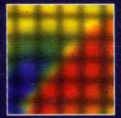


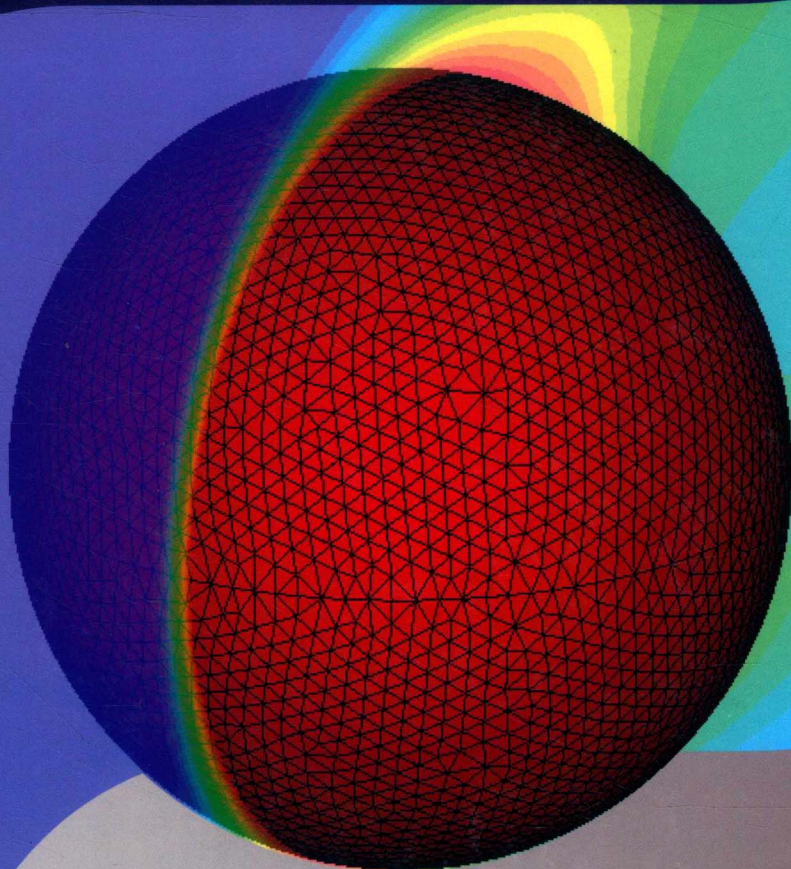
WILEY SERIES IN COMPUTATIONAL MECHANICS



Fundamentals of the Finite Element Method for Heat and Mass Transfer

Second Edition

P. Nithiarasu, R. W. Lewis,
and K. N. Seetharamu



WILEY

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Preface to the Second Edition

In this second and enhanced edition of the book, we provide the readers with a detailed step-by-step application of the finite element method to heat and mass transfer problems. In addition to the fundamentals of the finite element method and heat and mass transfer, we have attempted to take the readers through some advanced topics of heat and mass transfer. The first edition of the book covered only the application of the finite element method to heat conduction and flow aided laminar heat convection. The second edition of the book has been enhanced further with turbulent flow and heat transfer, and mass transfer, in addition to advanced topics such as fuel cells. We believe that the second edition provides a comprehensive text for students, engineers and scientists who would like to pursue a finite element based heat transfer analysis. This textbook is suitable for beginners, senior undergraduate students, postgraduate students, engineers and early career researchers.

The first three chapters of the book deal with the essential fundamentals of both the heat conduction and the finite element method. In the first chapter, the fundamentals of energy balance and the standard derivations of relevant equations for the heat conduction analysis are discussed. Chapter 2 deals with the basic discrete systems which provide a basis for the finite element method formulations in the following chapters. The discrete system analysis is demonstrated through a variety of simple heat transfer and fluid flow problems. The third chapter gives a comprehensive account of the finite element method formulations and relevant history. Several examples and exercises included in Chapter 3 give the readers a complete overview of the theory and practice associated with the finite element method.

The application of the finite element method to heat conduction problems are discussed in detail in Chapters 4, 5 and 6. The conduction analysis starts with a simple one-dimensional steady-state heat conduction in Chapter 4 and is extended to multi-dimensions in Chapter 5. Chapter 6 gives the transient solution procedures for heat conduction problems.

Chapters 7, 8 and 9 deal with heat transfer by convection. In Chapter 7, heat transfer aided by the laminar motion of a single phase flow is discussed in detail. All the relevant differential equations are derived from first principles. All the three types of convection modes; forced, mixed and natural convection, are discussed in detail. Several examples and comparisons are provided to support the accuracy and flexibility of the finite element procedures discussed. In Chapter 8 the turbulent flow and heat transfer are discussed in some detail. Some examples and comparisons provide the readers a chance to assess the accuracy of the methods employed. Chapter 9 utilizes the finite element method developed in Chapters 1, 7 and 8 to provide a solution approach to flow and heat transfer in compact heat exchangers. Chapter 10 provides an introduction to the application of the finite element to problems of mass transfer. A detailed

description of heat and mass transfer in porous media is then provide in Chapter 11. Two important applications of the finite element method for heat and mass transfer are explained in Chapters 12 and 13. Chapter 12 briefly introduces solidification problems using both heat conduction and convection approaches. Simple examples of solidification in this chapter may serve as a reference for students and researchers working in the area of solidification. In Chapter 13, we introduced a finite element solution approach to studying heat and mass transfer in fuel cells. Although the approach is only explained for solid oxide fuel cells, the method can be easily generalized to other types of fuel cells. Chapter 14 gives the reader sufficient information to understand the process of mesh generation. The main focus of this chapter is automatic and unstructured mesh generation. Some aspects of the adaptive mesh generation are also covered in this chapter. Finally, Chapter 15 briefly introduces the topic of computer implementation. The readers will be able to download the two-dimensional source codes and documentations from the website: **www.zetacomp.com**

Many people have assisted the authors either directly or indirectly during the preparation of this textbook. In particular, the authors wish to thank Dr Alessandro Mauro, Università degli Studi di Napoli Parthenope, for proofreading Chapter 13 and Dr Igor Sazonov, Swansea University, for helping the authors to put together part of Chapter 14. We would also like thank all our students, postdoctoral researchers and colleagues for providing help and support.

