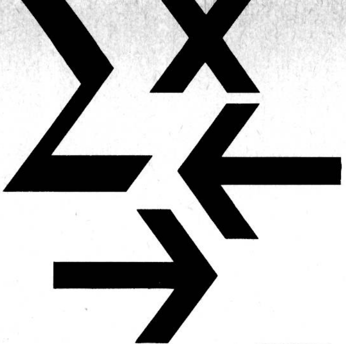
# Elementary Statistics

DONALD R. BURLESON



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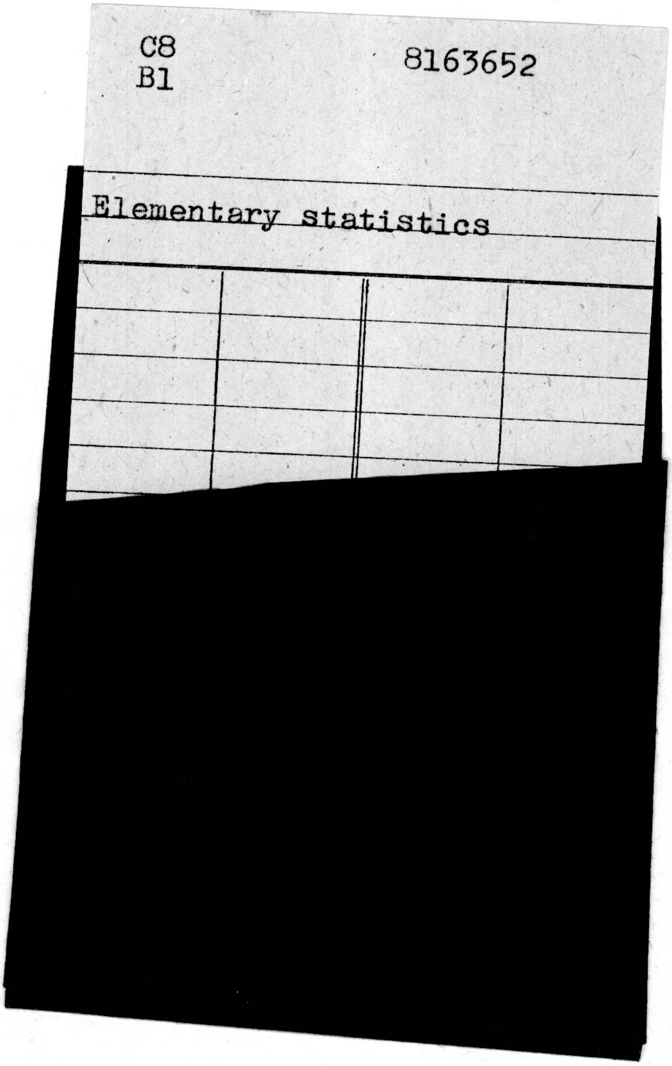
**Donald R. Burleson**

Middlesex Community College
Bedford, Massachusetts



# Elementary statistics

This book is dedicated to

HOWARD PHILLIPS LOVECRAFT

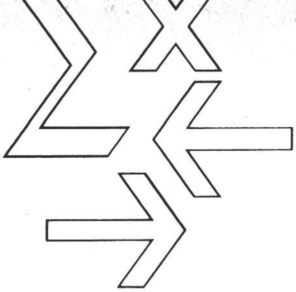
who is the central intellectual force in my life.

