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# Groups acting on graphs

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CAMBRIDGE UNIVERSITY PRESS
*Cambridge*
*New York New Rochelle Melbourne Sydney*

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi, Dubai, Tokyo, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521180009



First published 1989
First paperback edition 2010

*A catalogue record for this publication is available from the British Library*

*Library of Congress Cataloguing in Publication data*

Dicks, Warren, 1947–
Groups acting on graphs/Warren Dicks and M.J. Dunwoody.
p. cm. – – (Cambridge studies in advanced mathematics; 17)
Bibliography: p.
Includes indexes.
1. Groups, Theory of. 2. Graph theory. I. Dunwoody, M.J.
II. Title. III. Series.
QA171.D47 1989 88–3661 CIP

ISBN 978-0-521-23033-9 Hardback
ISBN 978-0-521-18000-9 Paperback

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ADVANCED MATHEMATICS 17



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To Alan, Elena and Xaro
To Jean, Luke and Megan

# *Preface*

The simplest instance of the interplay between group theory and topology occurs where a group acts on a graph and information is obtained about the group or about the graph; it is these occurrences which form the theme of this book.

Chapter I offers a review of the Bass–Serre theory of groups acting on trees and graphs, with some typical combinatorial group theoretic applications. For the sake of novelty, we have included a very recent result from the literature on the fixed group of an automorphism of a free group. Although we have attempted to make the account self-contained, it is rather brusque for initiation purposes, and the reader should ideally already have some familiarity with group theory, group actions, presentations and combinatorial theory.

Chapters II, III and IV are essentially new.

Chapter II, using Boolean rings, associates to each connected graph and positive integer $n$, a tree which explains how the graph disconnects when any $n$ edges are deleted. One application is the recent result from the literature characterizing infinite finite-valency distance-transitive graphs. This chapter is elementary in the sense that no background material is assumed.

Chapter III is devoted to drawing lines joining up functions in an equivariant way to get a previously unsuspected tree. The argument is technical and elementary. The result has some rather pleasing applications, which are collected together in Chapter IV. New results include the proof of a conjecture of Wall, and a characterization of arbitrary groups with more than one end; previously known results are the characterization of groups of cohomological dimension at most one over an arbitrary ring, and the characterization of groups which have a free subgroup of finite index. The reader is assumed to be familiar with module theory and exact sequences; a Sylow theorem is used in a remark; cohomology is introduced in a mild way, since the results can be phrased in terms of derivations to projective modules.

Chapters V and VI examine dimensions two and three, and consist of results which, having only recently appeared in the literature, are appearing in book form for the first time.

Chapter V is an algebraic account of the cohomological characterization of infinite surface groups as the groups which satisfy two-dimensional Poincaré duality. The cohomology and topology are more sophisticated than in the rest of the book. The reader is assumed to be familiar with the necessary homological algebra, which is quickly summarized without many proofs. The reader familiar with the topology of surfaces, or manifolds in general, will be able to appreciate the motivation behind the entire chapter; the reader without the background in topology will have to be sufficiently algebraically inclined to be motivated by the result in its own right.

Chapter VI examines groups acting on two-complexes and deduces that almost finitely presented groups are accessible in the sense of Wall. It concludes with a similar analysis of three-manifolds and deduces the equivariant loop and sphere theorems. Here the topological background is summarized without proofs.

There are no exercises, apart from four open conjectures and the occasional tedious argument left to the reader.

Each chapter concludes with some notes and comments citing our sources for results and ideas. The sources, which are listed in the absolutely minimal bibliography and author index, tend not to be primary, and our attributions should be taken lightly, especially by authors who have been omitted.

We are indebted to the many mathematicians who have made helpful comments and devoted much time and effort to helping us understand the literature; their sole reward is the knowledge that the book would have been even worse without their help.

We thank Ed Formanek and Peter Linnell for generously contributing unpublished results and arguments.

The first-named author thanks the Mathematics Department of Pennsylvania State University for providing a graduate course forum to air and develop some of the results, and the CRM in Barcelona for support and gracious hospitality during the summer of 1985. The second-named author thanks the Royal Society and the IHES for support and gracious hospitality during the first six months of 1983.

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# GROUPS ACTING ON GRAPHS, CAMBRIDGE STUDIES IN ADVANCED MATHEMATICS 17, CAMBRIDGE UNIVERSITY PRESS, 1989.

