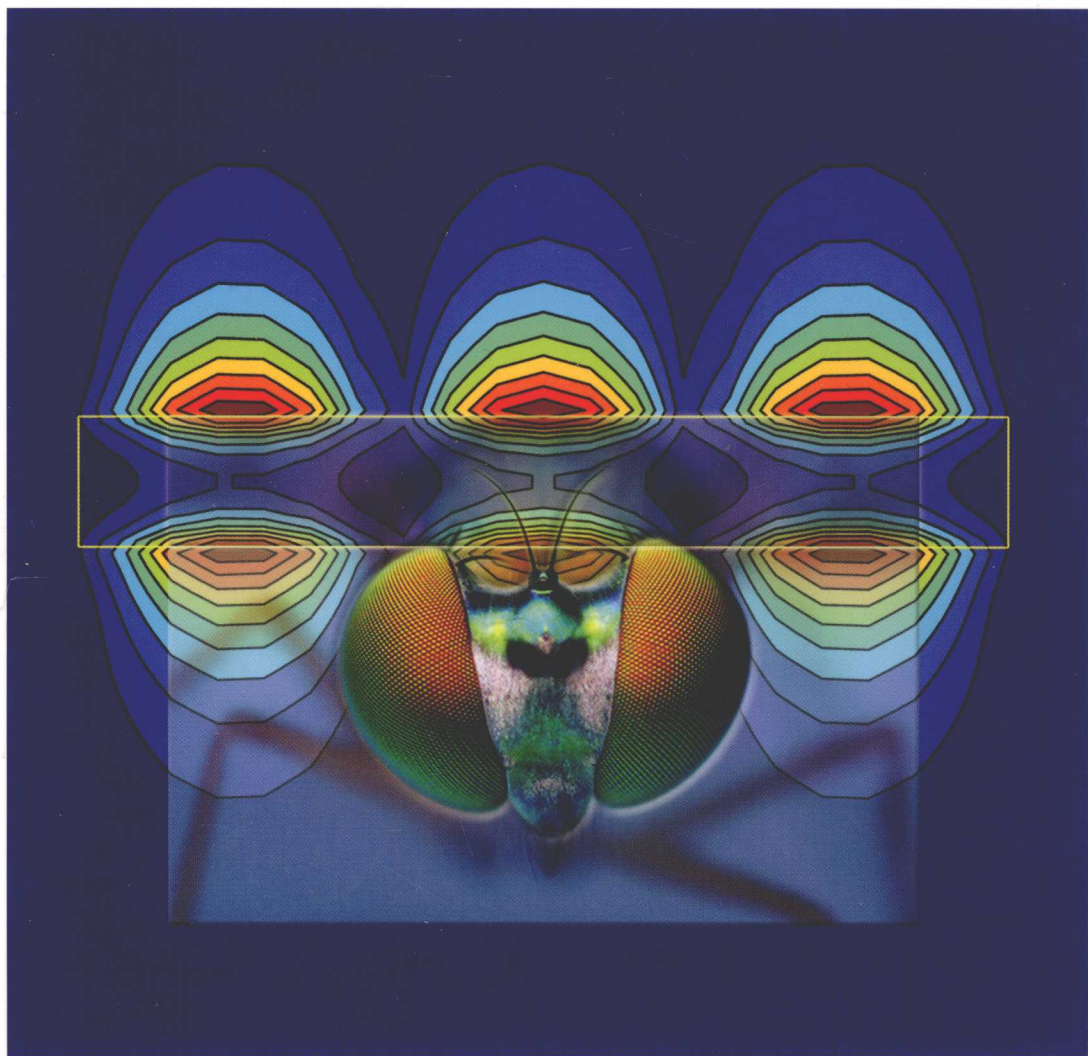


Jürgen Jahns, Stefan Helfert

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Introduction to Micro- and Nanooptics



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How to Study This Textbook

The aim of this book is to make the reader familiar with the physics and mathematics of micro- and nanooptics. The book is mainly intended to serve as a textbook for senior classes at universities, typically in Master and PhD programs. However, we also hope that scientists and engineers in industry will be able to benefit from the book. In general, the skills taught in a Bachelor program in the natural sciences and engineering should be sufficient to get started. The reader should have a certain level of familiarity with basic physics, in particular, optics. Furthermore, we assume fundamental knowledge of electromagnetic theory and mathematics such as vector analysis, differential equations and Fourier theory.

