SIXTH EDITION

MECHANICAL VIBRATIONS

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Mechanical Vibrations

Sixth Edition

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PEARSON

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To Lord Sri Venkateswara

Preface

Changes in This Edition

This book serves as an introduction to the subject of vibration engineering at the undergraduate level. The style of the prior editions has been retained, with the theory, computational aspects, and applications of vibration presented in as simple a manner as possible. As in the previous editions, computer techniques of analysis are emphasized. Expanded explanations of the fundamentals are given, emphasizing physical significance and interpretation that build upon previous experiences in undergraduate mechanics. Numerous examples and problems are used to illustrate principles and concepts. Favorable reactions and encouragement from professors and students have provided me with the impetus to prepare this sixth edition of the book.

Several additions and modifications are made to the fifth edition to make the coverage of vibration more comprehensive and presentation easier to follow in the sixth edition. Most of these additions and modifications were suggested by those who have used the text and by several reviewers. Some of these are indicated below.

- A brief discussion of the anatomy of the human ear and how vibrations are converted into sound by the ear. Experienced engineers can predict the specific cause of malfunction of a machine or engine just by hearing the sound generated by the malfunction.
- Several new applications of vibration are introduced through new problems. The problems are related to the vibration of a child restraint in a child-seat in an automobile, the prediction of injury to head in an automobile accident, the vibratory response of a diver on a high board, the transportation of a precision instrument, and new problems on vibration control.
- The solutions of five examples and eight illustrations are revised for improved presentation so that the reader understands the concept/solution process more easily.
- Additional details are included in the sections related to the response of an undamped system under initial conditions for improved presentation.
- The description and formulation of vibration problems in several different systems of units is considered to obtain the same response of the physical system.
- A new section titled Beams on Elastic Foundation is added along with illustrative examples and problems. This topic finds application in practical situations such as a railway track.
- The stability and vibration of branches of trees with birds sitting on them is considered using the basic principles of mechanics and vibration.
- Nine new examples, 54 new problems (including three new design projects) and 14 new illustrations are added in this edition.

Features of the Book

- Each topic in *Mechanical Vibrations* is self-contained, with all concepts explained fully and the derivations presented with complete details.
- Computational aspects are emphasized throughout the book. MATLAB-based examples as well as several general purpose MATLAB programs with illustrative examples are given in the last section of every chapter. Numerous problems requiring the use of MATLAB or MATLAB programs (given in the text) are included at the end of every chapter.
- Certain topics are presented in a somewhat unconventional manner. The topics of Chapters 9, 10, and 11 fall in this category. Most textbooks discuss isolators, absorbers, and balancing in different chapters. Since one of the main purposes of the study of vibrations is to control vibration response, all topics related to vibration control are given in Chapter 9. The vibration measuring instruments, along with vibration exciters, experimental modal analysis procedure, and machine condition monitoring, are together presented in Chapter 10. Similarly, all the numerical integration methods applicable to single- and multi-degree-of-freedom systems, as well as continuous systems, are unified in Chapter 11.
- Specific features include the following:
  - 252 illustrative examples are given to accompany most topics.
  - 988 review questions are included to help students in reviewing and testing their understanding of the text material. The review questions are in the form of multiple choice questions, questions with brief answers, true-false questions, questions involving matching of related descriptions, and fill-in-the-blank type questions.
  - An extensive set of problems is given in each chapter emphasizing a variety of applications of the material covered in that chapter. In total, there are 1214 problems, with solutions in the instructor's manual.
  - 34 design-project-type problems, many with no unique solution, are given at the end of various chapters.
  - 55 MATLAB programs are included to aid students in the numerical implementation of the methods discussed in the text.
  - Biographical information about 22 scientists and engineers who contributed to the development of the theory of vibrations is presented on the opening pages of all chapters and appendices.
  - MATLAB programs given in the book, answers to problems, and answers to review questions can be found on the Pearson Engineering Resources Portal: www.pearsonhighered.com/engineering-resources/. The *Solutions Manual*, with solutions to all problems and hints to design projects, is available to instructors who adopt the text for their courses via download from the Instructor Resource Center website at www.pearsonhighered.com.