## ERRATA

(JANUARY 31, 2008)

| PLACE | CHANGE |
|---|---|
| $5^4$ | Change "$(e)$" to "$(v)$" twice. |
| $6^7$ | Interchange labels "$srse$" and "$srsre$". |
| $9^{1-2}$ | Change to "repetitions of vertices and no repetitions of edges. Clearly such a path is reduced.". |
| $11_8$ | Change "$EX$" to "$VX$". |
| $11_5$ | Change boldface subscript "$\iota$" to ordinary subscript "$\iota$". |
| $11_5$ | Change "*and*" to "and". |
| $12^6$ | Change "elements 1,1" to "element 1". |
| $13^{11}$ | Change "$t_e(G(e)$" to "$G(e)^{t_e}$". |
| $17^{13}$ | Delete "so $G/N \approx \pi(G\backslash T)$". |
| $18_{11}$ | Change "$EX^{\pm 1}$" to "$ET^{\pm 1}$". |
| $27^1$ | Interchange "$G_{Te} = G_{\{p,p'\}}$" and "$G_e$". |
| $29_5$ | Interchange "$V$" and "$E$". |
| $32_{12}$ | Change "$u_1$" to "$g_1$". |
| $33^{13}$ | Change "$\alpha(e)(\alpha_{Te}(g^{t_e})u)$" to "$\alpha(e)(\alpha_{\bar{T}e}(g^{t_e})(u))$". |
| $35_9$ | Change "$\underset{v\in V}{\ast} G(v)$" to "$G = \underset{v\in V}{\ast} G(v)$". |
| $39^{10}$ | Change "69" to "71". |
| $39_{10}$ | Change "$e = EY_0$" to "$e \in EY_0$". |
| $40^7$ | Delete one "meet any". |
| $41_{17}$ | Change "thdn" to "then". |
| $41_{19}$ | Change "$g \cdots$" to "$g_2 \cdots$". |
| 44 | Stallings (1991) uses similar techniques to prove many other results. |
| $45^1$ | Change "$G$" to "$\ast_{i\in I} G_i$". |
| $45^7$ | Insert Corollary: If $\alpha : F \to G$ is a homomorphism of free groups then there exist subgroups $F_1$ and $F_2$ of $F$ such that $F = F_1 \ast F_2$, and $\alpha$ is injective on $F_1$, and $\alpha$ is trivial on $F_2$. (Herbert Federer and Bjarni Jónsson, Some properties of free groups, Trans. Amer. Math. Soc., 63 (1950), 1-27.) |
| $45_{10}$ | Change "take" to "takes". |
| $45_3$ | Change "Gerardin" to "Gérardin". |
| $46_{6,5}$ | Delete "from ... Brown". |
| $50_{10}$ | Change "$(v, v')$" to "$(v', v)$". |
| $53^5$ | Change "$= v(e^*)$" to "$= v(e)^*$". |

