



LUIS CASTAÑER

UNDERSTANDING MEMS

PRINCIPLES AND APPLICATIONS



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Luis Castañer

*Department of Electronic Engineering
Universitat Politècnica de Catalunya, Spain*

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UNDERSTANDING MEMS

To my wife Pamen, daughters Maya and Olga, grandsons Teo, Raul and
Marc-Eric and granddaughter Claudia

Preface

The field of microelectromechanical systems (MEMS) has greatly expanded in recent years since Richard Feynman's speech – 'there is plenty of room at the bottom' – opened the minds of researchers and companies to the possibility of exploring the potential of microstructures of minute dimensions.

The need for skilled professionals has steadily grown as businesses and research labs engage in challenging projects. Students are motivated because they are aware that they have, in their phones, watches or tablets, accelerometers, compasses and sensors and useful applications relying on them. Students see in the MEMS field, job opportunities and exciting career prospects.

Ultimately those devices have to work together with front-end electronics and digital processing to interface with the user. Students in electrical engineering and computer science departments in universities around the world are probably the most exposed to the field. Simultaneously, engineers and professionals working in the electronics and semiconductor industry face the challenge of integrating MEMS devices into chips, systems on chips or, broadly speaking, into electronic systems. The added value of those devices enables the expansion of high tech businesses.

In my experience of teaching MEMS in an electronic engineering department to engineering students from many countries I have faced the difficulty of selecting material for one-semester course and having to decide on the depth and breadth of the subjects covered.

The field is inherently multidisciplinary, and if the basics are not sufficiently covered, students will not achieve the intellectual satisfaction of a full understanding. However, if the coverage is too complex it cannot be extended to the various fundamental domains underlying the field. Solving problems is an important part of the learning process as it allows for concepts to be reviewed.

Those are the reasons why I have chosen to approach the subject using analytical solutions as far as possible, but with the help of two software tools: one very popular among science and engineering students, Matlab; and the other, very popular among electrical engineering students, PSpice. I have used Matlab to solve ordinary differential equations subject to boundary or initial conditions, applied to mechanical, electromechanical, electrokinetic and thermal problems. This allows numerical results to be found quickly which can then be discussed and put into context.

I have used PSpice to solve Laplace transforms of transfer functions and to solve electrical equivalent circuits of lumped thermal problems. Apart from the clarity of analytical solutions, this approach places the subject of MEMS in the same tool environment as other subjects the

student will already have taken. Commonality of tools is important at this level of the learning process, because it means that students do not have to spend a significant amount of time learning how to use new software.

The book includes 52 worked examples in the text and 100 solved problems in the appendices, organized by chapters. In my view this allows this textbook to be used not only as support material for a conventional course but also as a self-study resource for distance learning. I am very grateful to faculty colleagues, researchers and students with whom I have interacted all these years that have taken me to complete this book.

Luis Castañer
April 2015
Barcelona, Spain

About the Companion Website

This book is accompanied by a companion website:

www.wiley.com/go/castaner/understandingmems

The website includes:

- Matlab
- PSpice codes
- Chapter viewgraphs

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