Units and Notation

Both the SI and the English system of units are used in the examples and problems. A list of symbols, along with the associated units in SI and English systems, appear after the Acknowledgments. A brief discussion of SI units as they apply to the field of vibrations is given in Appendix E. Arrows are used over symbols to denote column vectors and square brackets are used to indicate matrices.

Organization of Material

Mechanical Vibrations is organized into 14 chapters and 6 appendices. The reader is assumed to have a basic knowledge of statics, dynamics, strength of materials, and differential equations. Although some background in matrix theory and Laplace transform is desirable, an overview of these topics is given in Appendices C and D, respectively. Chapter 1 starts with a brief discussion of the history and importance of vibrations. The modeling of practical systems for vibration analysis along with the various steps involved in the vibration analysis are discussed. A description of the elementary parts of a vibrating system—stiffness, damping, and mass (inertia)—is given. The basic concepts and terminology used in vibration analysis are introduced. The free vibration analysis of single-degree-of-freedom undamped and viscously damped translational and torsional systems is given in Chapter 2. The graphical representation of characteristic roots and corresponding solutions, the parameter variations, and root locus representations are discussed. Although the root locus method is commonly used in control systems, its use in vibration is illustrated in this chapter. The response under Coulomb and hysteretic damping is also considered. The undamped and damped responses of single-degree-of-freedom systems to harmonic excitations are considered in Chapter 3. The concepts of force and displacement transmissibilities and their application in practical systems are outlined. The transfer function approach, the Laplace transform solution of forced vibration problems, the frequency response, and Bode diagram are presented.

Chapter 4 is concerned with the response of a single-degree-of-freedom system under general forcing function. The roles of Fourier series expansion of a periodic function, convolution integral, Laplace transform, and numerical methods are outlined with illustrative examples. The specification of the response of an underdamped system in terms of peak time, rise time, and settling time is also discussed. The free and forced vibration of two-degree-of-freedom systems is considered in Chapter 5. The self-excited vibration and stability of the system are discussed. The transfer function approach and the Laplace transform solution of undamped and damped systems are also presented with illustrative examples. Chapter 6 presents the vibration analysis of multidegree-of-freedom systems. Matrix methods of analysis are used for the presentation of the theory. The modal analysis procedure is described for the solution of forced vibration problems in this chapter. Several methods of determining the natural frequencies and mode shapes of discrete systems are outlined in Chapter 7. The methods of Dunkerley, Rayleigh, Holzer, Jacobi, and matrix iteration are discussed with numerical examples.

While the equations of motion of discrete systems are in the form of ordinary differential equations, those of continuous or distributed systems are in the form of partial differential equations. The vibration analysis of continuous systems, including strings, bars, shafts, beams, and membranes is given in Chapter 8. The method of separation of variables is presented for the solution of the partial differential equations associated with continuous systems. The Rayleigh and Rayleigh-Ritz methods of finding the approximate natural frequencies are also described with examples. Chapter 9 discusses the various aspects of vibration control, including the problems of elimination, isolation, and absorption. The vibration nomograph and vibration criteria which indicate the acceptable levels of vibration are also presented. The balancing of rotating and reciprocating machines and the whirling of shafts are considered. The active control techniques are also outlined for controlling the response of vibrating systems. The experimental methods used for vibration response measurement are considered in Chapter 10. The hardware used for vibration measurements and signal analysis techniques are described. The machine condition monitoring and diagnosis techniques are also presented.

Chapter 11 presents several numerical integration techniques for finding the dynamic response of discrete and continuous systems. The central difference, Runge-Kutta, Houbolt, Wilson, and Newmark methods are discussed and illustrated. Finite element analysis, with applications involving one-dimensional elements, is discussed in Chapter 12. Bar, rod, and beam elements are used for the static and dynamic analysis of trusses, rods under torsion, and beams. The use of consistent and lumped mass matrices in the vibration analysis is also discussed in this chapter. Nonlinear vibration problems are governed by nonlinear differential equations and exhibit phenomena that are not predicted or even hinted by the corresponding linearized problems. An introductory treatment of nonlinear vibration, including a discussion of subharmonic and superharmonic oscillations, limit cycles, systems with time-dependent coefficients, and chaos, is given in Chapter 13. The random vibration of linear vibration systems is considered in Chapter 14. The concepts of random process, stationary process, power spectral density, autocorrelation, and wide- and narrow-band processes are explained. The random vibration response of single- and multidegree-of-freedom systems is discussed in this chapter.