## Exercises and Examples in Particular Areas of Application (P="Ponderables" Problems; E=Examples)

| Chapter, Section | Natural Sciences | Social Sciences | Business-Economics | Education | General Applications |
|---|---|---|---|---|---|
| 1 A | P8 | | P2 | 9 | 6 |
| B | 3, 4 | | | | E1, E2 |
| C | | | | | 1, 4 |
| 2 A | 17 | | 10 | 9, 18, 20, E6 | 1, 2, 3, 4, 5, 16, E7, E13 |
| B | | P10 | | 6, 9 | E7, E13 |
| C | E5, E8 | 3, 9, E7 | P1 | | 4, 12, P2 |
| 3 A | 5 | 7, 10, P2 | 8, P7 | | 9, 11, 12, P1 P3, E8, E9, E10 |
| B | 5 | 3, E1 | 1 | | 3, E3 |
| 4 A | 8, 11, P4 | 1, 7, 14, P6, P8 | 3, 9, 13, E6 | | 5, P9, E4, E8 E9 |
| B | 1, 5, 6 | P5, E1, E3 | 3, 4, E2, E4 | | 2, 7, 8, P2 |
| C | 5 | 8, 13, E3, E4 | 1, P1 | 10, 11 | E1, E2, E5, E6 |
| 5 A | 5, 7, 9, P1, P3 | 14, 15, P2 | 1, 11, 13, 16, 17, 19, 20, 21, 22, E2, E3, E7, E8 | | 3, E1, E4, E5 |
| B | P4, P7 | 11, 12, 13 | 1, 2, 7, 8, 9, 10, 15, 17, P1, P9 | | 3, 4, 5, 6, 19, 20, P3, P5, E1, E2, E3, E4, E5, E6, E7 |
| C | 14, E3, E4 | 7, 9, 12, 13, P1, P4 | 1, 2, 16, 18, P2, P6 | 10, 11 | 3, 4, 5, 6, 20, E1, E2, E5, E6 |
| D | | 3, 8 | 1, 2, 10 | | 4, 5, 9, E1, E2, E3 |
| 6 A | 2, 11 | 1, 12, 14, P1, P2, P3 | 5, 6, 7, 9, 16 | 15, E1 | 3, 8, 10, E2 |
| B | 12, 14 | 10, 13, 15, 16, 17, P3, E3 | 6, 7, 8, 18, E4 | 3, 11 | 1, 2, 4, P4, E1 |
| 7 A | 5, 9 | 6, 7, 8, 11, E1 | 4, E3 | 1, 3 | 2, 10, E4 |
| B | 1, 3, 4, 5, 6, 7, 10, 11, P1, P2 | 8, 9, E1 | | 2 | E2 |
| 8 A | | 8 | | 3, 6, 7, E1, E2 | 1, 2, 5 |
| B | 10 | E1 | 1, 2 | 5 | 3, 6, 7, 9, E2 |
| C | 6 | 4 | 3, E1 | | 1, 2, 5, 7, 9 |
| D | | 6, 10 | 1, 2 | | 3, 4, 5, 7, 8, 9, E2 |
| E | 6 | | 8 | 1, 2, 7 | 3, 4, 5, 9, 10, E1, E2 |

## Topics Introduced in "Ponderables"

| | | |
|---|---|---|
| **1** | **A** | Geometric mean, harmonic mean, midrange, quartiles, midquartile |
| | **B** | Square root interpolation, mean deviation, semi-interquartile range ($Q$) |
| | **C** | Frequency distribution standard deviation, frequency distribution mode |
| **2** | **B** | Odds, conditional probability, independent events |
| | **C** | Hypergeometric distribution |
| **3** | **A** | Tchebycheff's Theorem |
| **5** | **A** | Small-sample $t$-test |
| | **B** | Two-sample $t$-test |
| | **C** | Differences between means |
| | **D** | Double interpolation in $F$ tables |
| **6** | **A** | Factorial design, interaction, two-way ANOVA |
| | **B** | Yates' correction |
| **8** | **A** | Sign test for the median |



# Preface

This book is designed to serve a standard one-semester general statistics course at the college freshman level, for students with perhaps only a very modest background in mathematics.

Having written a number of mathematics textbooks and other books, and having taught statistics and a great variety of other courses for many years, I have again employed here an approach that I have long found effective both in writing and in teaching: an informal and conversational exposition, with examples worked out in detail at every turn to illustrate what has just been introduced and explained. I have made every effort to avoid "skipped steps" and "gaping holes" in the discussions, so that the student has the best chance of following the examples smoothly. With this discussion-and-examples format the student should be encouraged to go through all the examples carefully, because they have been designed to bring out various important aspects of the topics in question, as have the exercises. Many examples and exercises are object lessons in such matters as the effect of sample size on the computations.

