Solvable Theory of Nonlinear Differential Equations and Their Applications

（非线性微分方程的可解性理论及其应用）

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Preface

Many differential equations are motivated by problems arising from models of chemical reactors, neutron transport, population biology, infectious diseases, economics, engineering, control theory, economics and other systems. Usually, we need to discuss the existence of nonnegative solutions with certain desired qualitative properties. On the subject of differential equations many elementary books have been written. This book gives a systematic study to the exactness, existence and multiplicity of solutions of p-Laplacian type equations, prescribed mean curvature equations, fractional differential equations, impulsive differential equations, elasticity beam equations and functional differential equations. In light of the content and the methods used, this book is divided into eight chapters.

Chapter 1 gives a survey to the background, history and research development of p-Laplacian type equations, prescribed mean curvature equations, fractional differential equations, impulsive differential equations, elastic beam equations and functional differential equations.

The methods, theorems and definitions to be used are briefly summarized in Chapter 2.

By using time-map analysis and fixed point fixed point theorems, chapter 3 systematically studies the exactness and the existence of solutions for the one-dimensional p-Laplacian equations. In Section 3.1, the exact number and bifurcation diagrams of positive solutions are obtained for the one-dimensional p-Laplacian in a class of two-point boundary value problems under the nonlinearity f is general form $f(u) = \lambda g(u) + h(u)$. Meanwhile, some properties of the solutions are given in details. In Section 3.2, the exact number of pseudo-symmetric positive solutions is obtained for a class of three-point boundary value problems with one-dimensional p-Laplacian. In Section 3.3, using fixed point techniques combining with partially ordered structure of Banach space, we are concerned with determining values of λ, for which there exist positive solutions for a class of singular p-Laplacian differential equations. In particular, the dependence of positive solution $u_\lambda(t)$ on the parameter λ is also studied.

In Chapter 4, we employ the time-map analysis and Mawhin's continuation theorem to investigate the exactness of positive solutions and the existence of periodic solutions of prescribed mean curvature operator equations, respectively. In Section 4.1, we utilize time-map analysis to discuss the exact number and bifurcation diagram of positive solutions are obtained for the one-dimensional prescribed mean curvature equation when $f(u) = u^p + u^q$ under the case $0 < p < 1 < q < +\infty$. In Section 4.2, by using time-map analysis, we consider the exact number and bifurcation diagram of positive solutions for a one-dimensional prescribed mean curvature equation when $f(u) = u^p + u^q$ under six cases: $p = 0,\ 0 < q \leqslant 2$; $p = 1,\ 1 < q \leqslant 4$; $q = 1,\ 0 < p < 1$; $-1 < p < q \leqslant 0$; $-1 < p < 0,\ 1 < q < +\infty$ and $-1 < p < 0,\ 0 < q \leqslant 1$. In Section 4.3, we employ coincidence degree theory to study the existence of periodic solutions for a prescribed mean curvature Liénard equation with a deviating argument.

Chapter 5 utilizes fixed point theorems to investigate the existence of solution for fractional differential equations. In Section 5.1, the expression and properties of Green's function for a class of nonlinear fractional differential equations with integral boundary conditions are studied and employed to obtain some results on the existence of positive solutions by using fixed point theorem in cones. In Section 5.2, we investigate the existence and multiplicity of positive solutions for a class of higher-order nonlinear fractional differential equations with integral boundary conditions. In Section 5.3, we discuss the expression and properties of Green's function for boundary value problems of nonlinear Sturm-Liouville-type fractional order impulsive differential equations.

In Chapter 6, we shall investigate the existence and multiplicity of positive solutions for elastic beam equations with integral boundary conditions. In Section 6.1, using topological degree theory combining with partially ordered structure of space, some sufficient conditions for the existence and multiplicity of symmetric positive solutions for a beam equation are established. Meanwhile, the nonexistence of symmetric positive solutions is also studied. By applying a new technique for dealing with the bending term of beam equations, we consider the existence and multiplicity of positive solutions for a fourth order p-Laplacian elasticity problems in Section 6.2. It is interesting to point out that estimates on the norms of these solutions will also be provided. In section 6.3, we utilize the fixed point theory for completely continuous operator to investigate the existence of positive solutions for a class of fourth order impulsive beam equations with integral boundary conditions and one-dimensional p-Laplacian. Moreover, we offer some interesting discussion of the associated boundary value problems.