| PLACE | CHANGE |
|---|---|
| $54_{18}$ | Change "$s(v) = s(w)$ then $e(v) = e(w)$ for all $e \in E$" to "$e(v) = e(w)$ for all $e \in E$ then $s(v) = s(w)$". |
| $54_3$ | After "$\delta(ss') \subseteq \delta s$" insert "$\cup \delta s'$". |
| $55^2$ | Change "$s'(v)$" to "$s(v')$". |
| $55_{11}$ | Add "and not containing 0,1" after "$E_{n-1}$". |
| $57_{13,3}$ | Change "$E' \cup E$" to "$E \cup E'$". |
| $61^{17,23}$ | Change "$\iota s, \tau s = \tau s*, \iota s*$" to "$\iota s = \iota s^*, \tau s, \tau s^*$" twice. |
| $71_{10}$ | Change"Fredenthal" to "Freudenthal". |
| $77^{18}$ | Change"*and*" to "and". |
| $83^{10}$ | Change "$S'$" to"$E'$". |
| $83^{11}$ | Change "{e}" to "$\{e\}$". |
| $92^{16}$ | Delete "$\tilde{T}_1$ to a path of". |
| $99_7$ | Change "succesor" to "successor". |
| $100_{11}$ | Change "*H-H*" to "*G-H*". |
| $101_3$ | Change "amost" to "almost". |
| $102_5$ | Change "$<$" to "$\leq$". |
| $103^{9-13}$ | Change to "If $(p, q, r) = (2, 3, 6)$ then *acb*, *bac* generate a free abelian subgroup of rank two and index 6; see Magnus (1974),p.69." |
| $105^{19}$ | Bass (1993) gives a (short) proof that $G$ has a free subgroup of index $n$. |
| $107_9$ | Change "$G$ indexed by $A$" to "$A$ indexed by $G$". |
| $107_5$ | Change"of $AG$" to "of $(G, A)$". |
| $115^{14}$ | Change "Theorem 3.1" to "Theorem 3.13". |
| $134_9$ | Change "(1968)" to "(1971)". |
| $132_{14}$ | Change "$B$" to"$R$". |
| $134_5$ | Change to "Theorem 6.12 is due to Hopf for $G$ finitely generated, and the general case is new". |
| $134_3$ | Change "Theorem 4.12" to "Theorem 4.11". |
| $136^9$ | Change the first "$P$" to "$G$". |
| $136^{10}$ | Change "$H^*$" to "$H_*$" and "$H^i$" to "$H_i$". |
| $134^2$ | After "Brown(1982)" add "and Zimmerman (1981)". |
| $136^{12}$ | Change "$H_*$" to "$H^*$" and "$H_i$" to "$H^i$". |
| $140_2$ | Change "he" to "the". |
| $142_{13}$ | Change "$P$" to "$Q$" and "$P'$" to "$Q'$". |
| $142_{12}$ | Change "$P$" to "$Q$". |
| $142_{11}$ | Change "$P \to P'$" to "$Q \to Q'$". |
| $142_{10}$ | Change "$P \to P'$" to "$Q \to Q'$". |
| $142_7$ | Change "$Q$" to "$B$" twice. |
| $143^{11-13}$ | Change "$P$" to "$Q$", "$P'$" to "$Q'$","$P''$" to "$Q''$", twice each. |
| $143_3$ | Change "moduls" to "modules". |
| $144_{3,2}$ | Change the seven occurrences of "P" to "Q". |
| $146^{2,3}$ | Change "$\partial(p \otimes q) = \partial_P p \otimes q + (-1)^{\deg q} p \otimes \partial_q q$" to "$\partial(p \otimes q) = (-1)^{\deg q} \partial_P p \otimes q + p \otimes \partial_q q$". |

**PLACE** **CHANGE**

$146_{10-4}$ Should read
“$(\partial_{P\otimes Q}x)\cap\phi$
$= [(-1)^{\deg\, q}\partial_P p\otimes q + p\otimes \partial_Q q]\cap\phi$
$= (-1)^{\deg\, q}\partial_P p\otimes \phi q + p\otimes \phi\partial_Q q$
$= (-1)^{\deg\, q}\partial_{P\otimes C}(p\otimes \phi q) + [(p\otimes q)\cap\phi\partial_Q]$
$= (-1)^{\deg\, q}\partial_{P\otimes C}[(p\otimes q)\cap\phi] + [(p\otimes q)\cap\partial_{\mathrm{Hom}(Q,C)}\phi]$
$= (-1)^{\deg\, q}\partial_{P\otimes C}(x\cap\phi) + (x\cap\partial_{\mathrm{Hom}(Q,C)}\phi)$
If, further, $\phi$ is homogeneous, then either $\partial_{P\otimes C}(x\cap\phi)=0$ or $\deg\phi=-\deg q$, and in both cases we can write

(4) $\quad (\partial_{P\otimes Q}x)\cap\phi = ((-1)^{\deg\,\phi}\partial_{P\otimes C}(x\cap\phi)) + x\cap\partial_{\mathrm{Hom}(Q,C)}\phi$”.

$148^{13}$ In 2.16 Proposition, in the display change “$\mathrm{Ext}_R(B,C)$” to “$\mathrm{Ext}^n_R(B,C)$” twice, and after the display change “commutes” to “commutes with sign $(-1)^n$”.

$148_9$ In 2.17 Proposition, in the display change “$\mathrm{Ext}^n_R(B',C)$” to “$\mathrm{Ext}_R(B',C)$”, and change “$\mathrm{Ext}^{n+1}_R(B'',C)$” to “$\mathrm{Ext}_R(B'',C)$”, and after the display delete “with sign $(-1)^{n+1}$”.

