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Topology and Geometry for Physics

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（影印版）

〔德〕埃施里格（H. Eschrig）著

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序　言

物理学是研究物质、能量以及它们之间相互作用的科学。她不仅是化学、生命、材料、信息、能源和环境等相关学科的基础，同时还是许多新兴学科和交叉学科的前沿。在科技发展日新月异和国际竞争日趋激烈的今天，物理学不仅囿于基础科学和技术应用研究的范畴，而且在社会发展与人类进步的历史进程中发挥着越来越关键的作用。

我们欣喜地看到，改革开放三十多年来，随着中国政治、经济、教育、文化等领域各项事业的持续稳定发展，我国物理学取得了跨越式的进步，做出了很多为世界瞩目的研究成果。今日的中国物理正在经历一个历史上少有的黄金时代。

在我国物理学科快速发展的背景下，近年来物理学相关书籍也呈现百花齐放的良好态势，在知识传承、学术交流、人才培养等方面发挥着无可替代的作用。从另一方面看，尽管国内各出版社相继推出了一些质量很高的物理教材和图书，但系统总结物理学各门类知识和发展，深入浅出地介绍其与现代科学技术之间的渊源，并针对不同层次的读者提供有价值的教材和研究参考，仍是我国科学传播与出版界面临的一个极富挑战性的课题。

为有力推动我国物理学研究、加快相关学科的建设与发展，特别是展现近年来中国物理学者的研究水平和成果，北京大学出版社在国家出版基金的支持下推出了“中外物理学精品书系”，试图对以上难题进行大胆的尝试和探索。该书系编委会集结了数十位来自内地和香港顶尖高校及科研院所的知名专家学者。他们都是目前该领域十分活跃的专家，确保了整套丛书的权威性和前瞻性。

这套书系内容丰富，涵盖面广，可读性强，其中既有对我国传统物理学发展的梳理和总结，也有对正在蓬勃发展的物理学前沿的全面展示；既引进和介绍了世界物理学研究的发展动态，也面向国际主流领域传播中国物理的优秀专著。可以说，“中外物理学精品书系”力图完整呈现近现代世界和中国物理

科学发展的全貌，是一部目前国内为数不多的兼具学术价值和阅读乐趣的经典物理丛书。

“中外物理学精品书系”另一个突出特点是，在把西方物理的精华要义“请进来”的同时，也将我国近现代物理的优秀成果“送出去”。物理学科在世界范围内的重要性不言而喻，引进和翻译世界物理的经典著作和前沿动态，可以满足当前国内物理教学和科研工作的迫切需求。另一方面，改革开放几十年来，我国的物理学研究取得了长足发展，一大批具有较高学术价值的著作相继问世。这套丛书首次将一些中国物理学者的优秀论著以英文版的形式直接推向国际相关研究的主流领域，使世界对中国物理学的过去和现状有更多的深入了解，不仅充分展示出中国物理学研究和积累的“硬实力”，也向世界主动传播我国科技文化领域不断创新的“软实力”，对全面提升中国科学、教育和文化领域的国际形象起到重要的促进作用。

值得一提的是，“中外物理学精品书系”还对中国近现代物理学科的经典著作进行了全面收录。20 世纪以来，中国物理界诞生了很多经典作品，但当时大都分散出版，如今很多代表性的作品已经淹没在浩瀚的图书海洋中，读者们对这些论著也都是“只闻其声，未见其真”。该书系的编者们在这方面下了很大工夫，对中国物理学科不同时期、不同分支的经典著作进行了系统的整理和收录。这项工作具有非常重要的学术意义和社会价值，不仅可以很好地保护和传承我国物理学的经典文献，充分发挥其应有的传世育人的作用，更能使广大物理学人和青年学子切身体会我国物理学研究的发展脉络和优良传统，真正领悟到老一辈科学家严谨求实、追求卓越、博大精深的治学之美。

温家宝总理在 2006 年中国科学技术大会上指出，“加强基础研究是提升国家创新能力、积累智力资本的重要途径，是我国跻身世界科技强国的必要条件”。中国的发展在于创新，而基础研究正是一切创新的根本和源泉。我相信，这套“中外物理学精品书系”的出版，不仅可以使所有热爱和研究物理学的人们从中获取思维的启迪、智力的挑战和阅读的乐趣，也将进一步推动其他相关基础科学更好更快地发展，为我国今后的科技创新和社会进步做出应有的贡献。