I have sought to make the book flexible, both in terms of course content and level. The number of topics available should make it easy for the instructor to select whatever is desired, and the exercise sets are plentiful enough and the discussion inclusive enough to make it possible to stress any particular topic as needed. In fact, the exercises themselves provide considerable flexibility in course level, because each section includes a regular exercise set and special problems called "Ponderables." These addititional problems offer either more challenging questions about the section or an extension of the topics covered, or both. The instructor may assign these or not as preferred, or may use them for classroom discussion.

For their kind assistance I wish to thank all those at Winthrop Publishers who have worked with me on the book's production, as well as the reviewers who have made useful suggestions in manuscript. I want to express appreciation to my colleagues and students at Middlesex Community College for their words of encouragement. Finally, but especially, I want to thank my wife Sue and my sons Bruce and Brian for the patience and understanding that they have shown once again during a long and difficult time; I well know that when a community college instructor writes a textbook and keeps up with a heavy teaching load, the demands on the family are inordinate.

Donald R. Burleson

# Contents

# 1

# Statistics: A Beginning

Well begun is half done.—Horace

## A MEASURING CENTRAL TENDENCY

Probably it would surprise no one to say that we live in an increasingly complex world—and a world increasingly *numerical*. We are surrounded, in our daily lives, by numerical information. Advertisers hurl conflicting figures and claims at us from all sides, parading data before us to sell us their products. Our banking facilities and our insurance policies are founded on stunning amounts of numerical analysis. Our jobs often call for struggling through mazes of numerical information. Our physical health depends to no small degree upon the applications of numerical methods to medical science, to the production and distribution of food, to the preservation of the environment. Every day, weighty decisions are made in business and government—from the village store to the giant corporation, from the town level to the international level—by considerations of numerical data. Our cities flourish because of intelligent numerical planning and decision making, or crumble because of the lack of it. Our industries, our governments, all facets of our lives are shaped by the manipulation of numerical information.

It is therefore natural that the science of *statistics* has evolved as one of the most important of all human endeavors. In this book we will examine some of the concepts and procedures belonging to this far-reaching subject, to gain an acquaintance with the fundamentals of its applications to human life.

Statistics has so many aspects and so many uses that a brief definition of it, although necessary, is by itself unlikely to convey any true picture of its real nature. However, we may start by defining **statistics** as

*the science of organizing and analyzing numerical data for the purposes of description and decision making.*

That is, statistics concerns itself with certain numerical *descriptions* of the world, or parts of the world, and with the making of *decisions* on numerical grounds—intelligent decisions that often must be made earlier than we would have liked, and on the basis of only partial information.

Let us look at a few preliminary terms and ideas.

First, the total body of information that we can be concerned about in any given situation—the "sea of information" on whose shores we stand looking and wondering—is called a **population.**

In statistics a population may or may not have anything to do with people. For example, a population *may* be all the people in Los Angeles, or all the people in California, or in the United States, or in the world. Or it may be all the catsup bottles that come off a conveyor belt in a given year, or the heights of all the trees in a certain forest, or all the members of a certain rare family of butterflies, or all the possible readings on a meter for a chemical experiment. In short, a population may be a lot of things—it is, in any given situation, the *total* body of information that one would theoretically have to deal with.

The trouble with populations is that they tend to be *large.* Sometimes they are enormous, in fact. If the population in question, for example, is composed of all the actual weights of the schoolchildren in a given country, that population may contain millions of numbers. (A population, for that matter, may even be *infinite.* For example, if a population consists of all the *possible* weights of schoolchildren, then it literally contains infinitely many numbers.)

For this reason, statisticians concern themselves with *sampling.*

A **sample,** as one would expect, is a selection of items taken from a population. For example, a sample could consist of 100 schoolchildren selected from among *all* the schoolchildren in Massachusetts (a population consisting of many thousands). Or a sample could consist of 250 observations of an experiment that can be performed any number of times: a sample of size 250 taken from an infinite population.

The reason for sampling is clear: a population, typically, is very large, so that the statistician cannot possibly hope to handle *all* the information about it, but must be content with looking only at *some* of the information (a sample).

