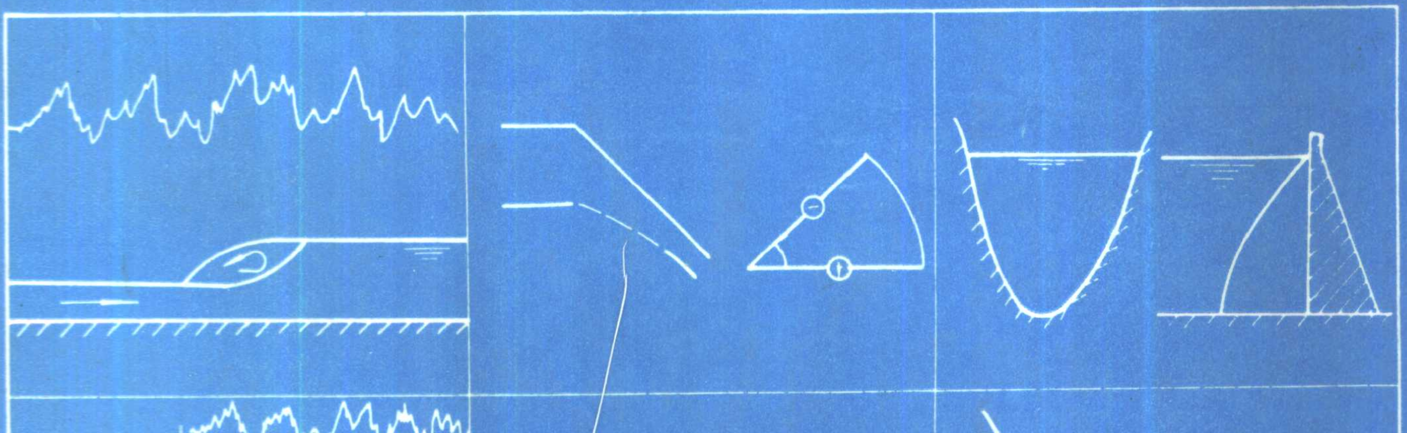
# 肖天铎科学论文集

# Selected Scientific Papers by Tiento Siao

（流体力学和水力学）

(Fluid Mechanics Hydraulics)

水利水电科学研究院水力学所
工业水力学及管道不恒定流研究室编



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## ——流体力学和水力学

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## 内 容 简 介

本论文选集反映出作者从事水利工程流体力学和水力学研究四十年的工作经验、共选入论文 31 篇，其中紊流包括边界层流动及掺气水流 11 篇、转换段中及绕阻体理想流 3 篇、地震动水压力 4 篇、水与弹性体耦合振动 4 篇、水与刚性体耦合振动 2 篇、水电站调节稳定性 4 篇、水锤及管道瞬变流 2 篇、突体和升台的初生空化数 1 篇．

论文编订按照发表时间的先后次序．前期较着重对紊流及其实验结果作理论分析性质的阐述，后期较着重针对具体工程中出现的问题求解析解，都具有普遍应用意义．论文集不仅数力学意味强，而且所得解析解可据以作一般的计算．此外，用英文写作的占 14 篇，其余用中文写作的，也附有英文摘要．

本选集可供水利、水能、以及数学力学专业的大、中专师生包括研究生，及有关科研、设计单位人员阅读，还可作教学的补充资料．

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# 序

教授级高级工程师肖天铎博士的科学论文选集流体力学和水力学出版了。清华大学和水利水电科学研究院同在北京，学术交往密切，大学的水利工程系很多同仁对作者肖博士是很熟悉的。他在国内水利工程界是流体力学和水力学方面最著名的学者之一。对他的论文选集出版，我感到很高兴，并乐为作序。

统观此集的内容，主要分为四个方面，即紊流、水动力学、水弹性振动、和管道瞬变流。论文编排次序按照发表时间的先后，体现出作者从事流体力学和水力学研究工作先后性质的改变与发展。内容较大部分是实验成果分析或是解析解的结果，较小部分是介绍性文章，但都属于基本研究性质，又都与工程实际有密切的结合。