“中外物理学精品书系”编委会 主任
中国科学院院士，北京大学教授
王恩哥
2010 年 5 月于燕园

Helmut Eschrig

Topology and Geometry for Physics

Springer

Preface

The real revolution in mathematical physics in the second half of twentieth century (and in pure mathematics itself) was algebraic topology and algebraic geometry. Meanwhile there is the Course in Mathematical Physics by W. Thirring, a large body of monographs and textbooks for mathematicians and of monographs for physicists on the subject, and field theorists in high-energy and particle physics are among the experts in the field, notably E. Witten. Nevertheless, I feel it still not to be easy for the average theoretical physicist to penetrate into the field in an effective manner. Textbooks and monographs for mathematicians are nowadays not easily accessible for physicists because of their purely deductive style of presentation and often also because of their level of abstraction, and they do not really introduce into physics applications even if they mention a number of them. Special texts addressed to physicists, written both by mathematicians or physicists in most cases lack a systematic introduction into the mathematical tools and rather present them as a patchwork of recipes. This text tries an intermediate approach. Written by a physicist, it still tries a rather systematic but more inductive introduction into the mathematics by avoiding the minimalistic deductive style of a sequence of theorems and proofs without much of commentary or even motivating text. Although theorems are highlighted by using italics, the text in between is considered equally important, while proofs are sketched to be spelled out as exercises in this branch of mathematics. The text also mainly addresses students in solid state and statistical physics rather than particle physicists by the focusses and the choice of examples of application.

Classical analysis was largely physics driven, and mathematical physics of the nineteens century was essentially the classical theory of ordinary and partial differential equations. Variational calculus, since the very beginning of theoretical mechanics a standard tool of physicists, was seen with great reservation by mathematicians until D. Hilbert initiated its rigorous foundation by pushing forward functional analysis. This marked the transition into the first half of twentieth century, where under the influence of quantum mechanics and relativity mathematical physics turned mainly into functional analysis (as for instance witnessed by the textbooks of M. Reed and B. Simon), complemented by the theory of Lie groups and by tensor analysis. Physicists, nowadays more or less familiar with these branches, still are on average mainly analytically and very little algebraically educated, to say nothing of topology. So it could happen that for nearly sixty years it was overlooked that not every quantum mechanical observable may be represented by an operator in Hilbert space, and only in the middle of the eighties of last century with Berry's phase, which is such an observable, it was realized how polarization in an infinitely extended crystal is correctly described and that textbooks even by most renowned authors contained meaningless statements about this question.

This author feels that all branches of theoretical physics still can expect the strongest impacts from use of the unprecedented wealth of results of algebraic topology and algebraic geometry of the second half of twentieth century, and to introduce theoretical physics students into its basics is the purpose of this text. It is still basically a text in mathematics, physics applications are included for illustration and are chosen mainly from the fields the author is familiar with. There are many important examples of application in physics left out of course. Also the cited literature is chosen just to give some sources for further study both in mathematics and physics. Unfortunately, this author did not find an English translation of the marvelous Analyse Mathématique by L. Schwartz,[1] which he considers (from the Russian edition) as one of the best textbooks of modern analysis. A rather encyclopedic text addressed to physicists is that by Choquet-Bruhat et al.,[2] however, a compromise between the wide scope and limitations in space made it in places somewhat sketchy.

The order of the material in the present text is chosen such that physics applications could be treated as early as possible without doing too much violence to the inner logic of the mathematical building. As already said, central results are highlighted in italics but purposely avoiding the structure of a sequence of theorems. Sketches of proofs are given, if they help understanding the matter. They are understood as exercises for the reader to spell them out in more detail. Purely technical proofs are omitted even if they prove central issues of the theory. A compendium is appended to the basic text for reference also of some concepts (for instance of general algebra) used in the text but not treated. This appendix is meant as an expanded glossary and, apart form very few exceptions, not covered by the index.

Finally, I would like to acknowledge many suggestions for improvement and corrections by people from the Springer-Verlag.

