

# *Solid State Physics*

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# Preface

We began this project in 1968 to fill a gap we each felt acutely after several years of teaching introductory solid state physics to Cornell students of physics, chemistry, engineering, and materials science. In both undergraduate and graduate courses we had to resort to a patchwork array of reading assignments, assembled from some half dozen texts and treatises. This was only partly because of the great diversity of the subject; the main problem lay in its dual nature. On the one hand an introduction to solid state physics must describe in some detail the vast range of real solids, with an emphasis on representative data and illustrative examples. On the other hand there is now a well-established basic theory of solids, with which any seriously interested student must become familiar.

Rather to our surprise, it has taken us seven years to produce what we needed: a single introductory text presenting both aspects of the subject, descriptive and analytical. Our aim has been to explore the variety of phenomena associated with the major forms of crystalline matter, while laying the foundation for a working understanding of solids through clear, detailed, and elementary treatments of fundamental theoretical concepts.

Our book is designed for introductory courses at either the undergraduate or graduate level.<sup>1</sup> Statistical mechanics and the quantum theory lie at the heart of solid state physics. Although these subjects are used as needed, we have tried, especially in the more elementary chapters, to recognize that many readers, particularly undergraduates, will not yet have acquired expertise. When it is natural to do so, we have clearly separated topics based entirely on classical methods from those demanding a quantum treatment. In the latter case, and in applications of statistical mechanics, we have proceeded carefully from explicitly stated first principles. The book is therefore suitable for an introductory course taken concurrently with first courses in quantum theory and statistical mechanics. Only in the more advanced chapters and appendices do we assume a more experienced readership.

The problems that follow each chapter are tied rather closely to the text, and are of three general kinds: (a) routine steps in analytical development are sometimes relegated to problems, partly to avoid burdening the text with formulas of no intrinsic interest, but, more importantly, because such steps are better understood if completed by the reader with the aid of hints and suggestions; (b) extensions of the chapter (which the spectre of a two volume work prevented us from including) are presented as problems when they lend themselves to this type of exposition; (c) further numerical and analytical applications are given as problems, either to communicate additional

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<sup>1</sup> Suggestions for how to use the text in courses of varying length and level are given on pp. xviii-xxi.

information or to exercise newly acquired skills. Readers should therefore examine the problems, even if they do not intend to attempt their solution.

Although we have respected the adage that one picture is worth a thousand words, we are also aware that an uninformative illustration, though decorative, takes up the space that could usefully be filled by several hundred. The reader will thus encounter stretches of expository prose unrelieved by figures, when none are necessary, as well as sections that can profitably be perused entirely by looking at the figures and their captions.

We anticipate use of the book at different levels with different areas of major emphasis. A particular course is unlikely to follow the chapters (or even selected chapters) in the order in which they are presented here, and we have written them in a way that permits easy selection and rearrangement.<sup>2</sup> Our particular choice of sequence follows certain major strands of the subject from their first elementary exposition to their more advanced aspects, with a minimum of digression.

We begin the book<sup>3</sup> with the elementary classical [1] and quantum [2] aspects of the free electron theory of metals because this requires a minimum of background and immediately introduces, through a particular class of examples, almost all of the phenomena with which theories of insulators, semiconductors, and metals must come to grips. The reader is thereby spared the impression that nothing can be understood until a host of arcane definitions (relating to periodic structures) and elaborate quantum mechanical explorations (of periodic systems) have been mastered.

Periodic structures are introduced only after a survey [3] of those metallic properties that can and cannot be understood without investigating the consequences of periodicity. We have tried to alleviate the tedium induced by a first exposure to the language of periodic systems by: (a) separating the very important consequences of purely translational symmetry [4, 5] from the remaining but rather less essential rotational aspects [7], (b) separating the description in ordinary space [4] from that in the less familiar reciprocal space [5], and (c) separating the abstract and descriptive treatment of periodicity from its elementary application to X-ray diffraction [6].

Armed with the terminology of periodic systems, readers can pursue to whatever point seems appropriate the resolution of the difficulties in the free electron model of metals or, alternatively, can embark directly upon the investigation of lattice vibrations. The book follows the first line. Bloch's theorem is described and its implications examined [8] in general terms, to emphasize that its consequences transcend the illustrative and very important practical cases of nearly free electrons [9] and tight binding [10]. Much of the content of these two chapters is suitable for a more advanced course, as is the following survey of methods used to compute real band structures [11]. The remarkable subject of semiclassical mechanics is introduced and given elementary applications [12] before being incorporated into the more elaborate semiclassical theory of transport [13]. The description of methods by which Fermi surfaces are measured [14] may be more suitable for advanced readers, but much of the survey

<sup>2</sup> The Table on pp. xix-xxi lists the prerequisites for each chapter, to aid those interested primarily in one aspect of the subject, or those preferring a different order of presentation.