Chapter 7 gives some new results of functional differential equations and impulsive functional differential equations. In Section 7.1, using well-known fixed point index theory in a cone, we deal with the existence of positive periodic solution for a functional differential equation with a parameter. The dependence of positive periodic solution $x_\lambda(t)$ on the parameter λ is also studied. In Section 7.2, we consider the existence of positive periodic solutions for the first-order impulsive functional differential equations with two parameters. Several new and more general existence and multiplicity results are derived in terms of different values of $\lambda > 0$ and $\mu > 0$. Here we not only consider the case that g is bounded, but the case that g is not necessarily bounded is also considered. Section 7.3 investigates the existence of periodic solutions, especially for the existence of nontrivial periodic solutions for a Rayleigh equation with two deviating arguments. The arguments are based upon Leray-Schauder index theorem and Leray-Schauder fixed point theorem. Meanwhile, two examples are worked out to demonstrate the main results. In Section 7.4, we apply Leray-Schauder fixed point theorem to investigate the existence of anti-periodic solutions for a Rayleigh equation with two deviating arguments. To illustrate how our main results can be used in practice we present an example. In Section 7.5, we investigates the expression and properties of Green's function for a second-order singular boundary value problem with integral boundary conditions and a delayed argument. Furthermore, several new and more general results are obtained for the existence of positive solutions for the above equation by using Krasnosel'skii's fixed point theorem. In Section 7.6, using a new method for dealing with second order singular p-Laplacian equations with impulsive effects and deviating arguments, several new and more general results are obtained for the existence of at least single, twin or triple positive solutions by using Krasnosel'skii and Zabreiko's fixed point theorem, fixed point theorem due to Avery and Henderson and Leggett-Williams's fixed point theorem.

In Chapter 8, we first consider the existence of positive solutions of impulsive differential equations with parameters. And then, introduce a new method for dealing with impulse term of second impulsive differential equations. In Section 8.1, using fixed point theorem in a cone, we consider a second order singular impulsive differential equation with a parameter and establish the dependence results of the solution on the parameter. In Section 8.2, we apply fixed point techniques combining with partially ordered structure of Banach space to investigate the existence and multiplicity results for a second-order impulsive differential equations involving the one-dimensional singular p-Laplacian. The exact upper and lower bounds for these positive solutions are also given. In Section 8.3, we discuss multi-parameter fourth order impulsive differential equations with one-dimensional m-Laplacian and deviating arguments. Using inequality techniques and fixed point theories, several new and more general existence and multiplicity results are derived in terms of different values of λ and μ. In Section 8.4, using a new method for dealing with impulse term of second impulsive differential equations, we are concerned with determining values of λ, for which there exist positive solutions. Section 8.5 investigates the existence of positive solutions for a second order impulsive differential equations with deviating arguments by using transformation technique and Krasnosel'skii's fixed point theorem. We discuss our problems under two cases when the deviating arguments are delayed and advanced. The approach to deal with the impulsive term is different from earlier approaches.

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Contents

Chapter 1

Progress in Nonlinear Differential Equations

This chapter gives a survey to the background, history and research development of p-Laplacian type equations, prescribed mean curvature equations, fractional differential equations, impulsive differential equations, elastic beam equations and functional differential equations.

1.1 p-Laplacian type equations

Differential equations with p-Laplacian arise naturally in non-Newtonian mechanics, nonlinear elasticity, glaciology, population biology, combustion theory, and nonlinear flow laws, see [1,2]. In recent years many cases of the exactness, existence, multiplicity and uniqueness of positive solution of differential equations with p-Laplacian have attracted considerable attention (see [3-15]).