至于论文集的内容细节，其中一些我是熟悉的。但为写这篇序文，作者向我补充了我未知的很多情况，使我能更好地写出以下对各篇论文的含义或事情经过。论文按编序 1 到 6 和 8、9 是他在留美期间发表或把写好的文稿留下回国后在美国发表的，以风洞实验和水动力学分析为主。论文 1 是讨论文，给出格栅的阻力系数表达式。论文 5 是硕士论文，4 是博士论文，H.Rouse 教授长作者十岁，是他这两篇论文的导师。以论文 4 为基础未选入文集的论文 3′发表后获美国土木工程师学会的年度优秀水力学论文奖。论文 4 的特点在于全面地用紊动细部各项来描述水跃的紊动特性；制定出有自由面的水流和封闭气流模型相似条件；用特殊形式毕托管量测掺气水跃的流速分布，继用渐近改正方法获得倾斜的流线图。论文 2、3 和 5 的一部分用保角变换求解较复杂的理想流动问题，是他从 J.McNown 教授授课中学习的结果。McNown 教授曾携论文 2 的解析解稿子和计算结果图示 G.Birkhoff 教授，其后在这位教授的专著书中列为页脚注。论文 6 为紊动介绍文，作了详细的发挥并给出示例。论文 8 是作者的边界层研究，用 Preston 管量测边壁上剪切应力同时求解析解，在工作中曾与 L.Landweber 教授讨论并经他部分指导，回国时留下的论文稿经他修改发表。论文 9 是作者参加依阿华大学力学与水力学系研究生班“高等流体力学”授课的讲稿，后同名书出版时作为书中的组成部分。该书一段时间内直至七十年代成为美国大学中水利工程专业流体力学课程主要采用课本之一。

论文集的其余部分是作者回国后在国内发表的。论文 7 对紊流的基本概念作了细部介绍，是论文 6 改用中文写成的。论文 4′是他的研究生读副博士学位时要求做两篇论文的首篇，经作者详细的指导。论文 10 有关高紊动度量测的校正是他在美国时写好，带回国内发表的。论文 11 和 12 体现了他原来在美国时对紊流研究在掺气水流问题上的应用，而论文 11 的前半部含有介绍文性质。论文 12 仍属于介绍性质，用水动力学方法详尽地给出 29 种转换段的理想流动解析解。他回国之后的工作前一段时间在水弹性振动和地震动水压力方面，后来又增加了管道瞬变流和水电站水轮机调节系统运行的稳定性方面。论文命题来自工程，均属基本研究性质，解析解结果具有普遍应用性，可据以作实际计算。论文 14 后来被对小浪底泄洪洞工程研究中参考。论文 16 其后在乌

江渡水电站大坝高速水流长跨度渡槽设计中考虑振动问题时曾引用。论文17为一般性的，特点是把不规则的河谷断面，用宽深比和断面充实度二参数来代表，求得解析解。论文18结合刘家峡水库工程，发表后梅强中教授曾致函"水利学报"颇多称道。论文19来源于西洱河二级和湖南龙源水电站工程。论文20与位山水库溢洪道结合，原有同题的英文稿，异于论文20在于用功和能求解，曾被顾兆勋教授60年代初携赴法国参加某水弹性学术会议，得到称赞。论文21结合四川映秀湾工程，22结合鲁布格和挪威的某水电站。论文23为解答阿尔巴尼亚的费尔泽水电站大型输水管道的问题而作，原为46页单行本，后来压缩成本文。论文24的解析解包含导致不稳定性的更广泛因素，对西洱河、鲁布格、回龙山、广东长湖，渔子溪一级五个水电站作了应用计算。论文25受京郊十三陵水库进水塔多年前提出的问题所启发。论文26结合黄河小浪底和京郊白河的倾斜坝面，27结合刘家峡泄洪洞，28结合贵州红枫和南湾二水电站。论文29结合海龙水库双进水塔，30受观察了黄河三圣宫闸门、海河挡潮闸门、和京郊三家店闸门漏水激振的启发，31仍是结合某水电站的问题。