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Series Editor's Preface

It is known that heat transfer provides a good context for teaching finite element methods and other computational mechanics topics. Fundamental concepts can be explained with such simple examples as heat conduction in 1D, then in 2D and 3D, and convective terms can be added to describe the special methods needed to deal with that class of partial differential equations. This book in our series does that, and with its distinguished, experienced authors, does it well. It not only teaches how to solve heat and mass transfer problems with finite element methods, but it also serves the purpose of teaching many different concepts in finite element methods. Readers from very diverse backgrounds will be able to benefit from this book. The book can be used by engineering undergraduate students to learn the fundamentals of heat and mass transfer and numerical methods, by graduate students in engineering and sciences to learn the advanced topics they need to know, and by practicing engineers and scientists as a good source and guide for research and development work in heat and mass transfer.

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Introduction

1.1 Importance of Heat and Mass Transfer

The subject of heat and mass transfer is of fundamental importance in many branches of engineering. A *mechanical engineer* may be interested to know the mechanisms of heat transfer involved in the operation of equipment, for example, boilers, condensers, air pre-heaters, economizers etc., in a thermal power plant in order to improve their performance. Nuclear power plants require precise information on heat transfer as safe operation is an important factor in their design. Refrigeration and air-conditioning systems also involve heat-exchanging devices, which need careful design. *Electrical engineers* are keen to avoid material damage in electric motors, generators and transformers due to hot spots, developed by improper heat transfer design. An *electronic engineer* is interested in knowing efficient methods of heat dissipation from chips and semi-conductor devices so that they function within safe operating temperatures. A *computer hardware engineer* is interested to know the cooling requirements of circuit-boards, as the miniaturization of computing devices is advancing at a rapid rate. *Chemical engineers* are interested in heat and mass transfer processes in various chemical reactions. A *metallurgical engineer* would be interested in knowing the rate of heat transfer required for a particular heat treatment process, e.g. the rate of cooling in a casting process has a profound influence on the quality of the final product. *Aeronautical engineers* are interested in knowing the heat transfer rate in rocket nozzles and in heat shields used in re-entry vehicles. An *agricultural engineer* would be interested in the drying of food grains, food processing and preservation. A *civil engineer* would need to be aware of the thermal stresses developed in quick setting concrete, the influence of heat and mass transfer on building and building materials as well as the effect of heat on nuclear containment and buildings etc. An *environmental engineer* is concerned with the effect of heat on dispersion of pollutants in air, transport of pollutants in soils, lakes and seas and their impact on life. A *bioengineer* is often interested in the heat and

mass transfer processes, such as hypothermia and hyperthermia associated with the human body.

The above-mentioned applications are only a sample of heat and mass transfer applications. The solar system and the associated energy transfer from the sun are the principal factors for existence of life on Earth. It is not untrue to say that it is extremely difficult, often impossible, to avoid some form of heat transfer in any process on Earth.

The study of heat and mass transfer provides economical and efficient solutions for many critical problems encountered in diverse engineering items of equipment. For example, we can consider the development of heat pipes which can transport heat at a much greater rate than that of copper or silver rods of the same dimensions and even at almost isothermal conditions. The development of present-day gas turbine blades, where the gas temperature exceeds the melting point of the blade material, is possible by providing efficient cooling systems. This is another example of the success of heat transfer design methods. The design of computer chips, which encounter heat flux of the order occurring in re-entry vehicles, especially when the surface temperature of the chips is limited to less than 100 °C, is again a success story of heat transfer design.

Although there are many successful heat transfer designs, further developments on heat and mass transfer studies are necessary in order to increase the life span and efficiency of the many devices discussed previously, which can lead to many more new inventions. Also, if we are to protect our environment, it is essential to understand the many heat and mass transfer processes involved and if necessary to take appropriate action.

1.2 Heat Transfer Modes

Heat transfer is that section of engineering science that studies the energy transport between material bodies due to temperature difference (Bejan 1993; Holman 1989; Incropera and Dewitt 1990; Sukhatme 1992). The three modes of heat transfer are:

- (a) conduction
- (b) convection and
- (c) radiation.

The conduction mode of heat transport occurs either because of an exchange of energy from one molecule to another without actual motion of the molecules, or is due to the motion of free electrons if they are present. Therefore, this form of heat transport depends heavily on the properties of the medium and takes place in solids, liquids and gases if a difference in temperature exists.

Molecules present in liquids and gases have freedom of motion and by moving from a hot to a cold region, they carry energy with them. The transfer of heat from one region to another due to such macroscopic motion in a liquid or gas, added to the energy transfer by conduction within the fluid, is called heat transfer by convection. Convection may be either free, forced or mixed. When fluid motion occurs due to a density variation caused by temperature differences, the situation is said to be a free or natural convection. When the fluid motion is caused by an external force, such as pumping or blowing, the state is defined as being forced convection.