Appendices A and B focus on mathematical relationships and deflection of beams and plates, respectively. The basics of matrix theory, Laplace transform, and SI units are presented in Appendices C, D, and E, respectively. Finally, Appendix F provides an introduction to MATLAB programming.

Typical Syllabi

The material of the book provides flexible options for different types of vibration courses. Chapters 1 through 5, Chapter 9, and portions of Chapters 6 constitute a basic course in mechanical vibration. Different emphases/orientations can be given to the course by covering, additionally, different chapters as indicated below:

- Chapter 8 for continuous or distributed systems.
- Chapters 7 and 11 for numerical solutions.
- Chapter 10 for experimental methods and signal analysis.
- Chapter 12 for finite element analysis.
- Chapter 13 for nonlinear analysis.
- Chapter 14 for random vibration.

Alternatively, in Chapters 1 through 14, the text has sufficient material for a one-year sequence of two vibration courses at the senior or dual-level.

Expected Course Outcomes

The material presented in the text helps achieve some of the program outcomes specified by ABET (Accreditation Board for Engineering and Technology):

- Ability to apply knowledge of mathematics, science, and engineering:
  The subject of vibration, as presented in the book, applies the knowledge of mathematics (differential equations, matrix algebra, vector methods, and complex numbers) and science (statics and dynamics) to solve engineering vibration problems.
- Ability to identify, formulate, and solve engineering problems:
  The numerous illustrative examples, problems for practice, and design projects help identify various types of practical vibration problems and develop mathematical models, analyze, solve to find the response, and interpret the results.
- Ability to use the techniques, skills, and modern engineering tools necessary for engineering practice:
  - The application of the modern software, MATLAB, for the solution of vibration problems is illustrated in the last section of each chapter. The basics of MATLAB programming are summarized in Appendix F.

    - The use of the modern analysis technique, Finite Element Method, for the solution of vibration problems is covered in a separate chapter (Chapter 12). The finite element method is a popular technique that is used in industry for the modeling, analysis, and solution of complex vibrating systems.
- Ability to design and conduct experiments, as well as to analyze and interpret data:
  The experimental methods and analysis of data related to vibration are presented in Chapter 10. The equipment used in conducting vibration experiments, signal analysis, and identification of system parameters from the data are discussed.

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List of Symbols

Symbol	Meaning	English Units	SI Units
$a, a_0, a_1, a_2, \ldots$	constants, lengths		
a_{ij}	flexibility coefficient	in./lb	m/N
$[a]$	flexibility matrix	in./lb	m/N
A	area	in.^{2}	m^2
$A, A_0, A_1, \ldots$	constants		
$b, b_1, b_2, \ldots$	constants, lengths		
$B, B_1, B_2, \ldots$	constants		
$\vec{B}$	balancing weight	lb	N
$c, \underset{\sim}{c}$	viscous damping coefficient	lb-sec/in.	N · s/m
$c, c_0, c_1, c_2, \ldots$	constants		
c	wave velocity	in./sec	m/s
C_c	critical viscous damping constant	lb-sec/in.	N · s/m
C_i	damping constant of i th damper	lb-sec/in.	N · s/m
C_{ij}	damping coefficient	lb-sec/in.	N · s/m
$[c]$	damping matrix	lb-sec/in.	N · s/m
$C, C_1, C_2, C_1', C_2' \ldots$	constants		
d	diameter, dimension	in.	m
D	diameter	in.	m
$[D]$	dynamical matrix	sec^2	s^2
e	base of natural logarithms		
e	eccentricity	in.	m
$\vec{e}_x, \vec{e}_y$	unit vectors parallel to x and y directions		
E	Young's modulus	lb/in.^{2}	Pa
$E[x]$	expected value of x		
f	linear frequency	Hz	Hz
f	force per unit length	lb/in.	N/m
$\underset{\sim}{f}, f$	unit impulse	lb-sec	N · s
F, F_d	force	lb	N
F_0	amplitude of force $F(t)$	lb	N

