研究所・高普考叢書

單元操作 輸送現象 解題技巧

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世界图书出版公司
北京・广州・上海・西安
1992

单元操作输送现象解题技巧

刘弘扬 编

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晓园出版社出版
世界图书出版公司北京公司 重印
北京朝阳门内大街 137 号
北京中西印刷厂印刷
新华书店北京发行所发行 各地新华书店经售

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1993 年 10 月第 一 版 开本：850×1168 1/32
1993 年 10 月第一次印刷 印张：16.75
印数：0001－500 字数：360 千字
ISBN：7－5062－1469－5/O·86
定价：11.00 元 (W_B9206/12)

序　　言

單操輸送學爲化工之基礎，亦爲研究所、高考、化工技師檢覈及升技術學院之必考科目，所以想在各種考試中順利過關，單操輸送是必讀的重要科目。

坊間有關單操輸送之書籍均著重於理論之陳述，無法對解題有深一層之解說，尤其理論陳述繁雜，往往讀完後似懂非懂，遑論解題？筆者有鑒於此，特集數年來補習班教學經驗與資料，編成此書以針對各項考試之快速正確解題。

本書以歷年各項考試爲準編寫而成，編列歷年各項考試中有出現之題型，並闡述解題技巧，使讀者在讀完本書後，對解題必有十足的信心，本書附有各校歷年研究所入學試題及解答，書後亦列出歷年高考試題以供參考。

筆者不佞，雖竭智窮慮，猶恐掛一漏萬，盼先進賢達不吝指正，俾作修正，以期更臻完美。

劉弘揚　謹識

Contens

Part(I) Transport phenomena

(A) The Simulation Problems and Solutions for Momentum transfer

The Problem-Solving Approach :

(一) Extracting the N-S Eq method

(1) Carefully read the problem statement

(2) Choose a coordination system (retangular, Cylinderical and Spherical)

(3) Sketch system diagam

(4) Extract the describing N-S Diff Eqs

(5) Set up BC & IC

(6) Solve the Eqs, Subject to BC & IC

(7) Analyse the Results

(二) Shell-balance method Analysis

(1) Choose a Shell size

(2) Force Balance

(3) Shear force related to velocity gradient

(4) Velocity distribution Solved (B.C Known)

(5) Average or Max. velocity

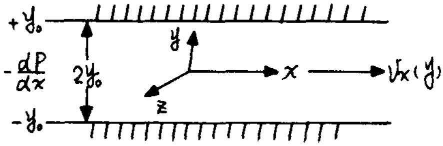
(6) Drag force

xample 1.

Steady state of laminar flow for
Newton's Incompressible fluids between two parallel flat
ates find

1. Velocity profile ?
2. Pressure drop per unit length of the conduit $\frac{-dp}{dx} = ?$
3. $V_{x\,max} = ?$
4. $< V_{x\,ave} > = ?$
5. $Q = ?$

풀】(一) By Simplifying Navier-Stokes eq



from eq of continuity $\nabla \cdot \vec{v} = 0$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0 \Rightarrow \frac{\partial V_x}{\partial x} = 0 \Rightarrow \frac{\partial^2 V_x}{\partial x^2} = 0$$

$$\rho \frac{D\vec{v}}{Dt} = -\nabla P + \rho \vec{g} + \mu \nabla^2 \vec{v}$$

$$\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}, \quad \vec{v} = V_x \vec{i} + V_y \vec{j} + V_z \vec{k}$$

from Eq of motion x − compoment

$$\frac{\rho}{g_c}\left(\cancelto{0}{\frac{\partial V_x}{\partial t}} + V_x \cancelto{0}{\frac{\partial V_x}{\partial x}} + V_y \cancelto{0}{\frac{\partial V_x}{\partial y}} + V_z \cancelto{0}{\frac{\partial V_x}{\partial z}}\right) = -\frac{\partial P}{\partial x}$$

$$\mu\left(\cancelto{0}{\frac{\partial^2 V_x}{\partial x^2}} + \frac{\partial^2 V_x}{\partial y^2} + \cancelto{0}{\frac{\partial^2 V_x}{\partial z^2}}\right) + \cancelto{0}{\frac{\rho}{g_c} g_x}$$

$$\Rightarrow \frac{\partial P}{\partial x} = \mu \frac{\partial^2 V_x}{\partial y^2} \Rightarrow \frac{dP}{dx} = \mu \frac{d^2 V_x}{dy^2} = \text{const}$$