Dresden, May 2010 Helmut Eschrig

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[1] Schwartz, L.: Analyse Mathématique. Hermann, Paris (1967).

[2] Choquet-Bruhat, Y., de Witt-Morette, C., Dillard-Bleick, M.: Analysis, Manifolds and Physics, Elsevier, Amsterdam, vol. I (1982), vol. II (1989).

Basic notations

Sets A, B, ..., X, Y, ... are subjects of the axioms of set theory. $A = \{x \mid P(x)\}$ denotes the family of elements x having the property P; if the elements x are members of a set X, $x \in X$, then the above family is a set, a subset (part) of the set X: $A \subset X$. X is a superset of $A, X \supset A$. $\subset, \supset$ will always be used to allow equality. A proper subset (superset) would be denoted by $A \subsetneq X (X \supsetneq A)$. Union, intersection and complement of A relative to X have their usual meaning. The product of n sets is in the usual manner the set of ordered n-tuples of elements, one of each factor.

Set and space as well as subset and part are used synonymously. Depending on context the elements of a space may be called points, n-tuples, vectors, functions, operators, or something else. **Mapping** and **function** are also used synonymously. A function f from the set A into the set B is denoted $f : A \to B : x \mapsto y$. It maps *each* point $x \in A$ *uniquely* to some point $y = f(x) \in B$. A is the domain of f and $f(A) = \{f(x) \mid x \in A\} \subset B$ is the range of f; if $U \subset A$, then $f(U) = \{f(x) \mid x \in U\}$is the image of U under f. The inverse image or preimage $U = f^{-1}(V) \subset A$ of $V \subset B$ under f is the set $f(U) = \{x \mid f(x) \in V\}$. V need not be a subset of the range $f(A)$; f^{-1} (V) may be empty. Depending on context, f may be called real, complex, vector-valued, function-valued, operator-valued, ...

The function $f : A \to B$ is called **surjective** or onto, if $f(A) = B$. It is called **injective** or one-one, if for each $y \in f(A)$, $f^{-1}(\{y\}) = f^{-1}(y)$ consists of a single point of A. In this case the inverse function $f^{-1} : f(A) \to A$ exists. A surjective and injective function is **bijective** or onto and one-one. If a bijection between A and B exists then the two sets have the same cardinality. A set is **countable** if it has the cardinality of the set of natural numbers or of one of its subsets.

The **identity mapping** $f : A \to A : x \mapsto x$ is denoted by Id_A. Extensions and restrictions of f are defined in the usual manner by extensions or restrictions of the domain. The restriction of $f : A \to B$ to $A' \subset A$ is denoted by $f|_{A'}$. If $f : A \to B$ and $g : B \to C$, then the **composite mapping** is denoted by $g \circ f : A \to C : x \mapsto g(f(x))$.

The monoid of natural numbers (non-negative integers, 0 included) is denoted by $\mathbb{N}$. The ring of integers is denoted by $\mathbb{Z}$, sometimes the notation $\mathbb{N} = \mathbb{Z}_+$ is used. The field of rational numbers is denoted by $\mathbb{Q}$, that of real numbers is denoted by $\mathbb{R}$ and that of complex numbers by $\mathbb{C}$. $\mathbb{R}_+$ is the non-negative ray of $\mathbb{R}$.

The symbol $\Rightarrow$ means 'implies', and $\Leftrightarrow$ means 'is equivalent to'. 'Iff' abbreviates 'if and only if' (that is, $\Leftrightarrow$), and $\Box$ denotes the end of a proof.

Contents

Chapter 1
Introduction

Topology and continuity on the one hand and geometry or metric and distance on the other hand are intimately connected pairs of concepts of central relevance both in analysis and physics. A totally non-trivial concept in this connection is parallelism.

As an example, consider a mapping f from some two-dimensional area into the real line as in Fig. 1.1a. Think of a temperature distribution on that area. We say that f is continuous at point $\boldsymbol{x}$, if for any neighborhood V of $y = f(\boldsymbol{x})$ there exists a neighborhood U of $\boldsymbol{x}$ (for instance U_1 in Fig. 1.1a for V indicated there) which is mapped into V by f. It is clear that the concept of neighborhood is central in the definition of continuity.

