# GROUP DECISION MAKING MODELS BASED ON MULTI-GRANULAR INFORMATION

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## Brief Introduction

Making a decision by a group is a widespread process in our daily life. The need of multiple views makes group decision making increasingly necessary in numerous societies and organizations today. In a group decision making scenario, each decision maker can express his/her preferences with various information granularities and different structures, depending on their different background knowledge and experience. Therefore, group decision making models need to deal with heterogeneous information to meet the demands stemming from modern societal and technological contexts. This book provides us some valuable ideas and approaches to deal with multi-granular information, including the ordinal, linguistic, fuzzy, interval information, and real numbers, which are useful to capture uncertainty, vagueness, imprecision in decision makers' opinions, preferences, or assessments in an effective manner. Focusing on such multi-granular information, the book proposed some improved group decision making models for each class of multi-granular information by combining with the probability theory, fuzzy set theory, optimization and rough set theory separately. For the heterogeneous information, especially when the number of decision makers is large, it is difficult to translate the heterogeneous information into unified one without loss of context. In this aspect, the book presents a new consensus-based group decision making model to manage heterogeneous information by employing the AIP (aggregation of individual priority)-based aggregation mechanism, which is able to utilize flexible methods for deriving each decision makers′ individual priority and to avoid information loss caused by unifying heterogeneous information. This book is meant for the researcher, student, or manager who uses mathematical modelling and optimization to solve the decision problems. Or more generally, this includes those working directly in group decision making, and also many others who use decision analysis, in fields like economics, finance, and many fields of science and management.


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To those

Who educate us;

Who support us;

Whom we loved and

With whom we collaborate

# Preface

A variety of information encapsulated in different representations is used by decision makers (DMs) to express their preferences during the decision making process. All such expressions (e. g., ordinal, linguistic, fuzzy and interval) constitute information with different granularity which is useful to capture uncertainty, vagueness, imprecision in DMs' opinions, preferences or assessments in an effective manner. Based on widely use of multi-granular information in group decision making (GDM) problems, a large number of GDM models have been proposed in the literature. Although these GDM models provide a valuable means to handle multi-granularity information, there are still some drawbacks associated with different GDM models that need improvements.

The purpose of this book is to provide some new GDM models and related methods for the improvement of existing GDM models from the viewpoint of multi-granular information. Specifically, GDM models based on four types of information, namely ordinal, linguistic, fuzzy and interval, are investigated in this research.

For the ordinal preferences, a two-stage dynamic GDM model based ona consensus reaching process is proposed, where a power average (PA) operator is employed to aggregate ordinal information with consideration of the relationship (e. g. agreement or disagreement) among the DMs. Additionally, a data cleaning process is integrated into the consensus reaching process to evade the bias caused by the conflicting opinions.

Owing to the internal linear ordinal property of linguistic term sets, a supplementary method of linguistic GDM models is obtained by extending the dominant-based rough set approach (DRSA) on support function to linguistic information. Since the group decisions generated by fuzzy GDM models are typically represented by fuzzy numbers, and the collective group decisions generated by intervals GDM models are normally represented by interval matrices, two new methods capable of handling fuzzy and interval information are then proposed to derive the final ranking from the collective group decisions.

The main contribution and innovation of this book lies in the improvements of various GDM models with multi-granular information and related methods. Mathematical proofs are presented, and efficiency of the resulting GDM models is demonstrated by using a number of examples and case studies.

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# Chapter 1 Introduction

## 1.1 Group Decision Making

Group decision making (GDM) is defined as a decision situation involving more than one individual. The group members have their own motivation and attitude, recognize the existence of a decision problem and attempt to reach a collective decision (Lu et al., 2007). GDM also can be defined as a participatory process, in which multiple individuals select one solution (or more than one) from the alternatives by determining the criteria, analyzing the problems, and evaluating the alternatives collectively (Barnett, 2014).

Our society has a long tradition of valuing the wisdom of a group, and GDM has been developed since the late 19th century, ranging from law-making to evaluation of a variety of projects and processes. Indeed, people prefer to rely on a group to make important decisions. The decision made by the group is normally no longer attributable to any single individual, since the outcome is influenced by all members' perspectives. Therefore, the decision made by a group is usually different from that made by individuals.