在以上的叙述里，只把肖天铎博士称为作者，其实许多文章还有其他作者。在文章被纳入论文集时都尽量征得了他们的同意。尤其第一作者教授级高级工程师董兴林是肖先生的长期密切合作者，高级工程师王念慎硕士和工程师杨开林博士既是肖天铎先生的研究生，也是科研中的密切合作者。

整个论文选集前后包括40年的时间，论文表现出作者工作认真和力求缜密详尽的精神，而且是始终一贯的。此选集曾受到三位学部委员黄文熙、严恺、和张光斗教授向出版社推荐。他们是他在大学时读书的老师，和去狮子滩暑假实习时的老师。还受到了李桂芬教授和我本人的推荐。

凡是个人论文集几乎都要对作者生平有点叙述。他的简单情况可以见1955年出版的"美国科学家"(American Scientists)和1991年先后出辽宁人民出版社出版的"二十世纪中国名人辞典"与上海人民出版社出版的"中国当代名人录"等书中的叙述。这里再作一些介绍。肖天铎博士1916年2月出生，籍贯江苏宿迁。自幼随父母住闽赣二省，后返原籍县的埠子集，入小学读二年半后，于1927年夏毕业。辍学两年，进入省立徐州中学。读至高一上学期后，考入免费的南京陆地测量学校简易科，一年毕业，进入陆地测量总局印刷股工作。1934年秋又返原中学重新读至高中毕业。1937年夏考取中央大学工学院水利工程系，经四年学习毕业后，在该系任助教四年。1945年考取公费研究生赴英国学习三年。首年在位于Sunderland的William Doxford and Sons造船厂实习，其后进入Newcastle-on-Tyne的King's College造船工程系学习，获一级优等造船工程工学士学位。继由该系C.Burrill教授介绍到美国依阿华大学水力学研究所工作八年，历任研究助理员、副研究员、和研究工程师。同时在该大学工学院力学和水力学系攻读研究生学位，1950年获硕士学位、1954年获博士学位。继二年中，还担任大学班流体力学实验和研究生班部分高等流体力学的授课。1956年夏辞去在美职务，回国参加社会主义建设。先在中国



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科学院水工研究室任研究员，后在水利水电科学研究院任高级工程师，其间1959年兼中国科技大学技术物理系理论力学大班授课一年。是博士研究生导师，1987年底退休后任水科院的咨询委员、学术委员会顾问、和学位委员会委员。

我校原水利工程系系主任张任教授曾经两次向他说，如果不是考虑到他身体较差，就要请他担任兼任教授了。我和肖先生除同行外，还因为我曾经赴依阿华大学水力学研究所作高级访问学者一年，他又在那里停留过较长时间，所以之间有一种亲切感，历年我们在学术上做过交流,喜欢为他的这个选集作序。

我相信肖先生的论文集出版后，对水力学和流体力学科研工作者，特别是对有兴趣于求解析解来解答工程实际问题者，将有很大的吸引力和参考应用价值。对我国流体力学和水力学的发展会起到推动的作用。

董曾南①

1992.1.10

①序作者是清华大学水利水电工程系教授、系主任，国际水力学研究协会（IAHR）理事，国家教委高等学校工科力学课程教学指导委员会委员，中国水利学会理事，《中国科学》编委，《中国大百科全书水利》编委，《水利学报》、《水力发电学报》编委，《水利水电科技》编委，水利水电科学研究院、水力学研究所顾问。

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## Discussion of Baines and Peterson's "An Investigation of Flow through Screens"