Symbol	Meaning	English Units	SI Units
F_t, F_T	force transmitted	lb	N
F_t	force acting on ith mass	lb	N
$\vec{F}$	force vector	lb	N
$\underset{\sim}{F}$, F	impulse	lb-sec	N · s
g	acceleration due to gravity	in./sec^2	m/s^2
$g(t)$	impulse response function		
G	shear modulus	lb/in.^{2}	N/m^2
h	hysteresis damping constant	lb/in.	N/m
$H(i\omega)$	frequency response function		
i	$\sqrt{-1}$		
I	area moment of inertia	in.^{4}	m^4
$[I]$	identity matrix		
Im()	imaginary part of ()		
j	integer		
J	polar moment of inertia	in.^{4}	m^4
$J, J_0, J_1, J_2, \ldots$	mass moment of inertia	lb-in./sec^2	kg · m^2
k, $\underset{\sim}{k}$	spring constant	lb/in.	N/m
k_i	spring constant of ith spring	lb/in.	N/m
k_t	torsional spring constant	lb-in./rad	N-m/rad
k_{ij}	stiffness coefficient	lb/in.	N/m
$[k]$	stiffness matrix	lb/in.	N/m
l, l_i	length	in.	m
m, $\underset{\sim}{m}$	mass	lb-sec^2/in.	kg
m_i	ith mass	lb-sec^2/in.	kg
m_{ij}	mass coefficient	lb-sec^2/in.	kg
$[m]$	mass matrix	lb-sec^2/in.	kg
M	mass	lb-sec^2/in.	kg
M	bending moment	lb-in.	N · m
$M_t, M_{t1}, M_{t2}, \ldots$	torque	lb-in.	N · m
$M\ t_0$	amplitude of $M_t(t)$	lb-in.	N · m
n	an integer		
n	number of degrees of freedom		
N	normal force	lb	N
N	total number of time steps		
p	pressure	lb/in.^{2}	N/m^2
$p(x)$	probability density function of x		
$P(x)$	probability distribution function of x		
P	force, tension	lb	N
q_j	jth generalized coordinate		
$\vec{q}$	vector of generalized displacements		
$\vec{q}$	vector of generalized velocities		
Q_j	jth generalized force		

Symbol	Meaning	English Units	SI Units
r	frequency ratio $= \omega/\omega_n$		
$\vec{r}$	radius vector	in.	m
Re( )	real part of ( )		
R(τ)	autocorrelation function		
R	electrical resistance	ohm	ohm
R	Rayleigh's dissipation function	lb-in./sec	$\text{N} \cdot \text{m/s}$
R	Rayleigh's quotient	l/sec^2	1 /s^2
s	root of equation, Laplace variable		
S_a, S_d, S_v	acceleration, displacement, velocity spectrum		
$S_x(\omega)$	spectrum of x		
t	time	sec	s
t_i	ith time station	sec	s
T	torque	lb-in.	N-m
T	kinetic energy	in.-lb	J
T_i	kinetic energy of ith mass	in.-lb	J
T_d, T_f	displacement, force transmissibility		
u_{ij}	an element of matrix $[U]$		
U,U_i	axial displacement	in.	m
U	potential energy	in.-lb	J
$\vec{U}$	unbalanced weight	lb	N
$[U]$	upper triangular matrix		
v, v_o	linear velocity	in./sec	m/s
V	shear force	lb	N
V	potential energy	in.-lb	J
V_i	potential energy of ith spring	in.-lb	I
w, w_1, w_2, ω_i	transverse deflections	in.	m
W_0	value of w at $t = 0$	in.	m
$\dot{w}_0$	value of $\dot{w}$ at $t = 0$	in./sec	m/s
w_n	nth mode of vibration		
W	weight of a mass	lb	N
W	total energy	in.-lb	J
W	transverse deflection	in.	m
W_i	value of W at $t = t_i$	in.	m
$W(x)$	a function of x		
x, y, z	cartesian coordinates, displacements	in.	m
x_0, $x(0)$	value of x at $t = 0$	in.	m
$\dot{x}_0$, $\dot{x}(0)$	value of $\dot{x}$ at $t = 0$	in./sec	m/s
x_j	displacement of jth mass	in.	m
x_j	value of x at $t = t_j$	in.	m
$\dot{x}_j$	value of $\dot{x}$ at $t = t_j$	in./sec	m/s
x_h	homogeneous part of $x(t)$	in.	m