Our main purpose is to provide the reader with a solid theoretical basis of micro- and nanooptical structures. The contents are organized as follows:

The fundamentals of optics like the wave equation and various aspects of light propagation are presented in the first chapters (Chapter 1–3). The basics of free-space propagation are described in Chapter 4. Specific topics of free-space microoptics are then described in Chapter 5 (*Refractive and reflective microoptics*), Chapter 6 (*Diffractive microoptics*), Chapter 8 (*Tunable microoptics*) and Chapter 9 (*Compound and integrated free-space optics*).

Fundamental aspects of waveguide propagation are presented in Chapter 10, specific examples of integrated waveguide optics in Chapter 11. In Chapters 12 and 13, we present novel areas of nanooptics are presented that have gained significant interest in recent years: Chapter 12 introduces the reader to the field of plasmonics, Chapter 13 is about photonic crystals. The list of modern topics is rounded up by Chapter 14 which deals with *left-handed materials*.

For a deeper understanding of the topics and for self-study, the reader will find additional material at the end of each chapter: a list of questions, that relate directly to the text, tells the reader which topics are relevant and offers the chance to test one's comprehension. Furthermore, a few exercises are provided to be solved together in class or individually. For further reading, we suggest a few references that refer to the topic of a chapter. These also have the purpose to introduce students to the world of scientific literature. We would like to remark that the purpose of the book is *not* to present latest results of research. Hence, we have abstained completely from presenting photographs of research results, for example. Lecturers might add suitable material of their own for teaching a class.

This text has to be seen in conjunction with the earlier book on “Microoptics” by S. Sinzinger and J. Jahns (Wiley-VCH Verlag GmbH, 1999 and 2003, 2nd edn). In comparison to that book, here certain topics appear either in a reduced form (like fabrication, for example) or were completely omitted (like characterization and applications of microoptics). Readers interested in those topics might use both books in a complementary fashion, this one for the description of the fundamentals and the earlier book for its presentation of research trends and applications.

Preface

Microlithography has changed the world. The impact of lithographic fabrication cannot be overemphasized: it has paved the way for mass fabrication at an unprecedented level of quality and reliability. The revolutionary development of microelectronics beginning in the 1950s is the foundation of the information society that is characterized by seemingly unlimited access to information as well as capabilities to exchange and store information, symbolized and characterized most of all by the Internet. Microlithographic fabrication has, for the first time in the development of technical evolution, changed a classical pattern of experience. Usually, quality and quantity are mutually exclusive or, in other words, high performance could only be achieved at a high price. As we all know, this is not true for lithographically fabricated devices. The performance and quality have increased by many orders of magnitude (for example, the processing power of electronic computers) while the price has remained constant or has even dropped.

For a long time, people have hoped for the same development in other areas, in particular, in optics and mechanics. Both areas are strongly related, almost like twins. Improvements in fine mechanics have improved classical optics and vice versa. It was in the 1960s when scientists also started to make use of then novel digital design and fabrication techniques for optics. Based on digital design and computer-controlled plotting facilities, computer-generated holography marks the beginning of (or, at least, the forerunner to) the field now known as microoptics. This development received a tremendous push in the 1980s and 1990s. That push was, to some extent, motivated by the rapid development of computing and communications which led to a general interest in novel optical techniques and hardware. As a consequence, the field of microoptics emerged as a new branch of optics and in the course of time has gradually led to numerous useful applications. The development has not been as revolutionary as the development of microelectronics for a number of reasons that will not be discussed here. Nonetheless, microoptics has become an important area of technology that is steadily growing.