$148_5$ Interchange “$\eta$” and “$\xi$”.

$149^{10}$ Change “$\partial_{P\otimes C}(x\cap\phi)-(-1)^{\deg\phi}(x\cap\partial_{\mathrm{Hom}(Q,C)}\phi$” to “$((-1)^{\deg\phi}\partial_{P\otimes C}(x\cap\phi))+x\cap\partial_{\mathrm{Hom}(Q,C)}\phi$”.

$149^{11,12}$ Delete “with sign $-(-1)^{\deg\phi}=(-1)^{n+1}$”.

$149^{15}$ In 2.18 Proposition, in the display change “$\mathrm{Ext}^{n-1}_R$” to “$\mathrm{Ext}^{n+1}_R$”.

$149^{17}$ In 2.18 Proposition, after the display change “commutes with sign $(-1)^n$” to “commutes with sign $(-1)^{n+1}$”.

$154^{16}$ Change the first “is” to “in”.

$155^1$ In top display change “$(-1)^{n+1}\xi\cap -$” to “$\xi\cap -$”.

$155_1$ Change “exact at $R$” to “exact at $RG$”.

$156^5$ Change “Theorem I.9.2” to “Corollary I.9.4”.

$158_4$ Change “contractible $n$-manifold $X$” to “$K$-orientable $K$-acyclic $K$-homology $n$-manifold $X$, as defined in Section 3 of Dicks-Leary (1995)”.

$163^{5,7}$ Change “$[1,R_p]$” to “$[2,R_p]$” twice.

$163^{13}$ Add “and $m_{p,1}$ denotes 1”.

$170_{13}$ Change “Thus $H$ is $FP_\infty$” to “Thus $G$ is $FP_\infty$”.

$171^{17-19}$ Change “ notice … .” to “notice that the $G$-action arises by embedding $G$ in $H\wr \mathrm{Sym}_n$ and defining actions of $H^n$ and $\mathrm{Sym}_n$ separately.”.

$173^9$ Delete “$G$-finite” after “locally finite”.

**PLACE** **CHANGE**

$173_{20-4}$ Replace with

"Fix a vertex $v_0$ of $Y_0$.

Consider any $w \in V_0$, and recursively construct an infinite reduced path $P_w$ as follows. Start with the vertex $w$, thought of as a base vertex, and take the neighbours of $w$ to be the base vertices of their respective components in the forest $Y_0 - \text{star}(w)$. Since $Y_0 - \text{star}(w)$ is infinite and has only finitely many components, one of the components is infinite. Choose one of these infinite components, and if there are more than one, choose one which does not contain $v_0$. This choice of infinite subtree corresponds to choosing an edge incident to $w$ to be the first edge in our infinite path. We now repeat the same procedure with our chosen infinite subtree with base vertex. In this way, we recursively construct an infinite reduced path $P_w$ which starts at $w$, and does not contain any edge $f$ such that $w$ lies in an infinite component of $Y_0 - \{f\}$ not containing $v_0$.

Let $e$ be an edge of $Y_0$. Let $Y_0(v_0, e)$ denote the $Y_0$-geodesic from $v_0$ to a vertex of $e$, but not passing through $e$. Let $\delta Y_0(v_0, e)$ denote the finite set of edges of $Y_0$ which have one vertex in $\delta Y_0(v_0, e)$ and the other vertex not in $Y_0(v_0, e)$. Thus $e$ lies in $\delta Y_0(v_0, e)$, and $Y_0(v_0, e)$ forms one of the finite components of $Y_0 - \delta Y_0(v_0, e)$. Let $Y_e$ denote the the subtree of $Y_0$ generated by the finitely many finite components of $Y_0 - \delta Y_0(v_0, e)$, so $Y_e$ is finite.

For $e \in EY_0$, $w \in VY_0$, we claim that if $e \in P_w$ then $w \in Y_e$. Suppose that $w$ does not lie in $Y_e$, so $w$ lies in an infinite component $Y_1$ of $Y_0 - \delta Y_0(v_0, e)$. Let $f$ denote the element of $\delta Y_0(v_0, e)$ incident to $Y_1$. Then $f$ lies between $w$ and $Y_0(v_0, e)$. Hence $f$ lies between $w$ and $v_0$. Also, $Y_1$ is an infinite component of $Y_0 - \{f\}$ containing $w$, so by its construction, $P_w$ stays in $Y_1$ and does not cross $f$. Hence $P_w$ does not meet $Y_0(v_0, e)$, so does not contain $e$. This proves the claim.