Addou et al. [3] and Sanchez and Ubilla [4] independently proved exact multiplicity of positive solutions for a more general k-Laplacian problem

$$\begin{cases} -(\phi_k(u'(x)))' = \lambda f(u), & -1 < x < 1, \\ u(-1) = u(1) = 0, \end{cases} \tag{1.1.1}$$

where $f(u) = u^q + u^p,\ k > 1, \phi_k(y) = |y|^{k-2}y$ and $(\phi_k(u'))'$ is one-dimensional k-Laplacian. Using shooting method, for $0 < q < k-1 < p$, they proved the existence of some $\lambda^* > 0$ such that (1.1.1) has exactly two positive solutions for $0 < \lambda < \lambda^*$, exactly one positive solution for $\lambda = \lambda^*$ and no positive solution for $\lambda > \lambda^*$.

In [7], Kajikiya et al. investigated the following one-dimensional p-Laplacian problem

$$\begin{cases} (\varphi_p(u'))' + \lambda\omega(t)f(u) = 0, & t \in (0,1), \\ u(0) = u(1) = 0, \end{cases} \tag{1.1.2}$$

and by virtue of the global bifurcation theory, they obtained the existence, nonexistence, uniqueness and multiplicity of positive solutions as well as sign-changing solutions under suitable conditions imposed on the nonlinear term f.

In [8], Dai et al. investigated the existence of one-sign solutions for the following periodic p-Laplacian problem

$$\begin{cases} -(\varphi_p(u'))' + q(t)\varphi_p(u) = \lambda\omega(t)f(u), & 0 < t < T, \\ u(0) = u(T), \quad u'(0) = u'(T). \end{cases} \tag{1.1.3}$$

The author also examined the uniqueness of the solution and its dependence on the parameter λ under condition

(H) $\dfrac{f(s)}{\varphi_p(s)}$ is strictly decreasing in $(0,\infty)$.

On the other hand, we notice that there has been a considerable attention on impulsive differential equations with one-dimensional p-Laplacian. For example, in [12], employing the classical fixed point index theorem for compact maps, X.Zhang et al. obtained some sufficient conditions for the existence of multiple positive solutions of the problem

$$\begin{cases} (\varphi_p(u'(t)))' = -f(t,u(t)), & 0 < t < 1, \quad t \neq t_k, \\ \Delta u|_{t=t_k} = -I_k(u(t_k)), \\ \Delta u'|_{t=t_k} = 0, \quad k = 1, 2, \cdots, m, \\ u(0) = \sum\limits_{i=1}^{m-2} a_i u(\xi_i), \quad u'(1) = 0. \end{cases} \tag{1.1.4}$$

Of course, some questions are

Q 1.1.1 Whether or not the exactness of positive solutions for problem (1.1.1) can be obtained when the nonlinearity f is general form: $f(u) = \lambda g(u) + h(u)$? Furthermore, can we generalize and improve the exactness results to nonlocal problems?

Q 1.2.2 Without a similar condition to that of (H), can we show the dependence of positive periodic solution $x_\lambda(t)$ on the parameter λ for problem (1.1.3)?

Q 1.2.3 If problem (1.1.4) with a parameter or multiple parameters, then can we investigate the existence, multiplicity and nonexistence of it?

1.2 Prescribed mean curvature equations

Mean curvature equations arise in differential geometry, physics and other applied subjects. For example, the negative solutions of prescribed mean curvature equations can describe pendent liquid drops in the equilibrium state(See[16]), or corneal shape(See[17]). In recent years, increasing attention has been paid to the study of the prescribed mean curvature equations by different methods(See [18-23]).

A typical model of prescribed mean curvature equation is

$$\begin{cases} -div\Big(\dfrac{\nabla u}{\sqrt{1+\|\nabla u\|^2}}\Big) = \lambda f(t,u), & t\in \mathbb{R}^+, u\in \ \Omega, \\ u=0 \ \text{ on } \partial\Omega, \end{cases} \tag{1.2.1}$$

where Ω is a bounded domain in $\mathbb{R}^N$ and $f : \bar{\Omega}\times\mathbb{R}^+ \to \mathbb{R}^+$ is continuous.