Since the 1990s, microlithographic structuring techniques have entered the sub-micron range. The entry into the nanoworld has the potential to lead to a development that might be as revolutionary as the initial beginnings in the microdomain. For one, it has become possible to control and interact with individual atoms rather than statistical ensembles of atoms. This allows one to observe and make use of

quantum effects that are much different in their physics as compared to “macro-physics.” Furthermore, the possibility to generate synthetic nanostructures allows one to engineer material properties, for example, the refractive index of a material. The physical properties of nanostructured materials often surpass those of bulk materials. In optics, so-called quarter-wave stacks are a good example. Used as mirrors, they can be designed and fabricated to reach reflectivities very close to 100%.

The fundamental idea of micro- and nanotechnology is to define the function of a device via its structure. In this book, we deal with micro- and *nanooptics*. Both are related due to the common technology platform used. And yet, they are fascinatingly different areas due to the physics involved: since for many applications, the wavelengths are just around one micrometer, microoptics typically uses structures that are several or many wavelengths in size, while nanodevices are usually in the subwavelength range. Hence, microoptics is still closer to classical phenomena, while nanooptics enters a new world that we do not yet fully oversee.

Hagen, December 2011

Jürgen Jahns, Stefan Helfert

List of Symbols

In the following we give a list of the principal symbols used in this book. Some variables are used for different physical quantities. However, their meaning becomes apparent from the context.

Scalar values, vectors and matrices occur and the following notations are used:

- scalar values are written italic: E_x or k_0
- function are written in roman: \sin , \cos
- physical vectors are written bold and italic: \mathbf{E}
- mathematical vectors and matrices are written in brackets: $[F]$

Scalar Quantities

$A_{I,II}$	amplitude of eigenmodes
a_m^f, a_m^b	amplitudes of forward (backward) propagating eigenmodes
$a(x)$	optical amplitude distribution in transverse direction
a	scaling parameter
a, b	object- and image-sided distances to respective principal planes
$B_{e,o}$	amplitude of even (odd) supermode
B_x, B_y, B_z	Cartesian components of \mathbf{B}
B	phase parameter
c_0	velocity of light in free vacuum
$C(x, t)$	concentration
D_x, D_y, D_z	Cartesian components of \mathbf{D}
$D(x, t)$	diffusion coefficient
d	diameter
ds	infinitesimal path length element
E_x, E_y, E_z	Cartesian components of \mathbf{E}
f	focal distance (occasionally, F is also used)
g	gradient constant
G	second Gaussian moment (also used instead of σ^2)
$G(x, x_0)$	Green's function

H_x, H_y, H_z	Cartesian components of H
$h(x)$	physical height profile
$I(x)$	optical intensity, usually simplified as $I = u ^2$
i	imaginary unit
j_x, j_y, j_z	Cartesian components of j
k_0	free space wavenumber
k_x, k_y, k_z	components of the k -vector
K	contrast
L	number of phase levels (of a diffractive element)
M	magnification
M	number of modes
M^2	beam parameter (of a laser beam)
n	refractive index
Δn	difference of refractive index
n_{eff}	effective index of refraction
N_F	Fresnel number
p_z	periodicity in z -direction
$p(x)$	local period
$p(x)$	point spread function
$\tilde{p}(\nu_x)$	pupil function
P	optical power
p	period (of a grating, for example)
p, p_0	pressure
r_c	radius of curvature
$R(\nu_x)$	aberrated wavefront
$R(z)$	radius of curvature of a Gaussian beam
R	radius
r	reflection coefficient
r, ϕ	polar coordinates
r, ϕ, z	cylindrical coordinates
$t(x)$	physical thickness ("sag")
t	time coordinate
U	(arbitrary) component of electric field
$u(x)$	complex amplitude of scalar field
\tilde{u}	angular frequency spectrum
V	film parameter, fiber parameter
V	voltage
V	volume
w	width, e.g., of a waveguide
w_0	radius of the waist of a Gaussian beam
w_g	1/e-width of Gaussian beam
$w(z)$	radius of a Gaussian beam in transverse direction at a distance z from the waist
$W(\nu_x)$	wavefront
x, y, z	Cartesian coordinates