As another example, consider the mapping g of Fig. 1.1b. The curve segment W_1 is mapped into V, but the segment W_2 is not: its part above the point $\boldsymbol{x}$ is mapped into an interval above $y = g(\boldsymbol{x})$ and its part below $\boldsymbol{x}$ is mapped disruptly into a lower interval. Hence, there is no segment of the curve W_2 which contains $\boldsymbol{x}$ as an inner point and which is mapped into V by g. The map g is continuous on the curve W_1 but is discontinuous at $\boldsymbol{x}$ on the curve W_2. (The function value makes a jump at $\boldsymbol{x}$.) Hence, it cannot be continuous at $\boldsymbol{x}$ as a function on the two-dimensional area. To avoid conflict with the above definition of continuity, the curve W_1 must not be considered a neighborhood of $\boldsymbol{x}$ in the two-dimensional area.

If f is a mapping from a metric space (a space in which the distance $d(\boldsymbol{x},\boldsymbol{x}')$ between any two points $\boldsymbol{x}$ and $\boldsymbol{x}'$ is defined) into another metric space, then it suffices to consider open balls $B_\varepsilon(\boldsymbol{x}) = \{\boldsymbol{x}' | d(\boldsymbol{x},\boldsymbol{x}') < \varepsilon\}$ of radius ε as neighborhoods of $\boldsymbol{x}$. The metric of the n-dimensional Euclidean space $\mathbb{R}^n$ is given by $d(\boldsymbol{x},\boldsymbol{x}') = (\sum_{i=1}^{n}(x^i - x'^i)^2)^{1/2}$ where the x^i are the Cartesian coordinates of $\boldsymbol{x}$. It also defines the usual topology of the $\mathbb{R}^n$. (The open balls form a base of that topology; no two-dimensional open ball is contained in the set W_1 above.)

Later on in Chap. 2 the topology of a space will be precisely defined. Intuitively any open interval containing the point x may be considered a neighborhood of x on the real line $\mathbb{R}$ (open intervals form again a base of the usual topology on $\mathbb{R}$). Recall that the product $X \times Y$ of two sets X and Y is the set of ordered pairs (x, y),

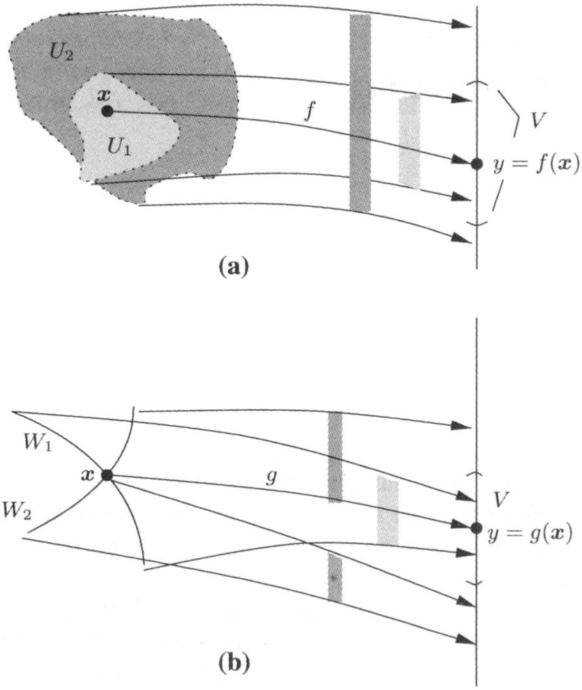

Fig. 1.1 Mappings from a two-dimensional area into the real line. **a** mapping f continuous at $\boldsymbol{x}$, **b** mapping g discontinuous at $\boldsymbol{x}$. The arrows and shaded bars indicate the range of the mapping of the sets U_1, U_2, W_1 and parts of W_2, respectively

$x \in X$, $y \in Y$. If X and Y are topological spaces, this leads naturally to the product topology in $X \times Y$ with a base of sets $\{(x, y)|x \in U, y \in V\}$ where U and V are in the base of the topology of X and Y, respectively. If this way the Cartesian plane is considered as the topological product of two real lines, $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$, then the corresponding base is the set of all open rectangles. (This base defines the same topology in $\mathbb{R}^2$ as the base of open balls.) Note that neither distances nor angles need be defined so far in $\mathbb{R} \times \mathbb{R}$: topology is insensitive to stretchings or skew distortions as long as they are continuous.

