值为真的概率的主观估计；也就是决策者本人的经验、直觉、有限的信息等因素在该人心目中所产生的信任程度的比例分配，因此，可以把主观概率和主观隶属度统一为"信比"，信比在论域上的分布可称为"信比分布"。

根据以上所述，我们可以把不确定信息分为强和弱二类：随机性和模糊性是强不确定性信息；未确知性是弱不确定性信息。未确知性的"弱"表现在两方面：首先，当未确知性与随机性和模糊性共存时，它就可以被后二者所掩盖或包含；其次，当未确知性单独存在时，它可以用主观概率和主观隶属度来描述，也就是采用随机性和模糊性的表达方式来表达。这样，未确知性的数学处理就得到了极大的简化，甚至在建立理论和具体方法时，在数学形式上就可以只考虑随机性和模糊性，在显式上不出现未确知性，而并不妨碍我们用这样的理论

和方法解决具有未确知信息的问题。

十多年来，我们一直在研究如何正确处理工程问题中的各种不确定性因素和信息。首先我们于1982年提出了地震烈度的模糊综合评定的方法；接着于1984年提出了结构模糊优化设计理论，以后发展为结构软设计理论，相应的在数学上建立了模糊随机规划的一般性理论；1989年提出了工程大系统的全局性优化理论；1990年建立了结构维修理论的框架。这些开创性的工作为全面地建立工程软设计理论奠定了基础。1990年以前在这个领域的工作已总结为专著《工程软设计理论》（科学出版社，1992）。目前这一理论仍在继续研究和发展中。

另一方面，从1983年开始，我和博士研究生欧进萍一起努力地从事于结构模糊随机振动理论的开拓性研究工作。首先，我们把地震时的地面运动模拟为具有模糊参数的随机过程，也就是说在这种随机过程的频谱密度中包含有模糊参数，它们分别代表地震烈度和场地分类这两种模糊因素。这样，在1985年就建立了抗震结构的模糊随机振动理论。

具有模糊参数的随机过程是一种最简单的模糊随机过程。失去模糊性时它蜕化为随机过程。但失去随机性时，模糊性将无所依托。这时的模糊性将如何表现？这个问题困扰了我两年，最后我发现要想解决这个理论问题，还必须把集合的概念加以拓展。

不论经典集合还是模糊集合都是静态的，只要涉及到模糊过程就需要把集合的概念推向动态，所以1987年我们提出了动态模糊集合的概念，这就为模糊随机过程和模糊随机振动的一般性理论的研究开拓了道路。