GDM problems have received prominent attention, and are well studied across many fields, such as complex social choice (Keeney and Kirkwood, 1975), bank or insurance company selection (Yücenur and Demirel, 2012; Dincer and Hacioglu, 2013), face recognition (Jing et al., 2003), supply chain coordination (Singh and Benyoucef, 2013), US army general's ranking (Retchless et al., 2007) and many other critical domains (Morais and de Almeida, 2012; Ebrahimnejad et al., 2012; Zhang and Liu, 2011). Some important technical domains that GDM has been attempting to solve including: ①group motion coordination (Couzin et al., 2005; de Lauwere et al., 2012); ② ability of a group to combine information (Lightle et al., 2009; Zhang and Guo, 2012; Herreraa et al., 2003); ③ optimal group size for reaching a consensus (Karotkin and Paroush, 2003; Zahedi, 1986; Wu and Xu, 2012a; Altuzarra et al., 2010; Guiqing et al., 2011); ④ performances of aggregation operators (Zhou et al., 2012a; Yang and Chen, 2012); ⑤integration of various GDM models (Ekel et al., 2009; Wang et al., 2012a).

From the perspective of data collection and processing of each decision maker (DM), existing GDM models range from well-defined situations (Limayem and de Sanctis, 2000; Keeney and Kirkwood, 1975) to uncertain environments (Çakır, 2012; Ebrahimnejad et al., 2012; Xu et al., 2013; Zhou and Chen, 2012). When DMs' preferences are represented in numerical values, it is common to construct deterministic GDM models (Limayem and de Sanctis, 2000). However, in real life applications, it is not reasonable to demand DMs to provide the preferences in exact numerical values. In addition, given that different group members with diverse cultures and backgrounds may provide different uncertain preferences and evaluations on the alternatives, it is important to examine GDM models with the capability of processing information under uncertain environments. This book focuses on the improvement of existing GDM models with multi-granular information. Specifically, the multi-granular information includes ordinal, interval, fuzzy information and linguistic variables.

## 1.2 Group Decision Making Versus Individual Decision Making

Since the late 19th century, many researchers have explored the efficiencies and deficiencies of GDM. Although some researches believed that group thinking caused distorted decisions (Janis, 1972), most researchers claimed that cooperative group could perform better than independent individuals on a wide range of tasks (de Andrés et al., 2010; Laughlin et al., 2006; Merigó et al., 2010). Recently, a study published in *Science* (Koriat, 2012) pointed out that for two decision makers (DMs) with nearly equal sensitivity, the outcome would be better than that if the decision was treated separately, assuming that each member could communicate with the other freely. According to Segal-Horn (2004), the decisions made by a group tend to be more effective than those made by single individual. This is in line with the common phenomenon of "wisdom of the crowds" in our daily life.

Groups are superior to individuals as decision-making entities owing to three main reasons (Tindale et al., 2003):

(1) A group is able to generate a large number of diverse alternatives or perspectives with higher quality than those from individuals by combining the unique

qualities of each member. Therefore, a group tends to be fairer by providing decisions from different perspectives.

(2) A group is perceived to be better than individuals when making important decisions as evidenced from empirical records.

(3) The collective decision-making ability of a group is considered to be more viable than individuals (Chen et al., 2007).

However, inpractice, not all the decisions made by a group are wise. According to Surowiecki (2004), there are four aspects pertaining to a group decision that is considered "wise":

(1) Diversity of opinions;

(2) Independence of opinions;

(3) Decentralization (specialization) of opinions;

(4) Appropriate aggregation mechanisms.

Based on these four aspects, Surowiecki (2004) presented numerous case studies to prove a general conclusion that group decisions often outperform those made by individual members in the group.

## 1.3 General Assumptions

In order to acquire a rational group decision, some basic assumptions are necessary to confine the research scope of this book. The assumptions are as follows:

(1) All DMs in a group are willing to cooperate with each other, i. e., the DMs are prone to re-consider their opinions after they have received new information related to the decision.

(2) All DMs can express their own initial opinions independently. Multi-granular information can be used owing to the diverse background of the DMs.