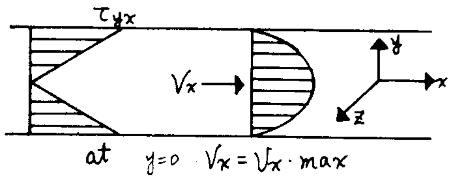
$$\text{B.C}\begin{cases}(1)\ y = \pm y_0, & V_x = 0 \\ (2)\ y = 0 & V_x = \text{finite} \Rightarrow \frac{dV_x}{dy} = 0\end{cases}$$

$$\frac{dv_x}{dy} = \frac{1}{\mu}\frac{dP}{dx}y + C_1 \quad \text{with B・C(2)} \Rightarrow C_1 = 0$$

$$\frac{dv_x}{dy} = \frac{1}{\mu}\frac{dP}{dx}y \quad \Rightarrow \quad \text{integrate} \Rightarrow V_x = \frac{1}{\mu}\frac{dP}{dx}\cdot\frac{1}{2}y^2 + C_2$$

$$\text{using B・C (1)} \Rightarrow \quad C_2 = \frac{-1}{\mu}\frac{dP}{dx}\cdot\frac{1}{2}\cdot y_0^2$$

$$1 \Rightarrow V_x = \frac{1}{\mu}\frac{dP}{dx}\cdot\frac{1}{2}(y^2 - y_0^2) \leftarrow V_x \text{ 對 } y \text{ 為一拋物線}$$



$$2 \Rightarrow V_{x\,max} = \frac{1}{-2\mu}\frac{dP}{dx}y_0^2$$

$$\Rightarrow V_x = \frac{1}{-2\mu}\frac{dP}{dx}y_0^2\left[1 - \left(\frac{y}{y_0}\right)^2\right]$$

$$= V_{x\,max}\left[1 - \left(\frac{y}{y_0}\right)^2\right]$$

$$3 \; \langle V_{x\,ave} \rangle = \frac{\int_{-y_0}^{y_0}\int_0^{w} V_x\,dy\,dz}{\int_{-y_0}^{y_0}\int_0^{w} dy\,dz} = \frac{Q}{A}$$

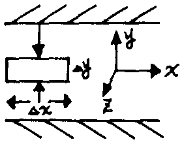
$$4.\; Q = \int dQ = \int V_x\,dA = \int_{-y_0}^{y_0} V_{x\,max}\left[1 - \left(\frac{y}{y_0}\right)^2\right]dy \;(\text{perunit width})$$

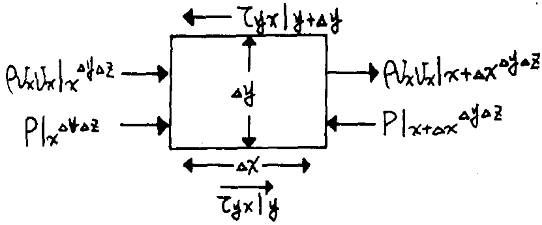
$$= \frac{1}{-2\mu}\frac{dP}{dx}\int_{-y_0}^{y_0}(y_0^2 - y^2)\,dy = \frac{1}{-2\mu}\frac{dP}{dx}\frac{4}{3}y_0^3$$

$$= \frac{-2}{3\mu}\frac{dP}{dx}y_0^3 = \frac{4}{3}V_{x\,max}\,y_0$$

$$\langle V_{x\,ave} \rangle = \frac{Q}{A} = \frac{\frac{4}{3}V_{x\,max}\,y_0}{2y_0} = \frac{2}{3}V_{x\,max}$$

(二)Byshell Balance：





Momentum Balance : $I_n - \text{Out} + \sum F^z = \text{Accum}$

(∵ fully dveloped)

$\rho(Vv|_x - vv|_{x+\Delta x})\Delta y \Delta z$ ← convective term (→ 0)

$+ (\tau_{yx}|_y - \tau_{yx}|_{y+\Delta y})\Delta x \Delta z$ ← shear force term

$+ (P|_x - P|_{x+\Delta x})\Delta y \Delta z$ ← pressure force term

$=$ acc (∵ s.s) (→ 0)