<sup>3</sup> References to chapter numbers are given in brackets.

of the band structures of actual metals [15] is readily incorporated into an elementary course.

Except for the discussion of screening, an elementary course might also bypass the essays on what is overlooked by the relaxation-time approximation [16] and by the neglect of electron-electron interactions [17].

Work functions and other surface properties [18] can be taken up at any time after the discussion of translational symmetry in real space. Our description of the conventional classification of solids [19] has been separated from the analysis of cohesive energies [20]. Both have been placed after the introduction to band structure, because it is in terms of electronic structure that the categories are most clearly distinguished.

To motivate the study of lattice vibrations (at whatever point after Chapter 5 readers choose to begin the subject) a summary [21] lists those solid properties that cannot be understood without their consideration. Lattice dynamics is given an elementary introduction, with the classical [22] and quantum [23] aspects of the harmonic crystal treated separately. The ways in which phonon spectra are measured [24], the consequences of anharmonicity [25], and the special problems associated with phonons in metals [26] and ionic crystals [27] are surveyed at an elementary level, though some parts of these last four chapters might well be reserved for a more advanced course. None of the chapters on lattice vibrations rely on the use of normal mode raising and lowering operators; these are described in several appendices for readers wanting a more advanced treatment.

Homogeneous [28] and inhomogeneous [29] semiconductors can be examined at any point after the introduction of Bloch's theorem and the elementary discussion of semiclassical mechanics. Crystalline defects [30] can be studied as soon as crystals themselves have been introduced, though parts of earlier chapters are occasionally referred to.

Following a review of atomic magnetism, we examine how it is modified in a solid environment [31], explore exchange and other magnetic interactions [32], and apply the resulting models to magnetic ordering [33]. This brief introduction to magnetism and the concluding essay on superconductivity [34] are largely self-contained. They are placed at the end of the book so the phenomena can be viewed, not in terms of abstract models but as striking properties of real solids.

To our dismay, we discovered that it is impossible at the end of a seven year project, labored upon not only at Cornell, but also during extended stays in Cambridge, London, Rome, Wellington, and Jülich, to recall all the occasions when students, postdoctoral fellows, visitors, and colleagues gave us invaluable criticism, advice, and instruction. Among others we are indebted to V. Ambegaokar, B. W. Batterman, D. Beaglehole, R. Bowers, A. B. Bringer, C. di Castro, R. G. Chambers, G. V. Chester, R. M. Cotts, R. A. Cowley, G. Eilenberger, D. B. Fitchen, C. Friedli, V. Heine, R. L. Henderson, D. F. Holcomb, R. O. Jones, B. D. Josephson, J. A. Krumhansl, C. A. Kukkonen, D. C. Langreth, W. L. McLean, H. Mahr, B. W. Maxfield, R. Monnier, L. G. Parratt, O. Penrose, R. O. Pohl, J. J. Quinn, J. J. Rehr, M. V. Romerio, A. L. Ruoff, G. Russakoff, H. S. Sack, W. L. Schaich, J. R. Schrieffer, J. W. Serene, A. J. Sievers, J. Silcox, R. H. Silsbee, J. P. Straley, D. M. Straus, D. Stroud, K. Sturm, and J. W. Wilkins.

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One person, however, has influenced almost every chapter. Michael E. Fisher, Horace White Professor of Chemistry, Physics, *and* Mathematics, friend and neighbor, gadfly and troubadour, began to read the manuscript six years ago and has followed ever since, hard upon our tracks, through chapter, and, on occasion, through revision and re-revision, pouncing on obscurities, condemning dishonesties, decrying omissions, labeling axes, correcting misspellings, redrawing figures, and often making our lives very much more difficult by his unrelenting insistence that we could be more literate, accurate, intelligible, and thorough. We hope he will be pleased at how many of his illegible red marginalia have found their way into our text, and expect to be hearing from him about those that have not.

One of us (NDM) is most grateful to the Alfred P. Sloan Foundation and the John Simon Guggenheim Foundation for their generous support at critical stages of this project, and to friends at Imperial College London and the Istituto di Fisica "G. Marconi," where parts of the book were written. He is also deeply indebted to R. E. Peierls, whose lectures converted him to the view that solid state physics is a discipline of beauty, clarity, and coherence. The other (NWA), having learnt the subject from J. M. Ziman and A. B. Pippard, has never been in need of conversion. He also wishes to acknowledge with gratitude the support and hospitality of the Kernforschungsanlage Jülich, the Victoria University of Wellington, and the Cavendish Laboratory and Clare Hall, Cambridge.

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# *Suggestions for Using the Book*

It is in the nature of books that chapters must be linearly ordered. We have tried to select a sequence that least obscures several interwoven lines of development. The accompanying Table (pp. xix–xxi) is designed to help readers with special interests (in lattice vibrations, semiconductors, or metals, for example) or teachers of courses with particular constraints on time or level.