(3) Each DM has his/her own specialized area, which allows decentralization of each member in the group.

(4) Not all DMs have the same initial opinions, which keep the diversity of the group.

(5) All DMs can communicate with each other freely.

## 1.4 Procedure of Group Decision Making

To establish a rational group decision making model, the following general steps are typically applied (Lu et al., 2007):

**Step 1.** Define the decision problem.

In this step, the problem is thoroughly analyzed, and any related questions can be put forward in order to identify and clarify the purpose of the decision.

**Step 2.** Determine the requirements and gather information.

Once the decision problem is defined, the DMs need to examine the resources they have in hand, and then identify what additional information they need.

**Step 3.** Establish the objectives.

Use a specific, measurable, attainable and realistic method to establish the relevant objectives. When some opinions on the objectives are in conflict with each other in a group, a discussion or negotiation process can be initiated until an agreement pertaining to the objectives is accepted by the group.

**Step 4.** Generate the alternatives.

In this step, a variety of alternatives for the potential solutions are generated by all DMs. Each DM is required to participate in the alternative generation process.

**Step 5.** Define the criteria.

In this step, the criteria used to evaluate the alternatives are set up. When defining the criteria, the group's goals as well as the culture of collaboration and cooperation should be taken into consideration. Different weights can be assigned to the criteria by different DMs. Similar criteria proposed by different DMs should be merged.

**Step 6.** Choose a group decision making model.

Based on the information obtained in GDM problems, e. g., coarse granular information (ordinal evaluations), or fine granular information (numerical evaluations), an appropriate GDM model can be chosen. In this step, choosing the appropriate aggregating operation is crucial for reaching a successful group decision.

**Step 7.** Evaluate the alternatives.

Based on the criteria defined in Step 5, all alternatives are ranked and analyzed for their positive and negative effects. As a result, the best alternative can be selected by using the model chosen in Step 6.

**Step 8.** Validate the solutions.

In this step, the group evaluates the outcome of the decision. If failures occur, the group returns to the earlier stage (Step 6).

**Step 9.** Implement the solutions.

This step involves discovering the necessary resources to put the group decision into practice.

The forementioned framework can be applied to most GDM problems. However, some modifications need to be made in accordance with different situations due to specific conditions of the problem domain.

## 1.5 Group Decision Making Models with Multi-granular Information

A variety of information encapsulated in different representations can be provided by different DMs during the decision making process, e.g., ordinal, interval, and fuzzy information, and linguistic expressions. In traditional GDM models, the DMs express their opinions by means of exact numerical values. However, in most realistic and pragmatic decision making situations, the DMs frequently provide qualitative assessments that are difficult to be measured by quantitative values. All such expressions (i.e., ordinal, linguistic, fuzzy and interval) constitute information with different granularities, which capture uncertainty, vagueness, imprecision in the DMs' opinions, preferences, or assessments in a more effective manner. Based on multi-granular information in GDM problems, a variety of GDM models with different methods to handle granular information have been proposed in the literature (Jabeur and Martel, 2007; Xu, 2010, 2011a; Li and Yang, 2004; Herrera et al., 1996a; Cabrerizo et al., 2013). These models provide valuable means for undertaking GDM problems with multi-granular information. However, there are some drawbacks associated with the existing GDM models that need improvements. The purpose of the book is to provide a number of new GDM models and related methods for tackling GDM problems from the viewpoint of multi-granular information. The proposed GDM models and related methods are based on a thorough analysis of the advantages and disadvantages of existing GDM models. As a result, four types of information, i.e., ordinal, linguistic, fuzzy and interval, are focused in this research.

To deal with ordinal information provided by the DMs, the most important problem is how to quantify the ranking information based on each individual's assessment. This is because ordinal information only provides ordering (ranking) of a set of criteria or alternatives. A number of methods (Stillwell et al., 1981; Lootsma and Bots, 1999; Hutton Barron, 1992; Noh and Lee, 2003) have been used to convert the ordinal ranking into numerical-valued information. A literature review on the methods for quantifying ordinal information is presented in Chapter 2. Another difficulty in using ordinal information is how to aggregate an individual's ordinal preference into a collective ranking to represent a consensus among the DMs. Many aggregation methods based on ordinal evaluation of individual rankings have been proposed in the literature for achieving a collective group preference. These aggregation methods can be divided into three categories, i. e. , parliamentary setting-based models, ranking weight-based models, and distance-based models. The details are presented in Chapter 2 (Section 2.3.1).