For any $v$ in $V$, there is a unique element $g$ of $G$ such that $gv \in V_0$, because $G$ acts freely on $V$, and we define $P_v = g^{-1}P_{gv}$. Thus $P_v$ is an infinite reduced path in $T$ which begins at $v$.

Consider any edge $e$ of $T$. We claim that there are only finitely many $v \in V$ such that $e$ belongs to $P_v$. Suppose then that $v \in V$ such that $e \in P_v$. There is a unique $g$ in $G$ such that $gv \in Y_0$, and then $ge \in gP_v = P_{gv}$. Hence $ge$ lies in $Y_0$, and $gv$ lies in the finite subtree $Y_{ge}$ of $Y_0$. Here $g$ is the unique element of $G$ such that $ge \in EY_0$, and we have $v \in g^{-1}Y_{ge}$, so there are only finitely many possibilities for $v$, as desired.".

$174_3$ Change "Thus we may assume that $n \geq 1$." to "If $n = 1$ then $G$ has an infinite cyclic subgroup of finite index by Theorem 4.4, and this case is easy. Thus we may assume that $n \geq 2$.".

$176_5$ Change "$K^k$" to "$H^k$.

$176_2$ Change "Thus in" to "Now let $(G, W)$ be a $PD^n$ pair, so, by ".

$177_{11}$ Change whole line to"$K$-orientable $K$-acyclic $K$-manifold $X$ of dimension $n$,

| PLACE | CHANGE |
|---|---|
| | whose boundary components are $K$-acyclic". |
| $177_6$ | Change "$\xi^\circ$" to "$\varepsilon^\circ$". |
| $178^{20}$ | Change "Definitions" to "Definition". |
| $183_9$ | There is a vertical arrow missing on the left of the diagram (1). |
| $185_{12}$ | In the display that comes two before (7), in the top row, change "$K\omega KE$" to "$\omega KE$". |
| $185_{11}$ | In the display that comes two before (7), in the label on the rightmost vertical arrow, delete "$(-1)^n$". |
| $185_7$ | In the display that comes before (7), in the label on the rightmost vertical arrow, delete "$(-1)^n$". |
| $186^{15}$ | In the display in mid-page, change the two rightmost "$\xi \cap -$" to "$-\xi \cap -$". |
| $186_{12}$ | In (8) change "$\xi \cap \eta_e$" to "$-\xi \cap \eta_e$". |
| $198^7$ | Change "$W - Gw$" to "$W - Gw_0$". |
| $198_{15}$ | Insert "$ET = Ge$" after "$G_e$ is finite". |
| $199^{14}$ | Insert "$ET = Ge$" after "$G_e$ is finite". |
| $202_{10}$ | Change "$\sum\limits_{j\in[1,N]}$" to "$\sum\limits_{i,j\in[1,N]}$". |
| $203^{10}$ | Change "$\|\|a\|\|\,\|\|b\|\| \geq \|\mathrm{tr}\,(\bar{b})\|$" to "$\|\|a\|\|\,\|\|b\|\| \geq \|\mathrm{tr}\,(a\bar{b})\|$". |
| $202_4$ | Change "then – induces" to "then $\overline{\phantom{x}}$ induces". |
| $203_8$ | Change to<br>"(1) $\|\|a_n e\|\|^2 = \mathrm{tr}(a_n e\overline{a_n e}) = \mathrm{tr}(a_n e\bar{e}\,\overline{a_n}) = \mathrm{tr}(a_n c\overline{a_n}) = \mathrm{tr}(a_n\bar{c}\,\overline{a_n})$.". |
| $203_{7,6}$ | Change "$\mathrm{tr}(a_n\overline{ca_n})$" to "$\mathrm{tr}(a_n\bar{c}\,\overline{a_n})$" twice. |
| $205^8$ | Change "$P \mapsto KG \otimes_K P$" to "$P \to KG \otimes_K P$". |
| $205^{14}$ | Change "for all $j \in [1,m]$" to "for all $i \in [1,m]$". |
| $205_{12}$ | Change "$\sum\limits_{i,g\in G}$" to "$\sum\limits_{i,g}$". |
| $206_{12,13}$ | Change "$[w_1 \cdots w_q] = [w_2 \cdots w_q w_1]$" to "$\mathrm{Tr}(w_1 \cdots w_q) = \mathrm{Tr}(w_2 \cdots w_q w_1)$". |
| $207^4$ | Before "invertible" insert "is". |
| $208_{14}$ | Change "$A *_C X_0$" to "$A *_C x_0$". |
| $209_2$ | Change "$V$-term" to "$E$-term". |
| $211_1$ | Change "$\to {}_0$" to "$\to P_0$". |
| $212_{22-21}$ | Change "$= K$ is right annihilated by $\omega KG$" to "$= K = KG/\omega KG$". |
| $212_{20}$ | Change "$\alpha^*(P^*) \subseteq \omega KG$" to "$\alpha^*(P^*) = \omega KG$". |
| $219^5$ | After "if" insert "and only if". |
| $219^{15}$ | Change "$Z_0(K,G)$" to "$\mathrm{Hom}(C_0(K),G)$". |
| $220_4$ | Change "$\mathbb{Z} \otimes \mathbb{Z}$" to "$\mathbb{Z} \times \mathbb{Z}$". |
| $222_{15}$ | Change "$G$" to "$K$". |
| $224^5$ | Change "$s$" to "$\sigma$". |
| $224_{14}$ | One can change "$P \cap \|K\|$" to "$P$", since $P \subseteq \|K\|$. |
| $224_7$ | Change "$j(\gamma_i)$" to "$j_P(\gamma_i)$". |
| $225^{11}$ | Change "$h^1$" to "$h_1$". |
| $225_2$ | After "colouring" insert "with two colours". |
| $229_3$ | Change "$X$" to "$S$". |
| $231^5$ | Change the second "$v_1$" to "$v_2$". |