Symbol	Meaning	English Units	SI Units
x_p	particular part of $x(t)$	in.	m
$\vec{x}$	vector of displacements	in.	m
$\vec{x}_i$	value of $\vec{x}$ at $t = t_i$	in.	m
$\dot{\vec{x}}_i$	value of $\dot{\vec{x}}$ at $t = t_i$	in./sec	m/s
$\ddot{\vec{x}}_i$	value of $\ddot{\vec{x}}$ at $t = t_i$	in./sec^2	m/s^2
$\vec{x}^{(i)}(t)$	ith mode		
X	amplitude of $x(t)$	in.	m
X_j	amplitude of $x_j(t)$	in.	m
$\vec{X}^{(i)}$	ith modal vector	in.	m
$\vec{X}_i^{(j)}$	ith component of jth mode	in.	m
$[X]$	modal matrix	in.	m
$\vec{X}_r$	rth approximation to a mode shape		
y	base displacement	in.	m
Y	amplitude of $y(t)$	in.	m
z	relative displacement, $x - y$	in.	m
Z	amplitude of $z(t)$	in.	m
$Z(i\omega)$	mechanical impedance	lb/in.	N/m
α	angle, constant		
β	angle, constant		
β	hysteresis damping constant		
γ	specific weight	lb/in.^{3}	N/m^3
δ	logarithmic decrement		
$\delta_1, \delta_2, \ldots$	deflections	in.	m
δ_{st}	static deflection	in.	m
δ_{ij}	Kronecker delta		
Δ	determinant		
ΔF	increment in F	lb	N
Δx	increment in x	in.	m
Δt	increment in time t	sec	s
ΔW	energy dissipated in a cycle	in.-lb	J
ε	a small quantity		
ε	strain		
ζ	damping ratio		
θ	constant, angular displacement		
θ_i	ith angular displacement	rad	rad
θ_0	value of θ at $t = 0$	rad	rad
$\dot{\theta}_0$	value of $\dot{\theta}$ at $t = 0$	rad/sec	rad/s
Θ	amplitude of $\theta(t)$	rad	rad
Θ_i	amplitude of $\theta_i(t)$	rad	rad
λ	eigenvalue $= 1/\omega^2$	sec^2	s^2

Symbol	Meaning	English Units	SI Units
$[\lambda]$	transformation matrix		
μ	viscosity of a fluid	lb-sec/in.^{2}	kg/m · s
μ	coefficient of friction		
μ_x	expected value of x		
ρ	mass density	lb-sec^2/in.^{4}	kg /m^3
η	loss factor		
σ_x	standard deviation of x		
σ	stress	lb/in.^{2}	N/m^2
τ	period of oscillation, time, time constant	sec	s
τ	shear stress	lb/in.^{2}	N/m^2
φ	angle, phase angle	rad	rad
φ_i	phase angle in ith mode	rad	rad
ω	frequency of oscillation	rad/sec	rad/s
ω_i	ith natural frequency	rad/sec	rad/s
ω_n	natural frequency	rad/sec	rad/s
ω_d	frequency of damped vibration	rad/sec	rad/s

Subscripts

Symbol	Meaning
cri	critical value
eq	equivalent value
i	ith value
L	left plane
max	maximum value
n	corresponding to natural frequency
R	right plane
0	specific or reference value
t	torsional