The one-dimensional version of (1.2.1) is

$$\begin{cases} -\Big(\dfrac{u'}{\sqrt{1+u'^2}}\Big)' = \lambda f(t,u), & a<t<b, \\ u(a)=u(b)=0. \end{cases} \tag{1.2.2}$$

There are some papers considering the exact number of positive solutions of (1.2.2) in special case of f (See[18, 19]). The study derived from an open problem proposed by A. Ambrosetti, H. Brezis and G. Cerami in [24], which concerned the exact number and the detailed property of solutions of the semilinear equation

$$\begin{cases} -u'' = \lambda(u^p+u^q), & u>0 \ \text{ in } \ (0,1), \\ u(0)=u(1)=0, \end{cases} \tag{1.2.3}$$

where $0<p<1<q<+\infty$. Since then, related problems have been studied by many authors, see [25,26] and the references cited therein.

Recently, P.Habets and P.Omari [18] considered the existence, nonexistence and multiplicity of positive solutions of the Dirichlet problem for the one-dimensional prescribed curvature equation

$$\begin{cases} -\Big(\dfrac{u'}{\sqrt{1+u'^2}}\Big)' = f(u), \ u>0 \ \text{ in } \ (0,1), \\ u(0)=u(1)=0, \end{cases} \tag{1.2.4}$$

in connection with the changes of concavity of the function f, such as $f(u) = \max\{\lambda u^p, \mu u^q\}$ and $f(u) = \min\{\lambda u^p, \mu u^q\}$, where $0<p<1<q$. In particular, they obtained the following results for $f(u)=\lambda u^p$, where $0<p<1$, $p=1$ or $p>1$.

Theorem 1.2.1 (See P.Habets and P.Omari [18])

(i) If $0<p<1$ then there exist λ_* and λ^* with $0<\lambda_*<\lambda^*$ such that (1.2.4) has exactly one solution for $\lambda\in(0,\lambda_*]\cup\{\lambda^*\}$, exactly two solutions for $\lambda\in(\lambda_*,\lambda^*)$, and no solution for $(\lambda^*,+\infty)$.

(ii) If $p=1$ then there exists λ_* with $0<\lambda_*<\lambda^*=\pi^2$ such that (1.2.4) has exactly one solution for $\lambda\in(\lambda_*,\lambda^*)$ and no solution for $\lambda\in(0,\lambda_*)\cup[\lambda^*,+\infty)$.

(iii) If $p>1$ then there exists $\lambda_*>0$ such that (1.2.4) has no solution for $\lambda\in(0,\lambda_*)$ and exactly one solution for $\lambda\in(\lambda_*,+\infty)$.

Moreover, P. Habets and P. Omari said their results still hold in the case $f(u) = \lambda u^p + \mu u^q$ (See, Remark 4.1 in [18]).

Very recently, W.Li and Z.Liu [19] examined problem (1.2.4) when $f(u) = u^p + u^q$, and obtained the following theorems.

Theorem 1.2.2 (See W. Li and Z. Liu [19]) Assume that $1 < p < q < +\infty$. The following conclusions hold:

(i) for any $\lambda > 0$, (1.2.4) has at most one solution;

(ii) there exist $0 < \lambda_1 < \lambda_2 < +\infty$ such that (1.2.4) has no solution for $0 < \lambda < \lambda_1$ and has exactly one solution for $\lambda > \lambda_2$;

(iii) if, in addition

$$q \leqslant \frac{p - 2 + \sqrt{p^2 + 20p + 20}}{2},$$

then there exists $0 < \lambda_* < +\infty$ such that (1.2.4) has no solution for $\lambda \leqslant \lambda_*$ and has exactly one solution for $\lambda > \lambda_*$.

Theorem 1.2.3 (See W. Li and Z. Liu [19]) Assume that $0 < p < q < 1$. The following conclusions hold:

(i) for any $\lambda > 0$, (1.2.4) has at most two solutions;

(ii) there exist $0 < \lambda_1 < \lambda_2 < +\infty$ such that (1.2.4) has exactly one solution for $0 < \lambda < \lambda_1$ and has no solution for $\lambda > \lambda_2$;

(iii) if, in addition

$$p \geqslant \left(\frac{\pi + 2}{\sqrt{2} \ln(\sqrt{2} + 1) + 2} \right)^{1/2} - 1,$$

then there exist λ_* and λ^* with $0 < \lambda_* < \lambda^*$ such that (1.2.4) has exactly one solution for $\lambda \in (0, \lambda_*] \cup \{\lambda^*\}$, exactly two solutions for $\lambda \in (\lambda_*, \lambda^*)$, and no solution for $(\lambda^*, +\infty)$.