| PLACE | CHANGE |
|---|---|
| $232^{15}$ | Change "$ET$" to "$VT$". |
| $232^{16}$ | Change the second "$ET$" to "$T$". |
| $236^{9}$ | At the end of the line add "Moreover it follows from the thinness of $b_2^*$ or $b_1^*$ that $\nu = \delta$." |
| $240_{13-12}$ | Change "$G$, the automorphism group of $K$, is generated" to "$G$ is the group of automorphisms of $K$ generated". |
| $245^{1-3}$ | Delete "$H^1(K, \mathbb{Z}_2)$ ... that". |
| $245_{3}$ | Change "$H^1(K, \mathbb{Z}_2) = 0$" to "every scc separates $M$". |
| $272^{7}$ | Insert in left hand column:<br>"Bass, H. {45, 46, 71}<br>1993. Covering theory for graphs of groups, *J. Pure and Appl. Algebra* **89**, 3–47. {105≈} ." |
| $272^{7}$ | In right hand column change the Burns entry to<br>"Burns, R.G.<br>1971. On the intersection of finitely generated subgroups of a free group, *Math. Z.* **119**, 121–130. {39} " |
| $273_{6}$ | In right hand column change "Gerardin" to "Gérardin". |
| $273_{5}$ | In left hand column change **"15" to "25".** |
| $273^{16-18}$ | In right hand column delete the entry. |
| $273^{25}$ | In right hand column change "Normal Flächen" to "Normalflächen". |
| $273_{4}$ | In right hand column change "Raüme" to "Räume". |
| $274_{22}$ | Delete from left hand column "134,". |
| $274^{20-45}$ | In the right hand column, interchange lines 20-32 with lines 33-45, to obtain alphabetic order. |
| $274^{27}$ | In left hand column change "dreidemensionalen" to "dreidimensionalen". |
| $274^{25}$ | In right hand column change "isomorphismen" to "Isomorphismen". |
| $275^{11}$ | Insert in right hand column<br>1991. Foldings of G-trees, pp. 355-368 in *Arboreal Group Theory* (Roger C. Alperin, Editor), MSRI Publications 19, Springer-Verlag, Berlin, 1991. {44≈} |
| $275_{7}$ | Change "Räume" to "Räumen". |
| $276^{3}$ | Insert in left hand column "(-)′". |
| $274_{7}$ | Insert in left hand column:<br>"Magnus, W.<br>*Noneuclidean Tesselations and their Groups*, Academic Press, New York, 1974. {103}" |
| $275^{10}$ | In the right hand column change " {71, 100} " to " {71, 100, 134}". |
| $275_{1}$ | In the right hand column add<br>"Zimmerman, B.,<br>1981. Über Homeömorphismen $n$-dimensionaler Henkelkörper und endliche Erweiterungen von Schottky-Gruppen, *Comm. Math. Helv.* **56**, 474–481. {134}" |
| $276_{12}$ | The triangle in the right hand column should be unshaded. |

# Conventions

$G$ denotes a group, fixed throughout the book.

