

Dick Hess



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The Population Explosion and Other Mathematical Puzzles

To the loving memory of my dear brother, Robert A. (Bob) Hess, 9 November 1940 to 2 March 2015.

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Preface

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This book is a sequel to *Mental Gymnastics: Recreational Mathematical Puzzles* and *Golf on the Moon*, written by me and published by Dover Publishing Co. in 2011 and 2014 respectively. The puzzles in all volumes are for the reader's enjoyment and should be passed on to others for their enjoyment as well. They are meant to challenge mathematical thinking processes, including logical thought, insight, geometrical, analytical and physical concepts, and may require considerable perseverance. While most of the puzzles can be solved by pencil and paper analysis, there are some that are best tackled with a computer to search for or calculate a solution. Be prepared to keep your wits about you.

I often encounter the ideas for many of these puzzles in publications or on-line sources that offer problem columns or puzzle sections. These include *Crux Mathematicorum with Mathematical Mayhem, Journal of Recreational Mathematics, Pi Mu Epsilon Journal, Puzzle Corner* in *Technology Review, Ponder This* and *Puzzle Up*. Other puzzle ideas were introduced to me by word of mouth through a delightful community of puzzle solvers. I owe a debt of gratitude to all enthusiasts who love to share their latest challenges and listen to mine.

Dick Hess

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Chapter 1

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Playful Puzzles

1 Word Mystery

What word has 8 letters, sometimes has 9 — it always contains 8 letters, occasionally uses 12 though! Find either of two answers.

2 Salary Secrecy

A group of 5 employees is at lunch and the subject of their average salary comes up. They all want to know the average but don't want to give information to any other about their own salary. Each has a pencil and piece of paper and there is no one else to assist them. How can they meet their objective?

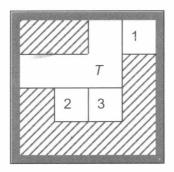
3 Relations Puzzles

- (a) A man points to another man and says: "Sons and daughters have I none but that man's father is my father's son." How are the two men related?
- (b) Ray's son-in-law is my Uncle Bob's father. If I am related by blood to Ray how is Ray related to me?

- 2 Population Explosion and Other Mathematical Puzzles
- (c) "Daughters and nephews have I none but Chris's father-in-law is my mother-in-law's son."
 - (i) What is the speaker's gender?
 - (ii) What is Chris's gender?
 - (iii) How are the speaker and Chris related?

4 Slider

Use only five sliding block moves to get the piece labeled T to the lower right corner. A move is one piece moved along any path.



5 Fastest Serve

A tennis player hits a serve that in kilometers per hour (kph) is exactly 100 more than when expressed in miles per hour (mph). How fast did he serve?

6 The Population Explosion

In March 2015 the estimated population of the earth reached 7.3 billion people. The average person is estimated to occupy a volume of 0.063 m³ so the volume of the total population is 0.4599 km³.

- (a) Model the earth as a sphere with a radius of 6,371 km and spread the volume of people over the surface of the earth in a shell of constant thickness. How thick is the shell?
- (b) The population currently grows geometrically at 1.14% a year. How long will it take at this rate for the population to fill a shell one meter thick covering the earth? What will the population be then?
- (c) At the 1.14% geometric rate how long will it take and what will the population be to occupy a sphere with a radius expanding at the speed of light (= 9.4605284×10^{12} km/yr)? Ignore relativistic effects.

Catenary

A 15 meter chain hangs from two vertical 10 meter poles placed d meters apart. The low point of the chain hangs 2.5 meters from the ground. What is *d*?

Chapter 2

8,3

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Geometric Puzzles

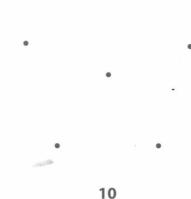
8 Mining on Rigel IV

An amazing thing about the planet Rigel IV is that it is a perfectly smooth sphere of radius 4,000 miles. Like the earth it rotates about a north pole so a latitude and longitude system of coordinates referenced to the poles serves to locate positions on Rigel IV just as it does on earth. Three prospectors make the following reports to headquarters.