Δx	spatial shift or width
δx	width (of a slit, for example)
x_0, y_0, z_0	coordinates in the object plane
Y_0	free space wave admittance
Y_W	wave admittance
z_L	longitudinal period in GRIN-lens imaging
z_R	Rayleigh parameter of a Gaussian beam
z_r	rod length of a GRIN lens
z_T	Talbot distance
Z_0	free space wave impedance
Z_W	wave impedance

Vectorial Quantities

B	magnetic flux density, magnetic induction
D	electric displacement
E	electric field
E_L	Lorentz field
F	arbitrary field
G	periodic electric field
G	reciprocal lattice vector
H	magnetic field intensity (strength)
j	electric current density
k	wave vector
M	magnetic polarization density
n	vector normal to a surface
P	electric polarization (density)
R	lattice vector
r	position vector
S	Poynting vector
S_{re}	real Poynting vector
ξ	ray aberration

Matrix Quantities

$[a]$	mathematical vector containing the amplitudes of various modes
$[X]$	eigenvector matrix

Greek Symbols:

α	angle
α	damping constant
α	electric polarizability
α_c	critical angle for total internal reflection
$\alpha_x, \alpha_y, \alpha_z$	angles relative to the Cartesian axes
β	propagation constant
η_0	free space wave impedance
η	diffraction efficiency
γ_L, γ_S	surface tensions
$\gamma_{SL}, \gamma_{WL}, \gamma_{CL}, \gamma_{WC}$	interfacial surface tensions
Γ	complex propagation constant ($\Gamma = -\alpha + i\beta$)
κ	coupling coefficient
λ	wavelength
λ_0	vacuum wavelength
Λ	period of a Bragg grating
μ_r	relative permeability
μ_0	free space permeability
μ	permeability
ν_x, ν_y, ν_z	spatial frequencies
ω	angular frequency
$\phi^{f,b}$	amplitudes of forward (backward) propagating Floquet–Bloch modes
$\Phi(\nu_x)$	wavefront aberration
ρ, θ	polar spatial frequency coordinates
σ^2	variance
σ	mechanical strain (in a membrane)
θ	contact angle
ε_r	relative permittivity
ε_0	free space permittivity
ε	angle
ε	permittivity
φ_g	Guoy phase
φ	optical phase

Other Quantities

A, B, C, D, E	coefficients describing wavefront aberration
\mathcal{E}	energy (of a photon)
\mathcal{L}	eikonal (optical path length)
$\mathcal{L}_x, \mathcal{L}_{xx}$	first and second derivative of eikonal
\mathcal{M}	scaling factor

Specific Mathematical Functions

$J_m, Y_m, I_m, K_m, H_m^{(1,2)}$ cylinder functions of order m

In particular:

J_m	Bessel function of the first kind
Y_m	Bessel function of the second kind (Neumann function)
I_m	modified Bessel function of the first kind
K_m	modified Bessel function of the second kind
$H_m^{(1,2)}$	Hankel function of the first, second kind

Acronyms

FWHM	full-width at half-maximum
LC	liquid crystal
MEMS	micro-electro-mechanical system
NA	numerical aperture
psf	point spread function
SBP	space-bandwidth product
SLM	spatial light modulator

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We would like to express our gratitude to Prof. em. Adolf Lohmann and Prof. em. Reinhold Pregla for many stimulating discussions throughout recent years.

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