T,T,Srao.[10] ①In this paper the authors give us an analysis of the pressure drop in flow through a screen spanning the entire section of a conduit or channel. The assumption that the pressure drop through a screen with a multiude of openings is the same as that through a plate with a single opening provided that the openings of both are of the same geometrical proportions, is reasonable. Certainly, the pattern of flow through a screen is simply a multiple of that for a single opening, considering that the center lines of the bars composing the screen merely represent axes of symmetry of the over-all flow. The authors' further assumption that a partial constriction in the form of a plate combines the features of an orifice and an abrupt enlargement, and hence is subject to simple, one-dimensional analysis, is equally plausible. The soundness of the resulting equation

$$\frac{\triangle p}{\rho V_0^2/2}=\left[\frac{1}{C_c(1-S)}-1\right]^2 \qquad [29]$$

is best reflected by its close agreement with the authors' experimental results for screens of solidity ratio $S>0.24$.

The pressure drop through a screen must be defined as the difference of the pressures at a section upstream from the screen, just before the flow becomes nonuniform, and at a section downstream from the screen, where the flow becomes uniform again. The pressure drop times the cross-sectional area $A_t$ of the conduit, however, must be balanced by the drag force of the screen, neglecting the resistance along the tunnel walls and other forces of secondary importance between the two sections. Defining the drag coefficient $C_D$ of a body as the ratio of the drag to the product of the dynamic pressue $\rho V_0^2/2$ and the projected (or solid) area $A_s$ of the body, we have from Equation [29] the drag coefficient for the screen

$$C_D=\frac{\Delta pA_t}{\rho V_0^2A_s/2}=\frac{\triangle p}{\rho V_0^2S/2}=\left[\frac{1}{C_c(1-S)}-1\right]^2\frac{1}{S} \qquad [30]$$

For the particular case of a screen composed of round bars, $C_c$ is then essentially epual to unity,and Equation [30] reduces to

$$C_D=\frac{S}{(1-S)^2} \qquad [31]$$

Since the foregoing analysis is based on the usual one-dimensional simpli-

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fications of closed–conduit flow, Equations [30] and [31] may be expected to apply only so long as such flow conditions are realized. When the solidity ratio of a screen is small, on the contrary, the pattern reduces to that of fiow around a series of essentially isolated (though continuous) elementary forms. The screen must then be regarded as an immersed body rather than a conduit constriction. This should be apparent from the fact that both Equations [30] and [31] require $C_D$ to approach