The prerequisites for each chapter are given in the Table according to the following conventions: (a) If “M” is listed after Chapter N, then the contents of Chapter M (as well, of course, as *its* essential prerequisites) are essential for understanding much or all of Chapter N; (b) If “(M)” appears after Chapter N, then Chapter M is not an *essential* prerequisite: either a small part of Chapter N is based on Chapter M, or a few parts of Chapter M may be of some help in reading Chapter N; (c) The absence of “M” or “(M)” after Chapter N does not mean that no reference back to Chapter M is made; however, such references as may occur are primarily because N illuminates the subject of M, rather than because M aids in the development of N.<sup>1</sup>

The rest of the Table indicates how the book might be used in a one semester (40 to 50 lectures) or two-semester (80 to 100 lectures) introductory undergraduate course. Chapters (or selections from a chapter) are listed for reading<sup>2</sup> if they are almost entirely descriptive, or, alternatively, if we felt that an introductory course should at least make students aware of a topic, even if time was not available for its careful exploration. The order of presentation is, of course, flexible. For example, although a two-semester course could follow the order of the book, one might well prefer to follow the pattern of the one-semester course for the first term, filling in the more advanced topics in the second.

An introductory course at the graduate level, or a graduate course following a one-semester undergraduate survey, would probably make use of sections omitted in the two-semester undergraduate course, as well as many of the sixteen appendices.

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<sup>1</sup> Thus to proceed to Chapter 12 with a minimum of digression it is necessary to read Chapters 8, 5, 4, 2, and 1.

<sup>2</sup> Students would presumably be asked to read the chapters bearing on the lectures as well.

<b>Chapter</b>	<b>Prerequisites</b>	<b>One-Semester Introduction</b>		<b>Two-Semester Introduction</b>	
		<b>LECTURES</b>	<b>READING</b>	<b>LECTURES</b>	<b>READING</b>
1. Drude	None	All		All	
2. Sommerfeld	1	All		All	
3. Failures of free-electron model	2		All		All
4. Crystal lattices	None	Summarize	All	All	
5. Reciprocal lattice	4	All		All	
6. X-ray diffraction	5	96-104		All	
7. Crystal symmetries	4				All
8. Bloch's theorem	5	132-143		All	
9. Nearly free electrons	8 (6)	152-166		All	
10. Tight binding	8		176	176-184	184-189
11. Computing band structure	8 (9)		192-193		All
12. Semiclassical dynamics	2, 8	214-233		214-233	
13. Semiclassical transport	12				244-246

<b>Chapter</b>	<b>Prerequisites</b>	<b>One-Semester Introduction</b>		<b>Two-Semester Introduction</b>	
		<b>LECTURES</b>	<b>READING</b>	<b>LECTURES</b>	<b>READING</b>
14. Measuring the Fermi surface	12	264-275		264-275	
15. Band structure of metals	8 (2, 9, 10, 11, 12)		All	All	
16. Beyond relaxation-time approximation	2 (13)				All
17. Beyond independent electron approximation	2		337-342	330-344	345-351
18. Surface effects	2, 4 (6, 8)		354-364	354-364	
19. Classification of solids	2, 4 (9, 10)		All		All
20. Cohesive energy	19 (17)	396-410		All	
21. Failures of static lattice model	(2, 4)		All		All
22. Classical harmonic crystal	5	422-437		All	
23. Quantum harmonic crystal	22	452-464		All	



24. Measuring phonons	2, 23		470-481	All	
25. Anharmonic effects	23		499-505	All	
26. Phonons in metals	17, 23 (16)	523-526		512-519 523-526	
27. Dielectric properties	19, 22		534-542	All	
28. Homogeneous semiconductors	2, 8, (12)	562-580		All	
29. Inhomogeneous semiconductors	28	590-600		All	
30. Defects	4 (8, 12, 19, 22, 28, 29)		628-636		All
31. Diamagnetism, Paramagnetism	(2, 4, 14)	661-665		All	
32. Magnetic interactions	31 (2, 8, 10, 16, 17)	672-682		672-684	
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# Important Tables

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The more important tables of data<sup>1</sup> or theoretical results are listed below. To aid the reader in hunting down a particular table we have grouped them into several broad categories. Theoretical results are listed only under that heading, and data on magnetic and superconducting metals are listed under magnetism and superconductivity, rather than under metals. Accurate values of the fundamental constants are given in the end paper and on page 757.

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<sup>1</sup> The data in the tables are presented with the aim of giving the reader an appreciation for orders of magnitude and relative sizes. We have therefore been content to quote numbers to one or two significant places and have not made special efforts to give the most precise values. Readers requiring data for fundamental research should consult the appropriate sources.

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# 1

## *The Drude Theory of Metals*

Basic Assumptions of the Model

Collision or Relaxation Times

DC Electrical Conductivity

Hall Effect and Magnetoresistance

AC Electrical Conductivity

Dielectric Function and Plasma Resonance

Thermal Conductivity

Thermoelectric Effects