ACOUSTICS AND AERODYNAMIC SOUND

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32 Avenue of the Americas, New York, NY 10013-2473, USA

Cambridge University Press is part of the University of Cambridge.

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www.cambridge.org
Information on this title: www.cambridge.org/9781107044401

First published 2015

Printed in the United States of America

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging in Publication Data
Howe, Michael (Acoustical engineer)
Acoustics and aerodynamic sound / Michael Howe, Boston University.
pages cm
Includes bibliographical references and index.
ISBN 978-1-107-04440-1 (hardback)
1. Acoustical engineering. I. Title.
TA365.H695 2014
620.2–dc23 2014024212

ISBN 978-1-107-04440-1

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Music, calm speech, whispering leaves fluttering in a breeze are pleasant and desirable sounds. Noise, howling gales, explosions and screeching traffic are less so. A quantitative understanding of the sources of all such sounds can be obtained by careful analysis of the mechanical equations of motion. This is provided by *Acoustics and Aerodynamic Sound*, which serves as a short, one-semester introduction to acoustics and aerodynamic sound at the advanced undergraduate and graduate levels.

Sound is treated as a branch of fluid mechanics, which is possible because students embarking on an advanced course in acoustics will be familiar with this topic. It is also desirable because an ability to relate acoustic events to hydrodynamic phenomena provides insight into acoustic principles, in particular into the role of vorticity in the mechanics of sound production by vibrating bodies and in the scattering and diffraction of sound.

MICHAEL HOWE has more than 30 years of experience in research on fluid mechanics, acoustics, random vibration, and structural mechanics. He is the author of more than 200 refereed journal articles on these and related subjects. His books include *Acoustics of Fluid-Structure Interactions*; *The Theory of Vortex Sound*; *Hydrodynamics and Sound*; and *Mathematical Methods for Mechanical Sciences.*

His expertise in acoustics has been recognised by the American Society of Mechanical Engineers, which awarded him the Per Bruel Gold Medal (2000); the American Institute of Aeronautics and Astronautics, which awarded him the Aero-Acoustics Medal (2001); and the Institute of Acoustics, which awarded him the Rayleigh Medal (2007).

In memoriam Dorothy Howe

1921–2014

Preface

An introductory account is given of the theory of the production and propagation of sound and its interactions with solid structures. It is intended for a one-semester course on acoustics at the advanced undergraduate or graduate level, and is therefore shorter than most of the standard texts – many important applications are omitted, such as speech and musical acoustics, ultrasonic imaging, and thermoacoustics. Sound is treated as a branch of fluid mechanics, which is possible because most students embarking on an advanced course are likely to have some familiarity with fluid mechanics or be sufficiently mature to assimilate the review material provided in the text. It is also desirable because an ability to relate acoustic events to hydrodynamic phenomena provides valuable insight into acoustic principles, in particular into the role of 'vorticity' in the mechanics of sound production by vibrating bodies and in the scattering and diffraction of sound. Any homogeneous fluid that has kinetic energy independent of moving boundaries must possess vorticity, a quantity that propagates by convection and molecular diffusion, and that therefore undergoes relatively little displacement during a typical acoustic cycle; the existence of vorticity signifies the presence of possible sources of sound, and its production occurs when sound is dissipated.

Chapter 1 forms a stand-alone introduction to theoretical acoustics, and is suitable together with material from §§2.1, 2.3, 2.4, 3.1, 3.2, 4.1–4.3, for a 'short course' on acoustics and aerodynamic sound. A general discussion of the Kirchhoff integral representation and generalisations is given in Chapter 2, with particular emphasis on sound sources of various types near an acoustically 'compact' body or section of a larger solid boundary, including the influence of surface vibration and scattering. Chapter 3 deals with the sound radiated by a baffled piston, Kirchhoff diffraction theory, and interactions of sound with apertures and perforated screens. Lighthill's theory of aerodynamic sound and its extensions to sound production by 'vortex-surface interactions' are presented in Chapter 4. Methods for treating the interaction of sound with small apertures, constrictions and duct open ends are discussed in Chapter 5, including nonlinear jetting and absorption by vorticity production – inspired by Rayleigh's application of Kelvin's general 'minimum energy' principle to impulsively started fluid motion. Sections numbered with an asterisk may be omitted on a first reading.

No discussion is given of experiments required to validate the theory, nor is there any account of applications to practical noise control problems. The latter are very well treated in several standard engineering handbooks. Similarly, little or no attention is given to specialities such as ray theory and the geometrical theory of diffraction, nonlinear effects of propagation, or the influence on propagation and sound generation of high-speed flows and combusting flows. Electrical analogies involving 'lumped parameter' models also are avoided. Nevertheless, the book provides a self-consistent treatment of the more familiar processes of propagation, scattering, diffraction and production of sound.