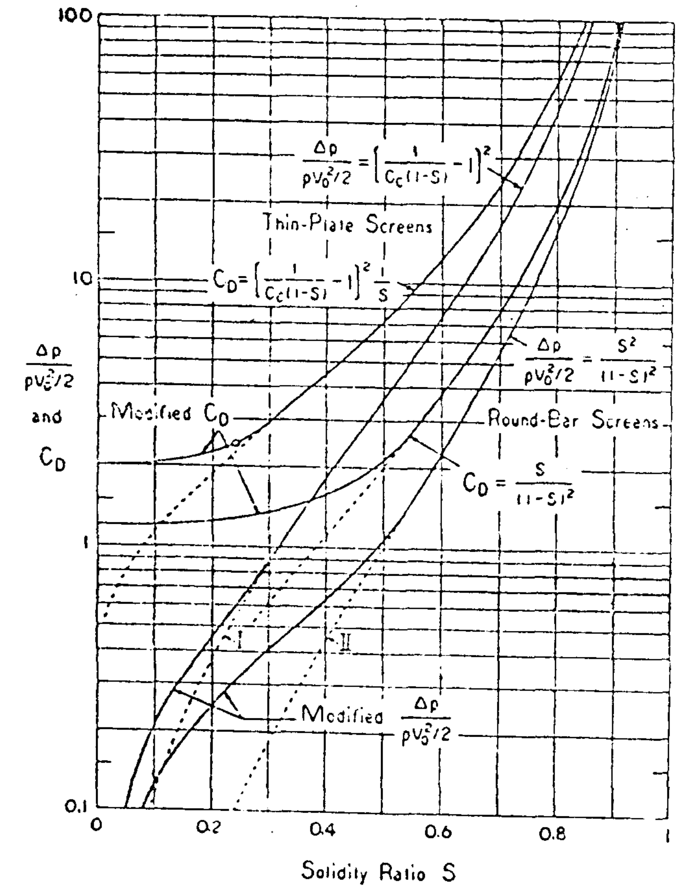


Fig. 14 Modified Coefficients of Pressure Drop and Drag For Thin–Plate And Round–Bar Lattice Screens

zero with decreasing values of S, which is far from true.For the same reason, Equation (30) cannot be correct at small solidity ratios, even though it correctly indicates that $\Delta p/(\rho V_0^2/2)=0$ when $S=0$. For example, as the solidity ratio approaches zero, a lattice screen takes the limiting form of a series of bars which are infinitely far apart; if the bars are round, the limiting value of $C_D$ will be about 1.2, and if flat (a first approximation to square, for which data are not at hand), $C_D$ will be about 2.0. As $S \to 0$, therefore, Equation [29] must be modified according to the true limits of Equations [30] and [31].

On the assumption that the pressure–drop coefficient for a given solidity ratio can be determined from the drag coefficient through multiplication of the latter by *1 / S*, the writer has sought to correct the authors' pressure curves in the region of low solidity. Curves for pressure drops and drag coefficients for screens of both the thin–plate and the round–bar type were first constructed, as shown in Fig. 14, in accordance with Equations [29],[30], and [31]. With the guide that the drag coefficient for the two types of screens must approach 2.0 and 1.2, respectively, as S approaches zero the curves for the drag coefficient were modified by eye (guided by the one available point determined by the authors in this region). Then by a simple conversion of the modified drag coefficient to the pressure drop the modifed curves for the latter were obtained. In the case of round bars(for which the correction is the less accurate), the difference between the two branches of the pressure curve is seen to bc quite marked.

# DEFLECTION OF JETS

## I. SYMMETRICALLY PLACED V-SHAPED OBSTACLE

by

T. T. Siao and P. G. Hubbard*

The deflection of a free jet by a solid boundary, which has long been utilized to develop power from flowing water, is well suited to free-streamline analysis because of the dominance of inertia and pressure intensity in the establishment of the flow pattern. The design of impulse machinery utilizing this momentum change could be facilitated greatly if the idealized geometry of the system under potential flow conditions were known, because such conditions represent asymptotic values which are approached as the effects of secondary variables are decreased. With such information available, refinements of design could be based upon a secure knowledge of the fundamentals, and many rules of thumb could be replaced with precise quantitative data in graphical or tabular form. Specifically, if the total angle through which the jet is deflected is determined for conditions of both partial and complete interception by the boundary, then the principle of impulse and momentum can be used to compute forces or other dynamic characteristics of the system.

This paper and the two which follow are devoted to a determination of the angles of deflection caused by certain idealized forms of solid boundaries placed either symmetrically or asymmetrically with respect to the axis of a two-dimensional jet. These patterns of flow correspond to those occurring as a bucket of an impulse machine passes through a circular jet. In this paper, the free-streamline method is used to find the angle through which a two-dimensional free jet will be deflected by a symmetrical V-shaped boundary placed on its axis.