- (a) Prospector A: "From my base camp I faced north and went 1 mile in that direction without turning. Then I went east for 1 mile. I rested for lunch before facing north again and going 1 mile in that direction without turning. Finally, I went west for 1 mile and arrived exactly at my base camp." What are the possible locations for base camp A?
- (b) Prospector *B*: "From my base camp I went 1 mile north; then I went 1 mile east. I next went 1 mile south and, finally, I went 1 mile west and arrived exactly at my base camp." What are the possible locations for base camp *B*?
- (c) Prospector *C*: "From my base camp I went 1 mile north; then I went 1 mile east. I next went 1 mile south and, finally, I went 1 mile west and arrived at the most distant point possible from my base camp under these conditions." What are the possible locations for base camp *C* and how far from base camp *C* does the prospector end up?

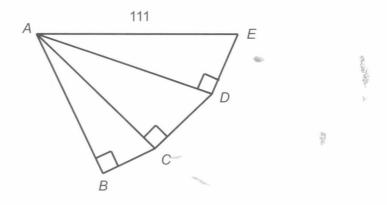
9 Linking Points

A straight line connecting any two points of the six points in the figure is a link. How many links can be placed without forming a triangle with three of the points as vertices?



10 Right Triangles

In the figure AE = 111 and other lengths are unknown. What is the value of $AB^2 + BC^2 + CD^2 + DE^2$?



6.00

11 The Clipped Polyhedron

A polyhedron, P_1 , has a small tip of each of its vertices sliced off by a plane to produce polyhedron P_2 . P_2 has F faces, V vertices and E edges.

- (a) One of F, V or E equals 11. What are the two possibilities for P_1 ?
- (b) One of F, V or E equals 13. What are the four possibilities for P_1 ?

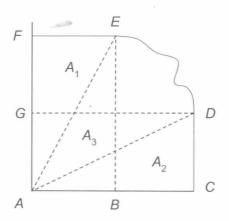
12 The Papered Boxes

- (a) Aksana states to her friend, Josh: "I have an ideal rectangular box with integer dimensions and no top. I have papered both inside and outside (ten surfaces) and notice that, amazingly, the area of paper in square units is the same as the volume of the box in cubic units. Furthermore, my box has the maximum volume for this situation." Josh replies: "My box has the same properties but the minimum volume possible." What are the dimensions of the two boxes?
- (b) Bob states to Kathy: "I have an ideal rectangular box with integer dimensions and no top. I have papered the five outside surfaces and notice that, amazingly, the area of paper in square units is the same as the volume of the box in cubic units. Furthermore, my box has the volume of a cube but is not a cube." Kathy replies: "My box also has an area of paper in square units that is the same as the volume of the box in cubic units but has half the volume of yours." What are the dimensions of the two boxes?
- (c) Chris states to her friend, Laurie: "I have an ideal rectangular box with integer dimensions. I have papered both inside and outside of the six surfaces and notice that, amazingly, the area of paper in square units is the same as the volume of the box in cubic units. Furthermore, the largest dimension of my box is odd." Laurie replies: "My box has the same properties but a smaller volume than yours." What are the dimensions of the two boxes?

(d) David states to his friend, Mary: "I have an ideal rectangular box with integer dimensions. I have papered the six outside surfaces and notice that, amazingly, the area of paper in square units is the same as the volume of the box in cubic units. Furthermore, the dimensions of my box are all different." Mary replies: "My box has the same properties but half the volume of yours." What are the dimensions of the two boxes?

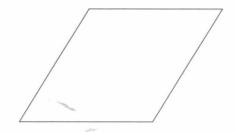
13 The Almost Rectangular Lake

The figure shows a lake ABCDEFG that is nearly rectangular except for the segment of its shoreline, DE. Let A_1 be the area of DEFG, let A_2 be the area of BCDE and let A_3 be the area of the wedge ADE. What is A_3 in terms of A_1 and A_2 ?



14
The Trifurcated Diamond

The tile shown is a diamond with 120° and 60° angles. Divide it into three tiles that are similar to each other, that is, they are the same shape but may be of different sizes. Tiles may be turned over. Find six solutions.



15 **Square Dissection**

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Completely dissect a square into the lowest number of different sized rectangles with integer edges and a length to width ratio of 3 to 1.