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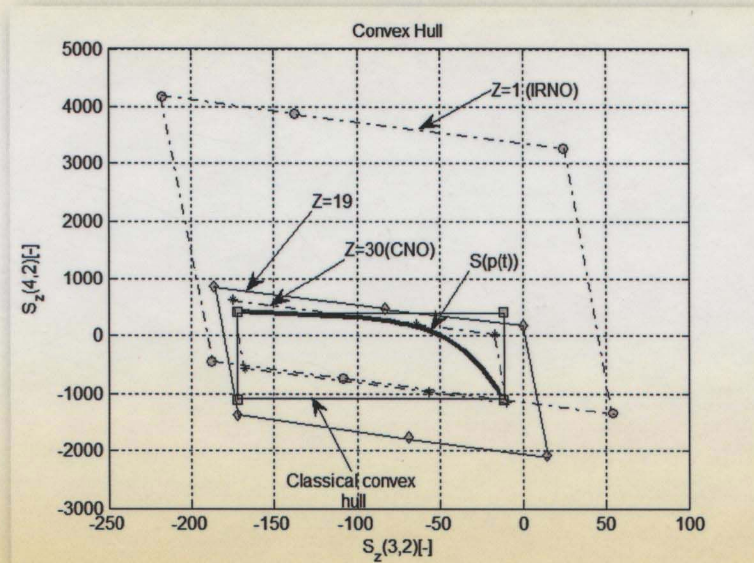

Péter Baranyi • Yeung Yam
Péter Várlaki

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Tensor Product Model Transformation in Polytopic Model-Based Control

Péter Baranyi
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Preface

This book introduces a methodology to numerically generate a class of invariant, canonical, and convex polytopic representations from any quasi-linear parameter-varying models. The class of polytopic representations is then readily executable for linear matrix inequality-based system control design. Our intention is to devise a standard design process that reduces analytical derivations as much as possible, echoing the recent paradigm shift toward the ready acceptance of numerical solutions as a valid form of output from control system problems. Our methodology is based on an extended version of singular value decomposition applicable to hyper-dimensional tensors. The resulting representations may exactly or approximately duplicate the original dynamics. Trade-offs between approximation accuracy and computational complexity can be performed through the singular values retained in the process. The book also proposes to manipulate the convexity of the resulting polytopic representations as a possible way to influence the subsequent linear matrix inequality-based design and control performance. This differs from the prevailing perspective in the literature, which focuses mainly on the proper formulation and conditioning of linear matrix inequalities to facilitate feasible designs. On the practical side, the book provides details about the application of the proposed methodology to design control systems with a number of real-life examples, including the aeroelastic wing section and the heavy vehicle rollover prevention problems. The book also introduces *TPtool*, a MATLAB™ Toolbox which contains the relevant algorithms of the proposed methodology. The Toolbox is available online for free downloading.

This book is not intended to be a textbook, but rather a reference book for graduate students, researchers, engineers, and practitioners who are dealing with nonlinear systems control applications. The book will be a practical tool to systematically generate controller design for a large class of nonlinear systems, especially with the help of the MATLAB Toolbox *TPtool*. More importantly, we sincerely hope that readers will also find the book stimulating and useful as a platform upon which new concepts and MATLAB functions may be defined and explored as needed for analyzing their control problems and carrying out the design processes at hand.

The authors owe the completion of this book to a number of people. We give our appreciation to our young researchers Dr. Béla Takarics, Dr. Stephen C.T. Yang, Patricia Gróf, and Péter Galambos for their help in conducting experimental case studies and developing the MATLAB Toolbox; to Dr. Pauline Lee for her help in English editing; and to our graduate students, Mr. Kerney Wu and Miss Heran Song, for proof-reading parts of the initial drafts. The content presented in this book is based on our works over many years in the areas of fuzzy approximation, multidimensional singular value decomposition, and tensor product polytopic structure and control. We would like to thank our collaborators and graduate students, past and present, for their input and contributions to the various subjects. We also would like to thank the agencies who funded our research projects over the years. They include, especially, the Hungarian National Development Agency (ERC-HU-09-1-2009-0004, MTASZ-TAK) (OMFB-01677/2009) and the Hong Kong Research Grant Council (Project Ref. 2050520, 2150750). Part of the research was also supported by the Janos Bolyai postdoctoral scholarship.

Finally, we thank our families for their support during the writing of this book.