$\Rightarrow (\tau_{yx}|_y - \tau_{yx}|_{y+\Delta x})\Delta x \Delta z + (P|_x - P|_{x+\Delta x})\Delta y \Delta z = 0$

$\div \Delta x \Delta y \Delta z$

$$\Rightarrow \frac{\tau_{yx}|_y - \tau_{yx}|_{y+\Delta x}}{\Delta y} + \frac{P|_x - P|_{x+\Delta x}}{\Delta x} = 0$$

take limit $\Delta x \rightarrow 0 \quad \Delta y \rightarrow 0$

$$\Rightarrow \frac{d\tau_{yx}}{dy} + \frac{dP}{dx} = 0$$

Beacuse it. is a Newton's fluid

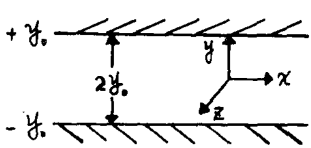
$$\tau_{yx} = -\mu \frac{dV_x}{dy}$$

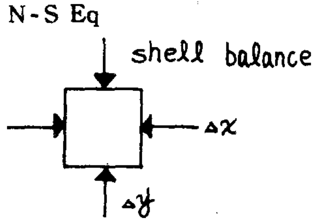
$$\Rightarrow \frac{d\tau_{yx}}{dy} = -\mu \frac{d^2V_x}{dy^2} = \frac{-dP}{dx}$$

$$\Rightarrow \frac{dP}{dx} = \mu \frac{d^2V_x}{dy^2}$$

just the same the result By N-S Eq

Detail discusion:





$$\frac{1}{\mu}\frac{d^2V_x}{dy^2}=\frac{dP}{dx}$$

$$\Rightarrow V_x=\frac{1}{\mu}\frac{dP}{dx}\frac{y^2}{2}+C_1y+C_2\rightarrow(A)$$

B、C discusion

(1) If both plates are stationary

$$V_x=(V_x)_{max}\left[1-\left(\frac{y}{y_0}\right)^2\right]$$

(2) Asufficient force is applied to the upper plate to maintain a velocity V_0

$$B\text{、}C\begin{cases}(1)\ y=-y_0\ , & V_x=0\\(2)\ y=y_0\ , & V_x=V_0\end{cases}$$

Substituting B、C into eq(A), We get C_1 , C_2

$$0=\frac{1}{2\mu}\frac{dP}{dx}y_0^2+C_1(-y_0)+C_2$$

$$V_0=\frac{1}{2\mu}\frac{dP}{dx}y_0^2+C_1(y_0)+C_2$$

聯立解：$C_1=\dfrac{V_0}{2y_0}$, $C_2=\dfrac{V_0}{2}-\dfrac{1}{2\mu}\dfrac{dP}{dx}y_0^2$

$$V_x=\frac{1}{2\mu}\frac{dP}{dx}y^2+\frac{V_0}{2y_0}y+\frac{V_0}{2}-\frac{1}{2\mu}\frac{dP}{dx}y_0^2$$

$$=\frac{1}{2\mu}\left(-\frac{dP}{dx}\right)(y_0^2-y^2)+\frac{V_0}{2}\left(\frac{y}{y_0}+1\right)$$

(3) If the Lower plate is set in motion $V=V_0^*$

$$B\text{、}C\begin{cases}(1)\ y=-y_0 & V_x=V_0^*\\(2)\ y=y_0 & V_x=0\end{cases}$$

代入 eq.(A) 聯立解 C_1 , C_2

$$C_1=\frac{-V_0^*}{2y_0}$$

$$C_2=\frac{V_0^*}{2}-\frac{1}{2\mu}\frac{dP}{dx}y_0^2$$

$$\Rightarrow V_x = \frac{1}{2\mu}\frac{dP}{dx}y^2 - \frac{V_0^*}{2y_0}y + \frac{V_0^*}{2} - \frac{1}{2\mu}\frac{dP}{dx}y_0^2$$

$$= \frac{1}{2\mu}\left(-\frac{dP}{dx}\right)(y_0^2 - y^2) + \frac{V_0^*}{2}\left(1 - \frac{y}{y_0}\right)$$

(4) If Both plate are moving

upper plate $= V_0$

Lower plate $= V_0^*$

a. if the direction is the same.