$\varnothing$ denotes the empty set.

Sets are indicated by $\{x \mid x \cdots\}$ or sometimes $\{x : x \cdots\}$ for typographical reasons.

$B \subseteq A$ means $B$ is a subset of $A$.

$B \subset A$ means $B$ is a *proper* subset of $A$, that is, distinct from $A$.

If $B \subseteq A$ then $A - B$ denotes the complement of $B$ in $A$.

$A \cup B$, $A \vee B$, $A \cap B$, $A \times B$, respectively, denote the union, the disjoint union, the intersection and the Cartesian product of two sets, $A$, $B$.

$\bigcup_{i \in I} A_i$, $\bigvee_{i \in I} A_i$, $\bigcap_{i \in I} A_i$, $\prod_{i \in I} A_i$, respectively, denote the union, the disjoint union, the intersection and the Cartesian product of a family of sets $A_i$ indexed by the elements $i$ of a set $I$.

$A^n$ denotes the Cartesian product of copies of a set $A$ indexed by a non-negative integer $n$, and the elements are written as $n$-tuples $(a_1, \ldots, a_n)$.

$|A|$ denotes the cardinal of a set $A$.

If $m$, $n$ are integers then $[m, n]$ denotes the set of integers $i$ such that $m \leqslant i \leqslant n$.

If $\alpha$, $\gamma$ are ordinals then $[\alpha, \gamma]$ and $[\alpha, \gamma)$ respectively denote the set of ordinals $\beta$ with $\alpha \leqslant \beta \leqslant \gamma$ and $\alpha \leqslant \beta < \gamma$.

If $I$ is a set and $m_i$, $i \in I$, are cardinals and $m$ is a cardinal then $m = \mathrm{HCF}_{i \in I} m_i$ means that $m$ is the largest cardinal which divides all the $m_i$. In practice, $m$ is an integer, or equivalently some $m_i$ is an integer.

$\mathbb{N}, \mathbb{Z}^+, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}, \mathbb{R}^n$, respectively, denote the positive integers, the non-negative integers, the integers, the rationals, the reals, the complex numbers, and Euclidean $n$-space.

$\mathbb{Z}_2$ denotes the set consisting of two elements 0 and 1; it performs as a set, a group, a ring, a Boolean ring, a field, and a discrete topological space.

Except where otherwise indicated, functions will be written on the left of the argument, and composed accordingly. We write $\alpha: X \to Y$ or $X \xrightarrow{\alpha} Y$ to denote a function, and $x \mapsto \alpha x$ to denote its action on elements. Here $\alpha^{-1}(y) = \{x \in X \mid \alpha x = y\}$ for any $y \in Y$.

Except where otherwise specified, groups will be written multiplicatively, and abelian groups will be written additively. If $x, y \in G$ then $[x, y]$ denotes the *commutator* $x^{-1}y^{-1}xy$; this should not be confused with the above interval notation.

$H \leqslant G$ means that $H$ is a subgroup of $G$.

Rings are associative and have a 1; in all situations of interest, the 0 and 1 are distinct.

(Left or right) module actions respect the 1 of the ring.

$M \oplus N$ denotes the direct sum of two modules, and $\bigoplus_{i \in I} M_i$ denotes the direct sum of a family of modules $M_i$ indexed by the elements $i$ of a set $I$.

If $R$ is a ring, $M$ a right $R$-module, and $N$ a left $R$-module, then $M \otimes_R N$ denotes the tensor product, viewed as an abelian group.

The numbering treats theorems, definitions, examples, remarks, etc. as subsections, labelled, for example, as 2.9 Remarks, in Section IV.2 in Chapter IV, and referred to as Remark 2.9 within Chapter IV, and as Remark IV.2.9 within all other chapters. The end of such a subsection is indicated by ▮.

References to the bibliography are by the author–date system, with primes to distinguish publications by the same author in the same year.