As represented in Fig. 1a (the $z$-plane), a two-dimensional jet with velocity $V_j$ and width $2a$ is deflected through an angle $\beta$ by

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the angular boundary of projected width $2b$. The sides of the boundary are inclined at an angle $\alpha$ relative to the original jet direction. In the hodograph plane (Fig. 1b), the bounding stream-

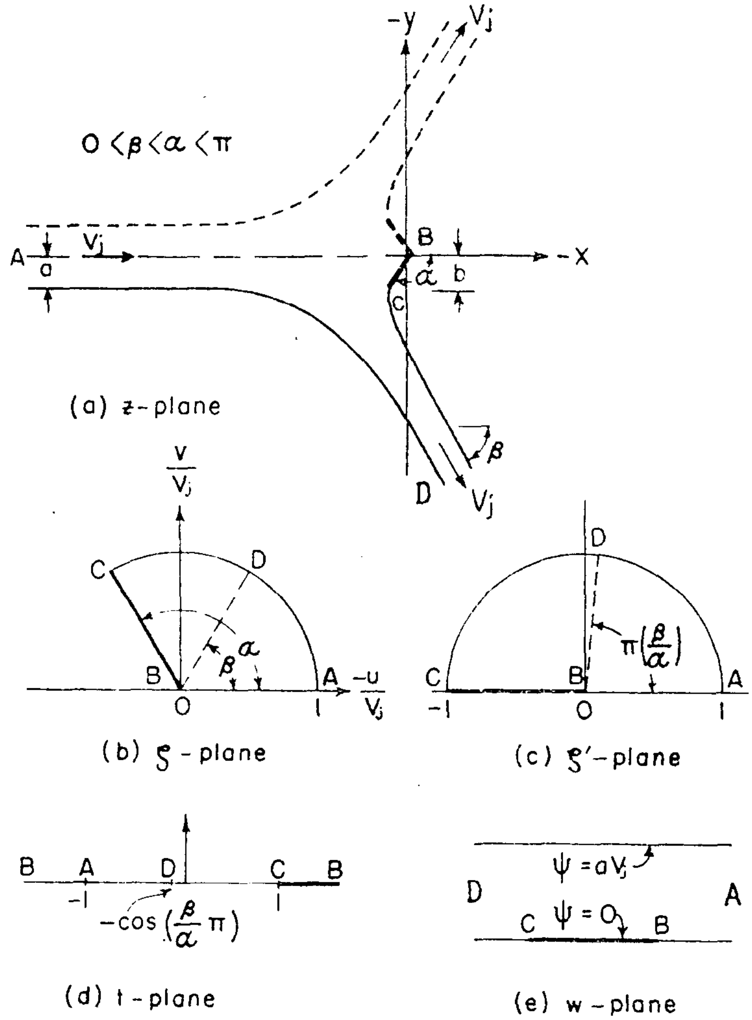


FIG. 1. TRANSFORMATION PLANES.

lines become a circular sector of angle $\alpha$ as can be shown from the definition relationship

$$\zeta = \frac{1}{V_j}\frac{dw}{dz} \tag{1}$$

in which $w$ is the complex potential. (Details of this *concept are* given in the first paper in this Bulletin.) This sector is expanded to a semicircle (Fig. 1c) by the transformation

$$\zeta' = \zeta^{\pi/\alpha} \tag{2}$$

and finally into the usual $t$-plane (Fig. 1d) by the transformation

$$t = -\frac{1}{2}\left(\zeta' + \frac{1}{\zeta'}\right) \tag{3}$$

In the $t$-plane, the flow pattern is simply that for a source with strength $aV_j$ at $A$ and a sink of equal strength at $D$ for the flow occurring in the upper half of the plane. This latter requirement is satisfied if the strengths of the source and sink are doubled to provide an image pattern in the lower half (which actually represents the omitted half of the original system). Application of the usual equations for a source and a sink on the real axis then results in the potential function (Fig. 1e)

$$w = \frac{V_j a}{\pi}\left[\ln\left(t + \cos\frac{\pi\beta}{\alpha}\right) - \ln(t+1)\right] \tag{4}$$