However, this research discovers that most of the GDM models that handle ordinal preferences suffer from three drawbacks: ① difficulty in managing conflicting opinions; ② neglecting the relationship among the preferences of the DMs; ③ assuming equal importance for all DMs. The details are elaborated in Chapter 2 (Section 2.3.1.4). In order to overcome these problems, a two-stage dynamic group decision making method for aggregating ordinal preferences is proposed in Chapter 3.

Fuzzy set theory provides a combination of qualitative and quantitative assessment in a single instrument, which makes it possible for DMs to address a variety of interpretations in an explicit manner. Therefore, fuzzy set has been widely used in various GDM problems. Many GDM models with fuzzy information have been proposed. Some of them are based on the aggregation of DMs' opinions that are represented as fuzzy numbers (Lan et al., 2005; Yager, 2003; Chen and Niou, 2011; Wu and Cao, 2011; Yager and Filev, 1999). Some GDM models are built by solving optimization problems to achieve the group opinion (Wang and Parkan, 2006; Xu and Zhai, 2009; Liu et al., 2012a; Li and Yang, 2004). Other GDM models are constructed based on generalization of fuzzy sets, for instance, type-2 fuzzy sets (Chen and Lee, 2010a), intuitionistic fuzzy sets (Atanassov et al., 2003), and hesitant fuzzy sets (Rodríguez et al., 2014). A review on fuzzy GDM models is presented in Chapter 2 (Section 2.3.3).

Summarizing existing fuzzy GDM models, it is found that the collective group opinions on alternatives are usually expressed by fuzzy numbers. Therefore, it is inevitable for each group to rank its respective fuzzy numbers. In order to utilize as much information pertaining to the DMs' preferences as possible, a new method is introduced for comparing fuzzy numbers based on a fuzzy probabilistic preference relation with each DM's confidence level. The ranking order of fuzzy numbers with the weighted confidence level is subsequently derived from the pair-wise comparison matrix based on 0.5-transitivity of the fuzzy probabilistic preference relation, as explained in Chapter 4 (Section 4.5). Proofs showing that the proposed method can retain more information are provided. The significance of the proposed method lies in its capability in overcoming the voting paradox caused by most existing GDM models.

Linguistic variable was first utilized in approximate reasoning by Zadeh (1975). It has been widely used to depict the qualitative aspects, especially in the realm of human-oriented systems. Linguistic GDM models introduce a flexible framework to represent the information in a direct, natural and adequate way when one is unable to express it precisely. In the literature, many linguistic GDM models have been established and analyzed (Herrera et al., 2008; Mata et al., 2009; Pérez et al., 2011; Cabrerizo et al., 2013). The most frequently used linguistic GDM models in the literature can be classified into two categories, i. e., the semantic-based models and the symbolic-based models. The details are presented in Chapter 2 (Section 2. 3. 4).

By investigating existing GDM models with linguistic information, this research discovers that in most linguistic GDM models, the importance of the DMs are usually assigned subjectively based on their experiences before the aggregation process starts. However, the DM's experience does not always lead to a correct judgment for the current problem. This is because the correctness of a judgment depends on the problem domain as well. As a result, the importance of the DMs is not only based on their experience and past accuracy (subjective weight) but also dependent ed on their current judgement on a specific problem (objective weight). Since the linguistic term set has an internal linear ordinal property, the support function of the DMs defined for ordinal information in Chapter 3 (Section 3. 3. 2) is also suitable for indicating the importance of each DM under the direct evaluation mechanism with linguistic information. Therefore, the objective weight of each DM in linguistic GDM models is determined by the DRSA-based support degree. The details are presented in Chapter 3 (Section 3. 7).