On the other hand, W.Li and Z.Liu [19] raised an open problem: It would be very interesting to study exact number of solution of (1.2.4) if p, q satisfy $0 < p < 1 < q < +\infty$ (See (d) of Remark 1.4 in [19]). The purpose here is to solve this problem.

Liénard type equations have a wide range of applications in applied science, such as physics, biology, mechanics, and the engineering technique fields (for instance, see [27-29]). However, to the best of our knowledge, the corresponding theory for prescribed mean curvature Liénard equation with a deviating argument is not investigated till now.

1.3 Impulsive differential equations

Impulsive differential equations, which provide a natural description of observed evolution processes, are regarded as important mathematical tools for the better understanding of several real world problems in applied sciences, such as population dynamics, ecology, biological systems, biotechnology, industrial robotic, pharmacokinetics, optimal control, etc. Therefore, the study of this class of impulsive differential equations has gained prominence and it is a rapidly growing field. For the general theory of impulsive differential equations, we refer the reader to the references [30-32] whereas the applications of impulsive differential equations can be found in [33-35].

Some classical tools such as bifurcation theory [36], fixed point theorems in cones [37], the method of lower and upper solutions [38] and the theory of critical point theory and variational methods [39,40] have been widely used to study impulsive differential equations.

At the same time, we notice that a new technique via appropriate transformation is proved to be very effective in studying the solvability of impulsive differential equations. Such techniques have attracted the attention of X. Zhang, J. Yan and A. Zhao [41] and X. Sun, H. Huo and C. Ma [42], etc.

In [41], X. Zhang, J. Yan and A. Zhao transformed the following problems

$$\begin{cases} x'(t) = -a(t)x(t) + p(t)f(t, x(t-\tau_0(t)), x(t-\tau_1(t)), \cdots, x(t-\tau_n(t))), \\ \qquad \text{a.e. } t > 0, \quad t \neq t_k, \\ x(t_k^+) - x(t_k) = b_k x(t_k), \quad k = 1, 2, \cdots \end{cases} \tag{1.3.1}$$

into

$$y'(t) = -a(t)y(t) + \lambda(t)g(t, y(t-\tau_0(t)), y(t-\tau_1(t)), \cdots, y(t-\tau_n(t))), \quad \text{a.e. } t \geqslant 0 \tag{1.3.2}$$

by using $x(t) = b(t)y(t)$, where

$$b(t) = \prod_{0<t_k<t} (1 + b_k),$$

here $\{b_k\}$ is a real sequence with $b_k > -1, \quad k = 1, 2, \cdots$. Then they considered the neutral differential equation (1.3.2) without impulses for establishing the existence of periodic solutions of problem (1.3.1).

Recently, by utilizing the same method as the papers [41], X. Sun, H. Huo and C. Ma [42] generalized the results of problem (1.3.1).

However it is quite difficult to apply this approach to second impulsive differential equation, especially for second impulsive differential equation with deviating arguments. Another question is that there are only a few articles which dealt with some impulsive differential equations with quasilinear operator.

1.4 Functional differential equations

Functional differential equations with periodic delays appear in some ecological models. For example, the model of the survival of red blood cells in an animal (See [43]), and the model of "dynamic disease" (See [44]), and so on. One of the important questions is whether these equations can support positive periodic solutions. In recent years, periodic population dynamics has become a very popular subject, and several different periodic models have been studied by many authors; see [45-47] and references therein.

An important functional differential equation is Rayleigh equation with two deviating arguments in the form of

$$x''(t) + f(x'(t)) + g_1(t, x(t - \tau_1(t))) + g_2(t, x(t - \tau_2(t))) = e(t), \tag{1.4.1}$$

which has been studied by authors [48-50] under the assumption of $f(0) = 0$ or $f(t, 0) = 0$. It is not difficult to see that if $g_1(t, 0) + g_2(t, 0) \not\equiv e(t)$, then the periodic solution obtained in [48-50] must be nontrivial. But if $g_1(t, 0) + g_2(t, 0) \equiv e(t)$, then the periodic solution obtained in [48-50] may be trivial under the assumption of $f(0) = 0$ or $f(t, 0) = 0$. And if the periodic solution is unique, then it must be trivial. Thus, it is worth discussing the existence of the nontrivial periodic of Rayleigh equations with two deviating arguments in this case.