From the purely analytical viewpoint, Eqs. (1), (2), (3), and (4) represent the solution to the problem, because proper manipulation of the variables will yield values of the velocity at any point in the physical plane. The explicit solution of these simultaneous algebraic and differential equations as they stand is quite involved, however, and the complexity of detail can be mitigated considerably by the introduction of an auxiliary variable $\tau$, defined by

$$\tau = \zeta^{1/n} \tag{5}$$

and two positive integers $m$ and $n$ ($n < m$) such that

$$\alpha = \frac{n}{m}\pi$$

Then, from Eq. (1)

$$dz = \frac{1}{V_j}\frac{dw}{\zeta} = \frac{1}{V_j \zeta}\frac{dw}{dt}\frac{dt}{d\zeta'}\frac{d\zeta'}{d\zeta}\frac{d\zeta}{d\tau}d\tau$$

and substitution of the appropriate derivatives from Eqs. (2-5) yields

$$dz = \frac{am}{\pi}\left[\frac{1}{\tau^m - e^{i\beta m/n}} + \frac{1}{\tau^m - e^{-i\beta m/n}} - \frac{2}{\tau^m - 1}\right]\tau^{m-n-1}\,d\tau$$

The resolving of this equation into partial fractions results in

$$dz = \frac{a}{\pi}\sum_{r=0}^{m-1}\left[\frac{e^{-i(2r\alpha+\beta)}}{\tau - e^{i(2r\alpha+\beta)/n}} + \frac{e^{-i(2r\alpha-\beta)}}{\tau - e^{i(2r\alpha-\beta)/n}} - \frac{2e^{-i2r\alpha}}{\tau - e^{ir\alpha/n}}\right] d\tau \quad (6)$$

Integration of this equation between proper limits will give the coordinates in the $z$-plane of any point corresponding to assigned values of $\alpha$ and $\beta$ and of $\zeta$. The differences in the coordinates of points $B$ and $C$, for example, can be found by noting that, for the points $B$ and $C$,

$$\zeta_B = 0 \quad ; \quad \zeta_C = e^{i\alpha}$$

or

$$\tau_B = 0 \quad , \quad \tau_C = e^{i\alpha/n}$$

Thus, since $B$ is the origin in the $z$-plane, integration of Eq. (6) between the limits of 0 and $e^{i\alpha/n}$ gives the coordinates of point $C$:

$$z_C = \frac{a}{\pi}\sum_{r=0}^{m-1}\Big\{ e^{-i(2r\alpha+\beta)} \ln\left[1 - e^{i(\alpha-2r\alpha-\beta)/n}\right] + e^{-i(2r\alpha-\beta)} \ln\left[1 - e^{i(\alpha-2r\alpha+\beta)/n}\right] - 2e^{-i2r\alpha} \ln\left[1 - e^{i(\alpha-2r\alpha)/n}\right]\Big\}$$

The imaginary part of this equation is the $y$-coordinate of point $C$, which is equal in absolute value to the distance denoted as $b$ in the $z$-plane:

$$b = \frac{a}{\pi}\sum_{r=1}^{m-1}\sin\frac{2rn}{m}\pi\left\{-\cos\beta \ln\left|\cos\frac{2r-1}{m}\pi - \cos\frac{\beta}{n}\right| + 2\ln\sin\frac{2r-1}{2m}\pi\right\}$$
$$+\frac{a}{\pi}\sum_{r=0}^{m-1}\cos\frac{2rn}{m}\pi\sin\beta\ln\left|\frac{\sin\frac{1}{2}\left(\frac{2r-1}{m}\pi - \frac{\beta}{n}\right)}{\sin\frac{1}{2}\left(\frac{2r-1}{m}\pi + \frac{\beta}{n}\right)}\right| + a(1-\cos\beta) \quad (7)$$

Vertical bars in Eq. (7) indicate that absolute values are to be used.

Although analysts have already presented general solutions of jet interceptions, and Cisotti [20] has completely solved the particular case of the normal plate, the method presented herein is considered more direct and the integrated solution is completed for the general case in Eq. (7). From this equation, values of $b/a$ corresponding to assumed values of $\beta$ have been computed for



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