Interval-valued information is another way to express preferences of the DMs with respect to the alternatives. The use of interval preferences is important when the DMs cannot clearly translate their preferences by precise numerical values with full confidence owing to limited information processing capabilities of the DMs, or time pressure in the real world. It can be regarded as a kind of information granularity that attempts to deal with uncertainty in a more precise manner. Existing GDM models that are based on interval preferences in the literature can be classified into two categories, i. e., the simple interval arithmetic-based GDM models and the optimization-based GDM models. The details are presented in Chapter 2 (Section 2. 3. 2).

For most existing interval GDM models, the collective group decision is normally expressed as an interval pairwise comparison matrix; as such, it is necessary to perform an additional procedure for deriving the priority weights from the collective pairwise comparison matrix. A new method to derive the interval priority weights from the interval pairwise comparison matrix is proposed in Chapter 5. The corresponding numerical-valued consistent matrices hidden in an interval fuzzy preference matrix are directly extracted based on consistency of the fuzzy preference relations. Then the priority weights are derived from the extracted numerical-valued consistent matrices. Compared with existing methods, the proposed method requires the adjustment of fewer intervals for an inconsistent matrix. As a result, it retains more original information in the given interval matrix. The priority weight vector derived by the proposed method is more valid as it exhibits a smaller degree of uncertainty, and is more reliable as ascertained by the shorter distance between the generated interval matrix and the original one. The details are presented in Chapter 5.

In GDM problems, every DM in a group is required to evaluate a set of alternatives based on some criteria independently. Each DM's preference can be expressed with multigranular information, for example, ordinal rankings, linguistic terms, fuzzy numbers, intervals, and real numbers, from coarse to fine granular information. As such, different GDM models are available in the literature to deal with different types of information employed by the DMs to provide their evaluations.

By investigating the advantages and disadvantages of existing GDM models in the literature, a number of research gaps are identified, as follows:

(1) Most GDM models that are based on ordinal information neglect the relationship between the preferences provided by the DMs.

(2) Most GDM models with linguistic information assume that all individuals are of equal importance, which cause the aggregated collective preference not to be an ideal representation of the group's opinion.

(3) Most GDM models with fuzzy information generate the collective group opinion in the form of fuzzy numbers. However, almost all of the existing fuzzy number ranking methods provide crisp results without considering the confidence degree of each DM during the evaluation process.

(4) In most GDM models with interval-valued information, the preference priority weights of each DM are derived by minimizing the difference between the weight-based consistent matrix and the original judgement matrix given by the DM. However, the priority weights generated by extensively minimizing the differences between the ratio of

the priority weights and the elements in the judgement matrix can distort the real solution of the problem under inconsistent circumstances (Saaty, 2000).

(5) Most GDM models cannot take into account conflicting opinions.

(6) Seldom GDM models are developed to manage the heterogenerous information, which are commonly expected when large number of DMs are involved.

In order to fill in the forementioned gaps, this book is going to treat the following objectives. The detailed elaboration and explanation of the objectives are provided in Chapter 3 to Chapter 6.

(1) To construct a dynamic GDM model for aggregating ordinal preferences by employing the power average (PA) operator, which takes the relationships of ordinal preferences into consideration and therefore overcomes the drawbacks of existing GDM models with ordinal information.

(2) To produce a new method for determining the objective weight of each DM by integrating a new support degree based consensus-reaching process into a GDM model, where the support degree of each DM can be reviewed as his/her relative weight in GDM problems.

(3) To devise a soft (fuzzy) comparison result attached with a membership function indicating the confidence degree of each DM for ranking the obtained collective preference in GDM models with fuzzy information, and a graph-based approach for ranking fuzzy numbers with a weighted confidence level is presented based on the 0.5-transitivity of the fuzzy probabilistic preference relation.

(4) To develop a new method for deriving the preference priority weight vector of each DM in GDM problems with interval information. This method directly extracts numerical-valued consistent fuzzy preferences hidden in an interval fuzzy preference relation.

(5) To integrate a new data cleansing process that is able to avoid prejudice caused by conflicting opinions, and integrate the cleansing process into GDM models.

(6) To handle the complexity pertaining to the unification of heterogeneous information from a large number of DMs, and provide optimal solutions based on unification methods.

## 1.6 Structure of This Book

This book consists of six chapters. Chapter 1 provides a general introduction, which