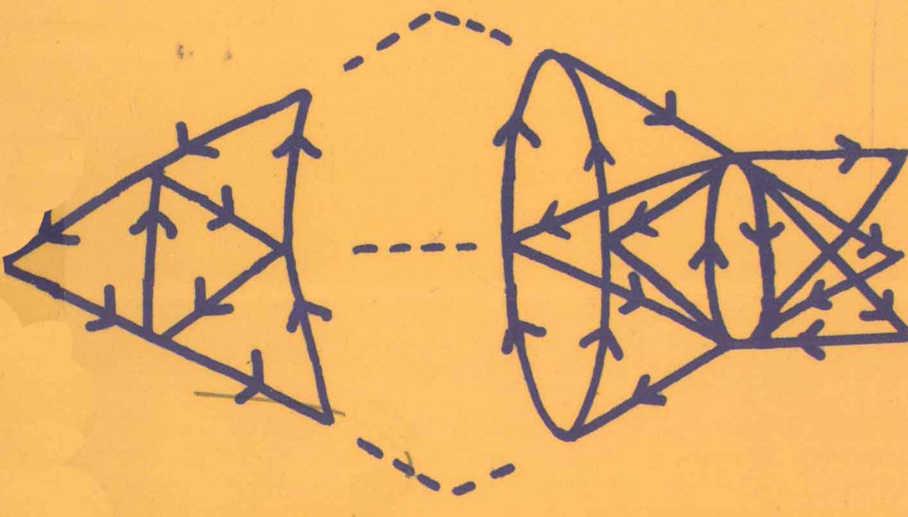
Philippe Di Francesco
Pierre Mathieu
David Sénéchal

Conformal Field Theory

Vol.1

共形场论

第1卷



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This book is dedicated to our families

影印版前言

所谓共形场论，就是共形群变换下不变的经典或量子场论。共形变换群是庞加莱群（Poincare Group）的推广。

共形场论(CFT)是过去 20 年里理论物理中最活跃且成果丰硕的研究领域之一。到目前为止，本书是第一部，也是唯一一部全面系统介绍共形场论的专著。

共形场论已经广泛应用于弦理论、统计物理、凝聚态物理和纯粹数学等诸多方面的研究。例如：弦的世界面（Worldsheet）所构成的黎曼面由二维共形场论来刻画；在数学理论中，如 Borcherds(菲尔兹奖获得者)提出的顶点算子代数（Vertex Operator Algebra），即为二维共形场论的代数理论，Drinfeld(菲尔兹奖获得者)等提出的所谓手征代数（Chiral Algebra），则是试图从代数几何的观点理解二维共形场论。

本书共 18 章，分为 3 个部分。

第 1 部分——简介。第 1 章中对本书涉及的相关概念进行了简单回顾。第 2 章是量子场论的一些基本概念，如自由玻色（费米）子，路径积分，关联函数，对称与守恒量，以及能动张量。第 3 章则涉及统计力学的一些基本概念，如玻尔兹曼分布，临界现象，重整化群和转移矩阵。

第 2 部分——基础理论。首先，第 4 章介绍了全局的共形不变。然后，第 5 章详细论述了有关二维共形不变基本而重要的概念，内容包括初级场、关联函数、Ward 恒等式、自由场、算子积展开和中心荷等等。第 6

章则是更为详细论述算子表述下的共形场论，此章的重点是 Virasoro 代数和顶点代数。随后两章论述了极小模型，极小模型是共形场论中最重要的模型之一。第 9 章和第 10 章分别介绍库仑气体和模不变，屏蔽算子和 Verlinde 公式等重要概念亦先后引入。第 11、12 两章分别介绍了 Q-态 Potts 模型和二维 Ising 模型。

第 3 部分——具有李群对称性的共形场论。第 13 章介绍了单李代数的一些基本内容，如单李代数的结构，最高权表示和特征标等等。第 14 章为仿射李代数（亦称 Kac-Moody 代数），内容基本与第 13 章平行。第 15~ 17 章，讨论的主题都是 WZW(Wess-Zumino-Witten)模型。WZW 模型是二维共形场论中另一个最重要的模型，它集中体现了二维共形场论的各种性质。最后一章，即 18 章为陪集构造。陪集构造是共形场论最重要的手段之一。对于物理学或是数学工作者而言，陪集构造方法将二维共形场论的研究带入到一个新的天地。

本书各章之后有大量的练习题，可检验和加深对所学内容的理解。

本书可作为高等院校理论物理和数学专业高年级本科生和研究生教材，也可供物理学和数学等相关学科研究人员参考。对于这些领域的研究人员和高校师生，这是一本不可多得的参考书。

Preface

This is the first extensive textbook on conformal field theory, one of the most active areas of research in theoretical physics over the last decade. Although a number of review articles and lecture notes have been published on the subject, the need for a comprehensive text featuring background material, in-depth discussion, and exercises has not been satisfied. The authors hope that this work will efficiently fill this gap.

Conformal field theory has found applications in string theory, statistical physics, condensed matter physics, and has been an inspiration for developments in pure mathematics as well. Consequently, a reasonable text on the subject must be adapted to a wide spectrum of readers, mostly graduate students and researchers in the above-mentioned areas. Background chapters on quantum field theory, statistical mechanics, Lie algebras and affine Lie algebras have been included to provide help to those readers unfamiliar with some of these subjects (a knowledge of quantum mechanics is assumed). This textbook may be used profitably in many graduate courses dealing with special topics of quantum field theory or statistical physics, string theory, and mathematical physics. It may also be an instrument of choice for self-teaching. At the end of each chapter several exercises have been added, some with hints and/or answers. The reader is encouraged to try many of them, since passive learning can rapidly become inefficient.

It is impossible to encompass the whole of conformal field theory in a pedagogical manner within a single volume. Therefore, this book is intentionally limited in scope. It contains some necessary background material, a description of the fundamental formalism of conformal field theory, minimal models, modular invariance, finite geometries, Wess-Zumino-Witten models, and the coset construction of conformal field theories. Chapter 1 provides a general introduction to the subject and a more detailed description of the role played by each chapter. In building the list of references listed at the end of this volume, the authors have tried to be as complete as possible and hope to have given appropriate credit to all.

The authors intend to complete this work with a second volume, that would deal with the following subjects: Superconformal field theory ($N = 1, 2$), parafermionic models, W-algebras, critical integrable lattice models, perturbed conformal field theories, applications to condensed matter physics, and two-dimensional quantum gravity.

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Modern, computerized book production minimizes the number of trivial errors, but one still has to rely on friendly humans to detect what the authors themselves have overlooked! We are grateful to Dave Allen, Luc Bégin, Denis Bernard, François David, André-Marie Tremblay, Mark Walton, and Jean-Bernard Zuber for their useful reading of various parts of the manuscript and, in many cases, their much-appreciated counsel. In particular, we thank M. Walton for numerous discussions on the subjects covered in part C of this volume and his constant interest in this project. P.D.F. is especially indebted to J.-B. Zuber, who patiently introduced him to the conformal world, and to the late C. Itzykson, who guided his steps through modern mathematics with his extraordinary and communicative enthusiasm. P.M. and D.S. acknowledge the support of the Natural Sciences and Engineering Research Council of Canada (NSERC) and of "le Fonds pour la Formation de Chercheurs et l'Aide à la Recherche" (F.C.A.R.) of Québec.

Philippe Di Francesco
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