At the same time, we notice that there exist only few results for the existence of anti-periodic solutions for Rayleigh equation and Rayleigh type equations with and without deviating arguments in the literature. The main difficulty lies in the middle term $f(x'(t))$ of Eq.(1.4.1), the existence of which obstructs the usual method of finding a priori bounds for delay Duffing or Liénard equations from working. Thus, it is worthwhile to continue to investigate the anti-periodic solutions of Rayleigh equation in this case.

Recently, differential equations with deviating arguments have received much attention. For example, in [51], C. Yang, C. Zhai and J. Yan studied the existence and multiplicity of positive solutions to a three-point boundary value problem with an advanced argument

$$\begin{cases} x''(t) + a(t)f(x(\alpha(t))) = 0, \quad t \in (0, 1), \\ x(0) = 0, \quad bx(\eta) = x(1), \end{cases} \tag{1.4.2}$$

where $0 < \eta < 1$, $b > 0$ and $1 - b\eta > 0$. The main tool is the fixed point index theory. It is clear that the solution of [51] is concave when $a(t) \geqslant 0$ on $[0, 1]$ and $f(x) \geqslant 0$ on $[0, \infty)$. However, few papers have been reported on the same problems when the solution without concavity.

Very recently, a class of p-Laplacian differential equations with deviating arguments both of an advanced or delayed type have received much attention. For example, in [52], T. Jankowski considered the following third order p-Laplacian differential equation

$$\begin{cases} (\varphi_p(u''(t)))' + h(t)f(t, u(t), u(\alpha(t))) = 0, \quad t \in J', \\ u'(t_k^+) = u'(t_k^-) + Q_k(u(t_k)), \quad k = 1, 2, \cdots, m, \\ \beta u(0) - \gamma u'(0) = 0, \quad \delta u(1) + \eta u'(1) = 0, \quad u''(0) = 0, \end{cases} \tag{1.4.3}$$

where $\alpha(t) \not\equiv t$ on J. The author obtained the existence of at least three positive solutions. The main tool is a fixed point theorem due to Avery [53] which is a generalization of the Leggett-Williams fixed point theorem.

Of course, a natural question is

Q 1.4.1 Whether or not the existence of positive solutions for a second order p-Laplacian differential equation with deviating arguments both of an advanced or delayed type can be proved?

Remark 1.4.1 In [52], by means of the properties of Green's function, T. Jankowski obtain the inequality

$$\min_{t \in [\xi, 1]} u(t) \geqslant \rho \|u\|,$$

where $\xi \in (0, 1)$ and $\rho \in (0, 1)$.

In fact, the calculation of ρ is very difficult when $t \in [\xi, 1]$. This is probably the main reason that there is almost no paper to study the existence of positive solutions for class of second order p-Laplacian impulsive differential equations with two parameters and deviating arguments both of an advanced or delayed type. In [52], T. Jankowski obtained a constant number ρ by means of the properties of Green's function. However, it is well known that there is not any Green's function in one-dimensional p-Laplacian boundary value problems of second order differential equations. This implies the following question.

Q 1.4.2 Whether or not a similar inequality can be obtained if there is no Green's function when $t \in [\xi, 1]$?

This needs to open a new technique to deal with second order p-Laplacian equations with deviating arguments, especially for second order p-Laplacian equations with impulsive effects.

1.5 Fractional differential equations

Fractional differential equations arise in many engineering and scientific disciplines as the mathematical modelling of systems and processes in the fields of physics, chemistry, aerodynamics, electrodynamics of complex medium, polymer rheology, Bode's analysis of feedback amplifiers, capacitor theory, electrical circuits, electron-analytical chemistry, biology, control theory, fitting of experimental data, and so