Péter Baranyi, Yeung Yam, and Péter Várlaki

Acronyms and Abbreviations

CHOSVD	Compact Higher-Order SVD
CNO	Close-to-Normality
DoF	Degrees of Freedom
GEVP	Generalized Eigenvalue Minimization Problem
HOOI	Higher-Order Orthogonal Iteration
HOSVD	Higher-Order Singular Value Decomposition
ICA	Independent Component Analysis
INO	Inverse Normality
IRNO	INO and RNO
LFR	Linear Fractional Representation
LMI	Linear Matrix Inequality
LMIs	Linear Matrix Inequalities
LPV	Linear Parameter-Varying
LTI	Linear Time-Invariant
NN	Nonnegativeness
NO	Normality
PDC	Parallel Distributed Compensation
qLPV	quasi-LPV
qNN	quasi-NN
qSN	quasi-SN
RHOSVD	Reduced Higher-Order SVD
RNO	Relaxed Normality
SN	Sum Normalization
SMC	Sliding Mode Control
SMO	Sliding Mode Observer
SVD	Singular Value Decomposition
TORA	Translational Oscillations with an Eccentric Rotational Proof-Mass Actuator
TP	Tensor Product
TP model	Finite Element TP Type Polytopic Model

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Chapter 1

Introduction

1.1 An overview

The book deals with two main areas of emerging importance in the field of systems and control, namely, the quasi-linear parameter-varying (qLPV) models and the linear matrix inequality (LMI) design method. The qLPV models efficiently treat nonlinear plants as linear time-invariant (LTI) systems with varying parameters that are functions of the state variables. Such formulation is applicable to a wide range of problems, for example, in the areas of aerospace control and gain scheduling. The LMI, on the other hand, is a very efficient computation-based method for multiobjective control system design. The integration of the two has been the subject of many ongoing works in recent years. This book, however, deviates from past approaches by bringing forth a number of novel concepts and perspectives.

Instead of using the given qLPV model directly, the book calls for the derivation and utilization of its finite element tensor product (TP) type polytopic model, or TP model for short, for design. Moreover, given that analytically deriving such representation will be a tedious or even impossible task, particularly for more complex systems, the book introduces a higher-order singular value decomposition (HOSVD)-based computational technique to numerically generate from any given qLPV model a TP model that is readily executable for the LMI control design. The TP model as generated may exactly or approximately duplicate the original dynamics, depending on the singular values retained in the process. In turn, the singular values serve as a measure to trade off the accuracy and complexity of the resulting model. For the subsequent LMI design process, the book adopts the parallel distributed compensation (PDC) framework, which uses the same TP form for the controller to readily incorporate multiple design objectives. The book also focuses on the generation of advantageous TP models with proper convexity properties as a means to facilitate controller designs. This so-called convex hull manipulation for design is a new concept introduced in this book.

The book also introduces a MATLAB Toolbox called *TPtool* specifically developed for the present TP model generation, convexity manipulations, and LMI-based controller design. The book also includes a number of practical examples to demonstrate the application of the proposed computational-based modeling and design approach.

The following sections contain more detailed descriptions of the salient features of the book.

1.2 TP model

In the Paris Conference of the International Congress of Mathematicians in 1900, D. Hilbert gave a famous speech culminating in the publication of 23 conjectures on unsolved problems, which he believed would provide the biggest challenge in the 20th century. Specifically, he hypothesized in the 13th conjecture that there exist continuous multivariable functions which cannot be decomposed as a finite superposition of continuous functions of a lesser number of variables [GG00], [Gra00], [Hil00], [Kap77]. This hypothesis was disproved by Arnold in 1957 [Arn57]. In the same year, Kolmogorov [Kol57] formulated a general representation theorem, along with constructive proof, to express continuous multivariable functions in terms of one dimensional functions. This proof justified the existence of *universal approximators*. Kolmogorov's representation theorem was further improved by several authors, including Sprecher [Spr65] and Lorentz [Lor66]. With these results, it has been proved from the 1980s that the approximation tools of biologically inspired neural networks and genetic algorithms are all valid universal approximators, and the same is true for the inference-based fuzzy logic mappings [BL91], [Cyb89], [HSW89], [Cas95], [Kos92], [Wan92], [NK92], [EHR94]. As such, these approximation tools have been widely adopted in the identification of system models. They turned out to be particularly useful for approximating systems that may not be describable by analytical equations.

The above approximation tools thus add to the number of effective identification techniques that we have today. However, the identified models they produced are quite different in form and description to the usual models derived from analytically closed formulas via physical considerations of a given system. For instance, the neural network model would be a graph with a huge numerical array of connection weights, or the fuzzy logic mapping would be a set of linguistic rules. Furthermore, some identification techniques would represent the model in the form of