Consider next the unit circle, 'the one-dimensional unit sphere' S^1, as a topological space with all open segments as base of topology, and the open unit interval $I =]0, 1[$ on the real line, with open subintervals as base of topology. Then, the topological product $S^1 \times I$ is the unit cylinder with its natural topology. Cut the cylinder on a line 'above one point of S^1', turn one cut edge around by 180° and glue the edges together again. A **Möbius band** is obtained (Fig. 1.2). This rises the question, can a Möbius band be considered as a topological product similar to the case of the unit cylinder? (Try it!) The true answer is no.

There are two important conclusions from that situation: (i) besides the local properties of a topology intuitively inferred from its base there are obviously important global properties of a topology, and (ii) a generalization of topological product is needed where gluings play a key role.

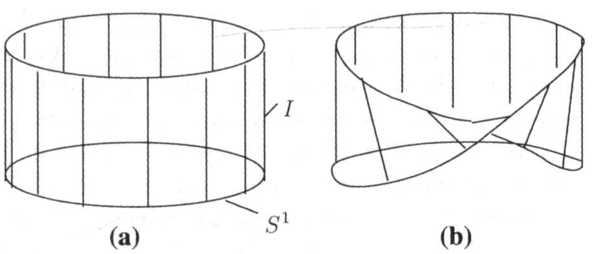

Fig. 1.2 **a** The unit cylinder and **b** the Möbius band

This latter generalization is precisely what a (topological) manifold is. The unit cylinder cut through in the above described way may be unfolded into an open rectangle of the plane $\mathbb{R}^2$. Locally, the topology of the unit cylinder and of the Möbius band and of $\mathbb{R}^2$ are the same. Globally they are all different. (The neighborhoods at the left and right edge of the rectangle are independent while on the unit cylinder they are connected.) Another example is the ordinary sphere S^2 embedded in the $\mathbb{R}^3$. Although its topology is locally the same as that of $\mathbb{R}^2$, globally it is different from any part of the $\mathbb{R}^2$. (From the stereographic projection which is a continuous one-one mapping it is known that the global topology of the sphere S^2 is the same as that of the completed or better compactified plane $\overline{\mathbb{R}^2}$ with the 'infinite point' and its neighborhoods added.) The S^2-problem was maybe first considered by Merkator (1512–1569) as the problem to project the surface of the earth onto planar charts. The key to describe manifolds are atlases of charts.

Topological space is a vast category, topological product is a construction of new topological spaces from simpler ones. Manifold is yet another construction to a similar goal. An m-dimensional manifold is a topological space the local topology of which is the same as that of $\mathbb{R}^m$. Not every topological space is a manifold. Since a manifold is a topological space, a topological product of manifolds is just a special case of topological product of spaces. A simple example is the two-dimensional torus $\mathbb{T}^2 = S^1 \times S^1$ of Fig. 1.3.

More special cases of topological spaces with richer structure are obtained by assigning to them additional algebraic and analytic structures. Algebraically, the

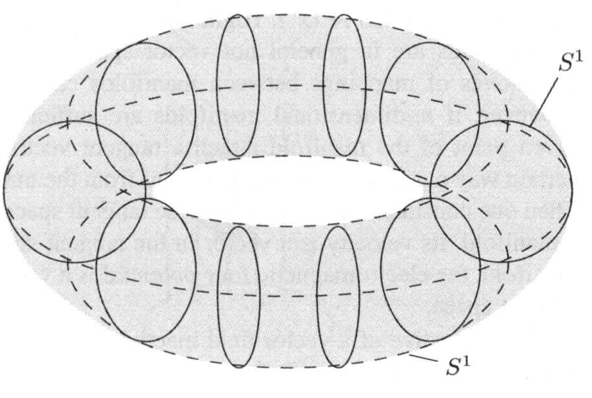

Fig. 1.3 The two-dimensional torus $\mathbb{T}^2 = S^1 \times S^1$