$$B\cdot C\begin{cases}(1)\ y = -y_0 & V_x = V_0^* \\ (2)\ y = y_0 & V_x = V_0\end{cases}$$

代入 eq(A) 聯立解

$$C_1 = \frac{V_0 - V_0^*}{2y_0}$$

$$C_2 = \frac{V_0^* + V_0}{2} - \frac{1}{2\mu}\frac{dP}{dx}y_0^2$$

$$\Rightarrow V_x = \frac{1}{2\mu}\left(-\frac{dP}{dx}\right)(y_0^2 - y^2) + \frac{y}{2y_0}(V_0 + V_0^*) + \frac{V_0 + V_0^*}{2}$$

b. if the direction is reversed

$$B\cdot C\begin{cases}(1)\ y = -y_0 & V_x = -V_0^* \\ (2)\ y = y_0 & V_x = V_0\end{cases}$$

$$C_1 = \frac{1}{2y_0}(V_0 + V_0^*)$$

$$C_2 = \frac{V_0 - V_0^*}{2} - \frac{1}{2\mu}\frac{dP}{dx}y_0^2$$

$$\Rightarrow V_x = \frac{1}{2\mu}\left(-\frac{dP}{dx}\right)(y_0^2 - y^2) + \frac{y}{y_0}(V_0 + V_0^*) + \frac{V_0 - V_0^*}{2}$$

(5) (Power Low) applied for Non-Newtowain fliud between parallel plate

$$B\cdot C\begin{cases}(1)\ y = \pm y_0, & V_x = 0 \\ (2)\ y = 0, & \tau_{yx} = 0\end{cases}$$

$$\frac{d\tau_{yx}}{dy} = -\left(\frac{dP}{dx}\right) \Rightarrow \tau_{yx} = \left(-\frac{dP}{dx}\right) y + C_1$$

$$B \cdot C\,(2) \Rightarrow C_1 = 0 \Rightarrow \tau_{yx} = \left(-\frac{dP}{dx}\right) y$$

in Power Law fliud

$$\tau_{yx} = m\left(-\frac{dV_x}{dy}\right)^n = \left(-\frac{dP}{dx}\right) y$$

$$\Rightarrow -\frac{dV_x}{dy} = \left[\frac{1}{m}\left(-\frac{dP}{dx}\right) y\right]^{1/n}$$

integral

$$\Rightarrow -V_x = \int \left[\frac{1}{m}\left(-\frac{dP}{dx}\right)\right]^{1/n} y^{1/n}\, dy$$

$$= \left[\frac{1}{m}\left(-\frac{dP}{dx}\right)\right]^{1/n} \int y^{1/n}\, dy$$

$$= \left[\frac{1}{m}\left(-\frac{dP}{dx}\right)\right]^{1/n} \left[\frac{y^{1/n+1}}{1+\frac{1}{n}} + C_2\right]$$

$$= \left[\frac{1}{m}\left(-\frac{dP}{dx}\right)\right]^{1/n} \left(\frac{n}{n+1} y^{1/n+1} + C_2\right)$$

$$B \cdot C\,(1) \Rightarrow C_2 = -\frac{n}{n+1} y_0^{\frac{1}{n}+1}$$

$$\Rightarrow V_x = \underbrace{\left[\frac{1}{m}\left(-\frac{dP}{dx}\right)\right]^{1/n} \left(\frac{n}{n+1}\right) y_0^{\frac{n+1}{n}}}_{(V_x)_{max} \text{ at } y=0} \left[1 - \left(\frac{y}{y_0}\right)^{\frac{n+1}{n}}\right]$$

$$\Rightarrow V_x = (V_x)_{max} \left[1 - \left(\frac{y}{y_0}\right)^{\frac{n+1}{n}}\right]$$

$$\langle V_x \rangle = \frac{\int_0^w \int_{-y_0}^{y_0} V_x\, dy\, dz}{\int_0^w \int_{y_0}^{y_0} dy\, dz} = \frac{2w \int_0^{y_0} \left[V_{x\max} \left(1 - \frac{y}{y_0}\right)^{\frac{n+1}{n}}\right] dy}{2y_0 \cdot w}$$

$$= (V_x)_{max} \int_0^{y_0} \left[1 - \left(\frac{y}{y_0}\right)^{\frac{n+1}{n}}\right] d\left(\frac{y}{y_0}\right)$$