The reader is assumed to possess a working knowledge of vector calculus including Cartesian tensor notation, and of rudimentary generalised functions ('delta' function, etc.). A good appreciation of the subject requires the student to work through the problems at the end of each chapter, which should be within the capabilities of most students working alone. Some of the problems are perhaps better treated as short end-of-term projects. Solutions of 'starred' problems are outlined in Chapter 6.

Michael Howe

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1
Introduction

1.1 Sources of sound

Music, calm speech, whispering leaves fluttering in a breeze are pleasant and desirable sounds. Noise, howling gales, explosions and screeching traffic are less so. A quantitative understanding of the *sources* of all such sounds can be obtained by careful analysis of the mechanical equations of motion. Most sources are very complex, frequently involving ill-defined turbulent and perhaps combusting flows and their interactions with vibrating structures, and the energy released as sound tends to be a tiny fraction of that of the structural and hydrodynamic motions. Our analysis must correctly and reliably account for this general *inefficiency* of sound generation, because small errors in source modelling can lead to very large errors in acoustic prediction.

In this book we shall consider only the simplest case in which the fluid can be regarded as continuous and locally homogeneous at all levels of subdivision. The motion of the fluid will be defined when the velocity and the thermodynamic state are specified for each of the *fluid particles* of which it may be supposed to consist. The distinctive fluid property possessed by both liquids and gases is that these fluid particles can move freely relative to one another under the influence of applied forces or other externally imposed changes at the boundaries of the fluid. Five scalar partial differential equations are required to determine these motions. They are statements of conservation of mass, momentum and energy, and they are to be solved subject to appropriate *boundary* and *initial* conditions. These equations will be used to formulate and analyse a wide range of problems; our main task will be to simplify these problems to obtain a thorough understanding of source mechanisms together with a quantitative description of the subsequent propagation of the sound including, possibly, its reflection, scattering and diffraction at solid boundaries.

A general introduction to acoustics is presented in this chapter; it forms the basis for the treatment of the fluid-structure interaction problems examined in the rest of the book. The classical acoustic wave equation is derived from the general equations of motion. Particular solutions, methods of solution and applications are then discussed, principally for situations in which acoustic sources radiate into an unbounded fluid.

1.2 Equations of motion of a fluid

The state of a fluid at time t and position $\mathbf{x} = (x_1, x_2, x_3)$ is defined when the velocity $\mathbf{v}$ and any two thermodynamic variables are specified. These quantities are governed by equations describing conservation of mass, momentum and energy, supplemented by a thermodynamic equation of state.

1.2.1 The material derivative

Let v_i denote the component of fluid velocity $\mathbf{v}$ in the x_i direction, and consider the rate at which any function $F(\mathbf{x}, t)$ varies *following the motion* of a fluid particle (a material point moving with the fluid). Let the particle be at $\mathbf{x}$ at time t, and at $\mathbf{x} + \delta\mathbf{x}$ a short time later at time $t + \delta t$, where $\delta\mathbf{x} = \mathbf{v}(\mathbf{x}, t)\delta t + \cdots$. At the new position of the fluid particle

$$F(\mathbf{x} + \delta\mathbf{x}, t + \delta t) = F(\mathbf{x}, t) + v_j \delta t \frac{\partial F}{\partial x_j}(\mathbf{x}, t) + \delta t \frac{\partial F}{\partial t}(\mathbf{x}, t) + \cdots,$$

where the repeated suffix j implies summation over $j = 1, 2, 3$. The limiting value of $(F(\mathbf{x} + \delta\mathbf{x}, t + \delta t) - F(\mathbf{x}, t))/\delta t$ as $\delta t \to 0$ defines the material (or 'Lagrangian') derivative DF/Dt of F:

$$\frac{DF}{Dt} = \frac{\partial F}{\partial t} + v_j \frac{\partial F}{\partial x_j} \equiv \frac{\partial F}{\partial t} + \mathbf{v} \cdot \nabla F. \tag{1.2.1}$$

DF/Dt measures the time rate of change of F as seen by an observer moving with the fluid particle that occupies position $\mathbf{x}$ at time t.

1.2.2 Equation of continuity

A fluid particle of volume V and mass density ρ has a total mass of $\rho V \equiv (\rho V)(\mathbf{x}, t)$, where $\mathbf{x}$ denotes the position of the centroid of V at time t (Figure 1.2.1). Conservation of mass requires that $D(\rho V)/Dt = 0$, that is, that

$$\frac{1}{\rho}\frac{D\rho}{Dt} + \frac{1}{V}\frac{DV}{Dt} = 0. \tag{1.2.2}$$

Now $\frac{1}{V}\frac{DV}{Dt} = \frac{1}{V}\oint_S \mathbf{v} \cdot d\mathbf{S}$, where the integration is over the closed material surface S forming the boundary of V, on which the vector surface element $d\mathbf{S}$ is directed *out* of V. It is the fractional rate of increase of the volume of the