A statistician hopes that the sample studied will paint some reasonably accurate and enlightening picture of the population from which it was taken. This is one of the ways in which statistics makes it possible to make informed decisions based upon partial information. But if this approach is to work sufficiently well, the sample taken should (if possible) be of good size, and should be *random.*

A sample is **random** if it is drawn from the population in such a way that no one could have predicted which items would be selected and which would not, each item having the same chance of being selected. This selection, needless to say, must be done with some care. For example, if out of a population of accounts in file folders we wish to select one folder at random (to audit that account, say), we may assign numbers to the folders and then take a number from a table of *random digits,* and select that corresponding folder (or keep taking random digits until one set of digits matches a folder number). This way, no one could say, "You picked my folder because you don't like me!" because no one could have predicted which folder we would select. On the other hand, it would *not* be a random procedure simply to close our eyes and reach into the drawer containing the alphabetized folders and pick one, because (for example) we would be more likely to choose a folder starting with M in the middle of the drawer than to pick one starting with A or Z.

If a sample is selected in such a way that it is *not* random, then we may say that it is **biased**. For a very obvious example, suppose we are sampling people and measuring their heights to gain an idea of the public's heights in general. If we were to take our sample by stopping basketball players on their way to and from the locker room, our sample would be so thoroughly biased that it would provide no real idea of most people's heights.

Now, suppose that from some population, a random sample of numbers is taken.

One aspect of the sample which we may wish to measure is **central tendency**: We may wish to determine, in some sense, where the "middle" of the data is. And to do so, we should compute some sort of *statistic* that measures central tendency. In popular terminology, we want to compute some sort of "average" for the sample numbers. It turns out that there are a number of different ways to do this. (In computing an "average" of some sort for a sample of numbers drawn from a population, we become involved with **descriptive statistics**: computations which simply *describe* some aspect of a sample or a population. This descriptive aspect of statistics we will look at in various forms before pursuing **inferential statistics**: the science of drawing probable inferences or conclusions about populations by analyzing samples. More about this later.)

The most common kind of "average"—the better term being "measure of central tendency"—is called the **mean** (also sometimes called the **arithmetic mean** to distinguish it from certain other means).

Given a set of sample numbers

$$\{x_1, x_2, x_3, \ldots, x_n\}$$

(where *n* is the number of items in the sample), the *mean* of the sample is denoted $\bar{x}$ and is defined to be

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}.$$

That is, we simply add the numbers and divide by the number of things added. (This is what most people naturally think of when the term "average" is used.) Because it is a little awkward to have to write the sum of the $x$'s "strung out" as shown, we may more compactly write

$$x_1 + x_2 + x_3 + \ldots + x_n = \sum x$$

where the $\sum$ (Greek capital letter *sigma*) simply means "the sum of. . . ." Thus

$\sum x$ (read "sigma $x$") means "the sum of all the $x$'s." Thus we can write the formula for the sample mean $\bar{x}$ more simply as

$$\bar{x} = \frac{\sum x}{n}.$$

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**EXAMPLES**

**1.** Given the eleven sample numbers

$$\{52, 57, 47, 50, 40, 41, 52, 42, 40, 59, 47\}$$

the sample mean is computed as

$$\bar{x} = \frac{\sum x}{n} = \frac{\sum x}{11}$$

$$= \frac{52+57+47+50+40+41+52+42+40+59+47}{11}$$

$$= \frac{527}{11} = 47.909090\ldots \approx 47.9$$

(where $\approx$ means "is approximately equal to"). We have retained the mean to the nearest tenth here in keeping with a general principle:

Always compute a sample statistic correct to one *more* decimal place than the given data.

(Thus, having given data in integer form calls for a mean to the nearest tenth.)

**2.** Given the eight sample numbers

$$\{23.09, 38.20, 29.71, 24.58, 31.34, 28.12, 34.50, 22.76\}$$

the sample mean is

$$\bar{x} = \frac{\sum x}{n} = \frac{232.30}{8} = 29.0375,$$

which must be rounded off. Because the given data are in hundredths, we should round this result off to the nearest thousandth; again, to one more decimal place than the given numbers. The question here is whether to round off to 29.037 or 29.038 (because 29.0375 is exactly halfway between them). The general practice, when a number occurs halfway like this, is to round the